Dengue epidemic spread: modeling and optimal containment strategies

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***Abstract*—One of the emergencies related to climate change concerns the spread of epidemics; even in areas not linked to a humid-tropical climate, Dengue, West Nile, Zika are spreading, to name the most common. Possible containment measures are linked to human behavior, such as the use of appropriate clothing, mosquito nets, repellents. Other measures concern actions to limit the reproduction of mosquitoes, for example avoiding stagnant water, and, in some cases, they are more invasive, such as acting with targeted disinfestations. The combined action of these strategies, taking into account the limitations related to their applicability and the protection of the ecosystem, is implemented in this work, within the framework of the optimal control theory. The proposed results underline the importance of prevention action and propose suggestions on how to implement the most invasive actions.**

***Index Terms*—epidemic modeling, Dengue epidemic spread, optimal control.**

I. Introduction

Climate change and globalization are causing worse arboviruses spread, allowing contagions not necessarily related to travel, [1], [2], [3], [4], [5], [6], [7]. Characteristic of arboviruses is that they need a vector to be transmitted, typically the mosquitos; among the arboviruses the ones that constitute an emergency, mainly in tropical areas (involving about 4 billions of people), are Dengue, Yellow Fever, Chikungunya and Zika. The importance of tackling arboviruses has been established by the World Health Organization (WHO) in 2022 suggesting a plan to face in a coordinated way such kind of epidemic emergency, [8]. In this paper Dengue virus is considered, even if the general aspects of modeling and control can be extended to other arbovirus spreads. The transmission occurs by the bite of infected mosquitos, mainly the Aedes aegypti one; the bite allows feeding for the virus with replication in the mosquito midgut. With the bite of mosquitos also humans can transmit the virus, especially when the patients have significant viremia. It is estimated that there are nearly 4 billion people at risk of contracting Dengue; in 2023 alone, more than 6,000 deaths were recorded in 92 countries, [9]. Dengue is usually asymptomatic but, in some cases, can be fatal. Fever is accompanied by severe headache, pain around and behind the eyes, severe muscle and joint pain, nausea and vomiting, skin rashes that may appear over most of the body 3-4 days after the onset of fever. Dengue is caused by one of the four viruses (Den-1, Den-2, Den-3 and Den-4) and yields immunity only for the specific virus involved in the infection; the second infection generally provides severe Dengue. Climate conditions play a fundamental role in arboviruses spread due to the special humidity conditions that favour the proliferation of vector larvae, [10], [11], [12] . No specific treatment is available until now, and early detection with access to medical care helps in reducing severe cases. To face such an emergency, suitable modeling can help in understanding dynamics between subjects and vectors and possibly suggests containment strategies. The study of arboviruses, and, in particular, of Dengue, can be effectively conducted from a modeling point of view, taking into account the peculiarity of transmission that occurs through a vector and the seasonality that, as seen, strongly influences the diffusion.

In epidemiology, typical models are compartmental ones, in which the population of individuals, humans, vectors and mosquitoes, are distributed in classes according to their specific situation with respect to the infection. Three main classes of models may be considered: host to host transmission models, generally an extension of the SIR one, [14], vector host transmission, useful to deal with containment measures, [15], and within-host transmission, studying the interaction between free virus and susceptible target cells, [19]. In [16] an extensive review of the last ten years research on Dengue fever epidemiology is presented, whereas in [13], the proposed survey stresses the specific characteristics of different regions in which Dengue has significant impact. Particular effort has been devoted for estimating and forecasting Dengue outbreak, making use of Kalman filter, [21], improving the analysis of Dengue spatial transmission, [22], validating the results considering data from Mexico, [23]. Among compartmental models, a recent complex model is proposed in [24], where 10 groups are considered, including host and mosquitos; referring to a specific country, Nepal, in which climate and social conditions make dengue a significant risk, the Authors study a model in which in the human population are distinguished high-risk and low risk susceptible individuals, as well as asymptomatic, symptomatic and hospitalized patients. It is particularly interesting the analysis of economic and social cost, in terms of lost productivity and mental health needs, due to dengue spread.

Recently, a new model has been proposed, considering two populations, the humans, for which an extended SIR model is considered, and the mosquitos, split into healthy and infected, [17]. In the human population the infected people, that can transmit the virus, are distinguished between asymptomatic and the ones that can have severe symptoms. In the study it is analysed the impact of model parameters, such as the death rates of mosquitos and the recovery rates of humans, on the epidemic spread; those considerations are interesting also as a guide for the introduction of containment measures. What has been deduced is that acting directly on the death rates of mosquitos is an effective method to reduce the reproduction number. Ad hoc vaccines have been developed in the last few years, but the most common containment strategies involve both preventive actions (mosquitos nets, suitable wearing, repellents) and specific measure to increase mosquitos death rate (disinfestations campaign) and to reduce their reproduction capability. Moreover, also limitations from logistic and economic point of view must be taken into account, along with attention to the preservation of the ecosystem. The suitable framework to determine the containment measures considering also the above mentioned limitations is optimal control, [18]. In this paper, the Dengue epidemic spread in Brasil is considered adopting the model in [17] and implementing optimal containment strategies with suitable resource allocations. Controls involving human behavior and prevention are introduced along with direct action on vector dynamics. The paper is organized as follows: in section II the model proposed in [17] is equipped with control actions. In Section III the optimal containment measures are determined, whereas, in the numerical results in Section IV, the proposed description is applied to real data determining and discussing the obtained results. In the conclusion section V the main results of the paper are summarized, outlining also future work.

II. Materials and methods

As stated in the Introduction, Dengue virus requires a vector, typically the mosquitos, that transmit the infection to humans; on the other hand, also infected people can transmit the virus to mosquitos when bitten. There are four viruses that can cause Dengue; a permanent immunity is obtained only if the infection depends on the same virus; therefore it is assumed that reinfection is statistically possible. Human population and the vectors' one are here modeled by means of a compartmental description in which all the subjects (humans and mosquitos) are partitioned into groups depending on their condition with respect to the disease; therefore there are susceptible people in S_H and susceptible mosquitos S_V, infected subjects I_H, people without symptoms I_{HF}, fragile infected patients, infected vectors I_V, removed patients R_H. In the following model, it is assumed that both humans and mosquitos can infect and be infected. Possible control actions to reduce the infection aim at 1) reducing the contacts between human population and mosquitos, $u_1(t)$, 2) reduce the growth rate of population of mosquitos, $u_2(t)$, 3) improve the death rate of mosquitos. More precisely:

- u_1 refers to informative campaign to suggest people wearing protective clothes, or using repellents, or build mosquitos nets;
- u_2 aims at reducing mosquitos growth by means of proper disinfestation campaign or informative one to reduce favourable conditions for mosquitos reproduction, like stagnant water;
- u_3 aims at reducing mosquitos number, thus increasing their death rate

The mathematical relation

$$\dot{X}(t) = F(X, U) \tag{1}$$

describing the evolution of the state variables $X(t) = (S_H(t) \quad I_H(t) \quad I_{H_F}(t) \quad R_H(t) \quad S_V(t) \quad I_V(t))^T$ under the control action $U(t) = (u_1(t) \quad u_2(t) \quad u_3(t))^T$ is given by

$$\dot{S}_H = - \beta_{HV}(1-u_1)S_H I_V - d_{S_H} S_H + A_H + rR \tag{2}$$

$$\dot{I}_H = \alpha\beta_{HV}(1-u_1)S_H I_V - \gamma_H I_H - d_{I_H} I_H \tag{3}$$

$$\dot{I}_{HF} = (1-\alpha)\beta_{HV}(1-u_1)S_H I_V - \gamma_{I_{HF}} I_{HF} - d_{I_{HF}} I_{HF} \tag{4}$$

$$\dot{R} = \gamma_H I_H + \gamma_{I_{HF}} I_{HF} - d_R R - rR \tag{5}$$

$$\dot{S}_V = a(1-u_2)S_V - d_{S_V} u_3 S_V - \beta_{VH}(1-u_1)S_V I_H \tag{6}$$

$$- \varphi\beta_{VH}(1-u_1)S_V I_{HF} + A_V \tag{7}$$

$$\dot{I}_V = \beta_{VH}(1-u_1)S_V I_H + \varphi\beta_{VH}(1-u_1)S_V I_{HF} \tag{8}$$

$$- d_{I_V} u_3 I_V \tag{9}$$

with

- β_{HV} and β_{VH} are the contact rates; they are coefficients that include the probability to be infected, the viral load, the robustness of the *subject* with respect to the contact with the virus. More precisely, β_{HV} is the contact rate occurring when a healthy individual meets an infected mosquito, whereas β_{VH} is the contact rate between a healthy mosquito and an infected person;
- a is the growth rate of the mosquitos population, assuming an exponential behavior;
- α: fraction of the infected patients that does not show symptoms;
- $1/r$ is the time required to a human to become susceptible again, once recovered;
- d_{S_H}, d_{I_H}, $d_{I_{HF}}$ and d_R are the death rates, where the subscripts indicate the corresponding compartment;
- ϕ reduces the possibility of a fragile infected patient to infect a mosquito, assuming that such a patient should be almost isolated, under medication or even hospitalized;
- $\frac{1}{\gamma_H}$ and $\frac{1}{\gamma_{HF}}$ are the time required for a patient to recover from the I_H and I_{HF} compartment respectively
- A_H denotes the rate of new incomers in the healthy populations, assuming as negligible the corresponding rates in the other classes
- A_V is the rate of new incomers in the healthy populations of mosquitos, again assuming as negligible the corresponding rate in I_V.

The control is assumed bounded by logistic, economic, technical reasons, but also to take into account limitations due to protection of the ecosystem; it is therefore assumed: $U^{(}t) \in [U^{min}, U^{max}]$, meaning that each control is subject to a box constraint.

In Fig. 1, the block diagram describing the infection is shown.

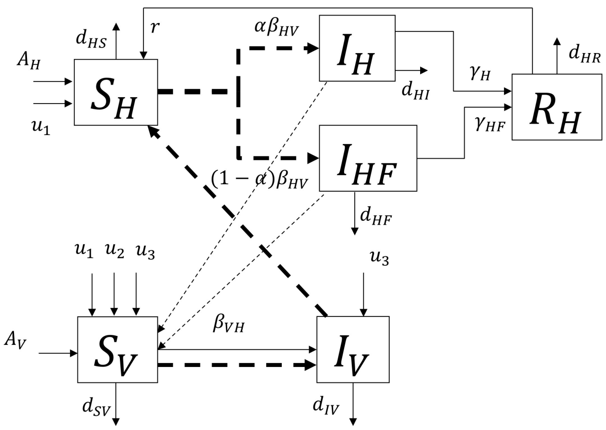

Fig. 1. Block diagram of the adopted model with the introduced control.

III. The optimal control strategy

As already stated in the previous section, the containment measures introduced aim both at reducing the contacts between humans and mosquitos and at acting directly on mosquitos' life and growth. The choice of control action $U(t)$ is here implemented in the framework of optimal control theory aiming at determining the best containment measures able to reduce the impact of the epidemic minimizing, at the same time, the required effort. Moreover, the control is assumed bounded both for technical, economic and logistic reasons and for ecosystem protection.

The following optimal control problem is stated:

Problem 1: Consider the system

$$\dot{X}(t) = F(X, U) \tag{10}$$

representing, in compact form, the state equations (2)–(9), with initial condition $X(t_i) = X_i$ and the constraint $U(t) \in [U^{min}, U^{max}]$; assume fixed the final time t_f, determine the optimal control $U^o(t) = (u_1^o(t) \quad u_2^o(t) \quad u_3^o(t))^T$ satisfying the constraints on the control, the initial conditions on the state, the evolution system (10) and minimizing the cost index:

$$J(X, U) = \frac{1}{2}\int_{t_i}^{t_f} (X^T(t)Q(t)X(t) + U^T(t)R(t)U(t))dt \tag{11}$$

being $Q(t) \geq 0$ and $R(t) > 0$.

The problem can be solved by applying the Pontryagin principle; the system (1), that is (2) - (9), is nonlinear and only necessary conditions are available. The following proposition holds:

Proposition 1: If a solution of Problem 1 exists, it is given by: $U^o(t) = (u_1^o(t) \quad u_2^o(t) \quad u_3^o(t))^T$ with

$$u_i^o(t) = min(max(\frac{-\Phi_i(t)}{2b_i}, U_i^{min}), U_i^{max}), \quad i = 1, 2, 3 \tag{12}$$

with

$$\begin{aligned}\Phi_1(t) = (\lambda_1(t) - \lambda_2(t)\alpha - \lambda_3(t)(1-\alpha))\beta_{H,V} S_H I_V \\ +\lambda_5(t)\beta_{V,H} S_V I_H + \lambda_5(t)\phi\beta_{V,H} S_V I_{H,F} \\ -\lambda_6\beta_{V,H}(\phi S_V I_{H,F}) - \lambda_6(t)\beta_{V,H} S_V I_H\end{aligned} \tag{13}$$

$$\Phi_2(t) = -\lambda_5(t) a S_V(t) \tag{14}$$

$$\Phi_3(t) = -\lambda_5(t) d_{S_V} S_V - \lambda_6(t) d_{I_V} I_V \tag{15}$$

where λ_i, $i = 1, ...6$ are the costate functions satisfying the costate equations:

$$\dot{\lambda}(t) = -\left.\frac{\partial H}{\partial X}\right|^T \tag{16}$$

with zero terminal value at the final instant t_f.

Proof Proposition 1 is proved by applying the Pontryagin principle; to this aim the Hamiltonian function is introduced:

$$\begin{aligned}H(X, U, \lambda) = \frac{1}{2}(X^T(t)Q(t)X(t) \\ +U^T(t)R(t)U(t)) + \lambda^T(t)F(X, U)\end{aligned} \tag{17}$$

Necessary conditions are the costate equation (16) and the Pontryagin inequality

$$H(X, U, \lambda) \leq H(X, W, \lambda) \quad W \in [U^{min}, U^{max}] \tag{18}$$

thus yielding

$$u_i^o(t) = min(max(\frac{-\Phi_i(t)}{2b_i}, I_i^{min}), U_i^{max}), \quad i = 1, 2, 3 \tag{19}$$

with $\phi_i(t)$, $i = 1, 2, 3$ are given by the formula (13)- (15).

As far as the costate function, the null value must be imposed in the final instant t_f, being the state in t_f not fixed.

IV. Numerical results and discussion

The model (2)–(9), in absence of control, has been identified in [17]. Some of the parameters depend on the characteristics of the human and mosquitos populations and on the specificity of Dengue; in particular, the following values have been chosen:

- death rates of human population: $d_{S_H} = d_{I_H} = d_{R_H} = 10^{-6}$, $d_{I_{HF}} = \cdot 10^{-6}$;
- recovery rates: $\gamma_{I_H} = 1$, $\gamma_{HF} = \frac{1}{3}$;
- immunity loss: $r = 0.5$;
- $A_H = 50$: rate of new incomers in the human population;
- $\alpha = 0.99$ percentage of subjects that got Dengue with mild symptoms.

As far as the parameters regarding the vector population, the values adopted are:

- $d_{S_V} = d_{I_V} = \frac{1}{3}$: death rates of mosquitos;
- $a = \frac{1}{4}$: reproduction time of mosquitos;

- $A_V = 10^6$: rate for new incoming mosquitos;
- $\phi = 0.01$: percentage of severely infected patients non adequately protected and isolated and that can still spread the virus.

The last parameter ϕ can be interpreted also as a control factor, meaning that the patients with severe symptoms are isolated (or even hospitalized) and therefore have reduced probability to spread the virus.

As far as the contact rates β_{H_V} and β_{V_H} are concerned, they have been identified by using real date regarding the impact of Dengue in Brazil between January 1 2014 and January 1 2024, [8]. The underlying assumption is that β_{H_V} and β_{V_H} can be assumed piecewise constant for period of 6 months, corresponding to *Autumn-Winter* and *Spring-Summer*; this choice is motivated by the seasonality characteristic of Dengue. The identification of β_{HV} and of β_{VH} has been obtained by minimizing the distance between the output of the model and real data:

$$J(\beta_{HV}, \beta_{VH}) = \int_{t_i}^{t_f} (w(I_R - I_{HF})^2 + max(I_R - I_{HF}))dt$$

being w a weight chosen equal to 10^{-3}, used to normalize the two elements of the cost index.

In Table I the identified values are reported.

TABLE I
VALUES OF TRANSMISSION RATES β_{HV} AND β_{VH}

Semester	β_{HV} (10^{-5})	β_{VH} (10^{-5})
10/1/20 – 3/31/21	0.1332	0.0174
4/1/21 – 9/30/21	0.0270	0.001
10/1/21 – 3/31/22	0.3346	0.0074
4/1/22 – 9/30/22	0.0010	0.0010
10/1/22 – 3/31/23	0.0930	0.105
4/1/23 – 9/30/23	0.0010	0.0010

With these numerical values an acceptable model identification is obtained, as shown in Fig.2, where a period of 3 years is considered.

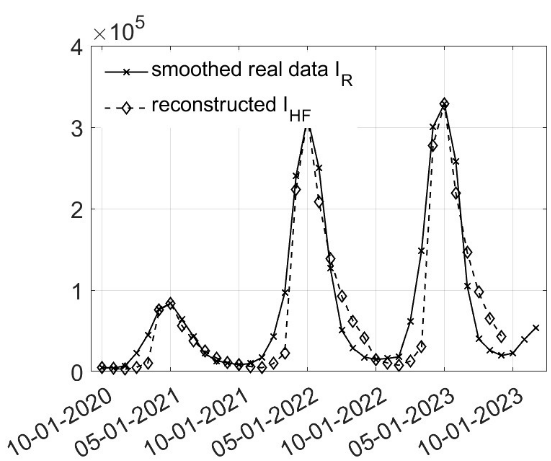

Fig. 2. Example of the effectiveness of the reconstruction of the number of severe infected patients by using the identified values of the transmission rates in the period October 2020 - December 2023.

Once the proposed model is identified to describe the evolution of Dengue in Brasil, it is possible to determine the best control strategy to reduce its impact on the population, trying to apply as few control action as possible. Being the goal the reduction of the infected patients, the ones in I_H and $I_{H,F}$, the cost index (11) is chosen with

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10^{-2} & 0 & 0 & 0 \\ 0 & 0 & 10^{-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The aim of using as less containment measures as possible, saving also the ecosystem, can be pursued by choosing the weighting matrix R:

$$R = \begin{bmatrix} 10^3 & 0 & 0 \\ 0 & 10^6 & 0 \\ 0 & 0 & 10^6 \end{bmatrix}$$

The lower weight for control u_1 is motivated by the fact that it is mainly related to habits that could reduce the direct contact between humans and mosquitos, that is wearing appropriate clothes, use repellents or mosquitos net: all actions that don't have impact on the ecosystem and on life of any subject. Box constraints for the control actions are introduced, taking into account realistic limitations, like the effectiveness of informative campaigns, of mosquitos nets and of repellents, for control u_1, or, for the control u_2 and u_3, ecological reasons to save ecosystem, as well as the efficacy of disinfestation campaigns; more precisely, it is chosen:

- $u_1 \in [0, 0.5]$, meaning that the use of repellents, of suitable clothes and mosquitos nets can reduce by 50% the contact rates $\beta_{H,V}$ and $\beta_{V,H}$;
- $u_2 \in [0, 0.3]$, allowing a reduction by 30% of the growth rate;
- $u_3 \in [1, 2]$, assuming the possibility of doubling the death rate of mosquitos.

The optimal control obtained with equation (12) requires the determination of the costate λ solving the equation (16) backwardly, whereas the state equations (2)–(9) are solved starting from the initial state condition. Numerically, it is solved by using the interior point algorithm. By adopting the weight matrices Q and R, the optimal controls u_i, $i = 1, 2, 3$ obtained are shown in Fig. 3, thus yielding the evolutions of the total number of infected patients, $I_H(t) + I_{H,F}(t)$, in Fig. 4, and the evolutions of the total number of mosquitos, $S_V(t) + I_V(t)$, in Fig. 5.

The strong control action $u_1(t)$, almost for the entire control period, and mainly for the first 18 months, allows to strongly reduce the total number of infected patients, see Fig. 4, while preserving the mosquitos, Fig. 5.

It is interesting to study the effect of different limitations in control u_1, that is allowing higher values for the constraints, $u_1 \in [0, 0.9]$; this suggests on one side that available repellents could be more effective, and also the adoption of more restrictive human habits. Therefore, also control u_1 has a

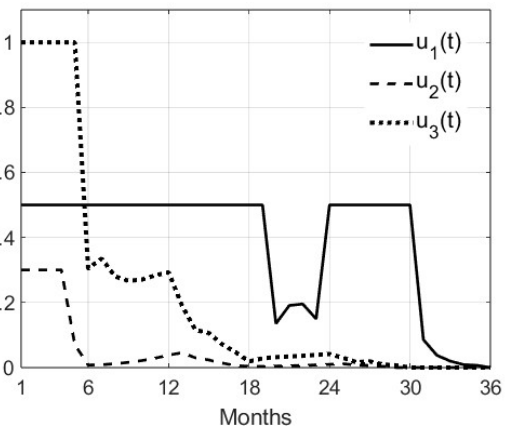

Fig. 3. Optimal control actions

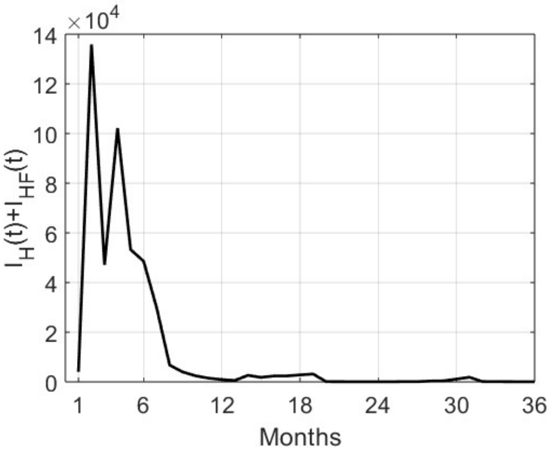

Fig. 4. Evolution of the total number of infected humans

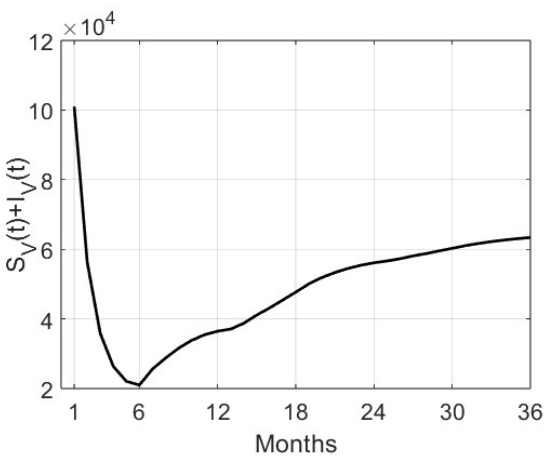

Fig. 5. Evolution of the total number of mosquitos, infected and non infected

significant social cost, that can be modeled by augmenting the value of the element $r_{11} = 10^6$ in matrix R, equating it to the others $r_{22} = r_{33}$.

The obtained controls are shown in Fig. 6; it can be noted that the control u_1 is at its maximum value 1 about for the first 6 months, then it decreases with a sort of periodical behaviour, without reaching again the maximum value and with the minimum values corresponding to the period in which the epidemics is less severe, see Fig. 2. The control u_2 has almost constant very low values, whereas the control u_3 is generally decreasing but with higher values with respect to the ones in Fig. 3. The interesting suggestion is that the disinfestation action u_3 needs to be applied especially before the beginning of the spring-summer seasons, corresponding to the higher number of documented cases. The number of infected patients is generally decreasing since the very beginning of the control period, Fig. 7. The strongest action of control u_3 has the direct effect on the total number of infected patients, Fig. 8, that reach the value of $6x10^6$ only at the end of the control period, with a reduced total number of mosquitos. This second strategy, while requiring more severe effort and social costs in avoiding contacts between humans and mosquitos, must be associated to important disinfestation action on mature mosquitos, while containment measures to prevent reproduction can be relaxed.

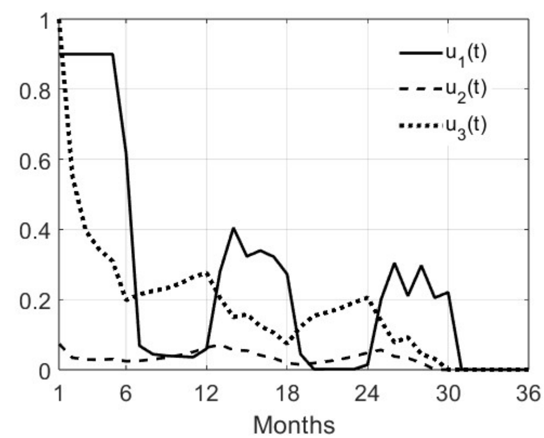

Fig. 6. Optimal control actions

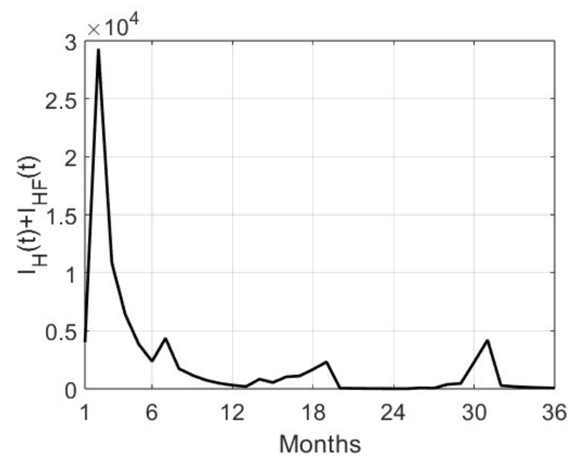

Fig. 7. Evolution of the total number of infected humans

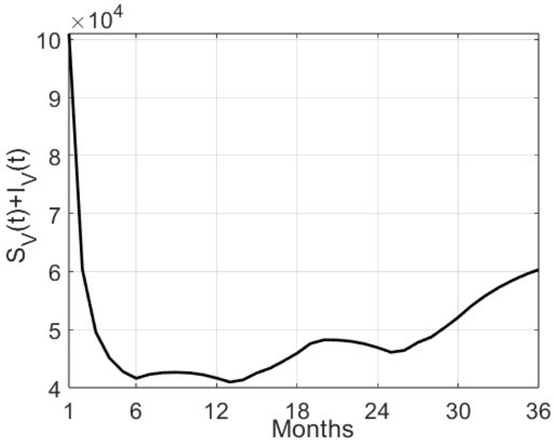

Fig. 8. Evolution of the total number of mosquitos, infected and non infected

V. CONCLUSIONS AND FUTURE WORK

Arboviruses represent a serious problem for global health, aggravated by climate change and globalization. Even areas not generally affected by these events are becoming susceptible to these kind of viruses, developing contagions not necessarily related to travel. In this work, using a recently proposed model, the possibility of intervention to reduce infections among populations is studied, but safeguarding the ecosystem; therefore, the proposed actions concern indications on human behaviours aimed at avoiding contact between subject and vector and targeted disinfestation actions, including containment measures to avoid mosquito reproduction. The results show the importance of prevention, that has a lower impact on population with respect to the other actions.

Future work will be devoted in including oscillatory aspects in the model to capture better the seasonality of arboviruses disease; this would imply a different modeling of the vector dynamics. Data analysis will be deeply implemented, analysing data from different countries, so trying to capture the intrinsic characteristics of the disease.

From the control point of view, the introduction of vaccination (available since 2022) could be discussed at least in those country with severe spread.

REFERENCES

[1] S.T. Mai, A. Abubakar, P. Kilpatrick, H. Nguyen, H. Vandierendonck, "Dengue Fever: From Extreme Climates to Outbreak Prediction", 2022 IEEE International Conference on Data Mining (ICDM), pp. 1083-1088, 2022.

[2] N. Giri, R. Joseph, S. Chavan, R. Heda, R. Israni, R. Sethiya, "AI-based prediction for early detection of Tuberculosis in India based on environmental factors", 2020 19th IEEE International Conference on Machine Learning and Applications (ICMLA), pp. 278-285l, 2020.

[3] R.Nirwantono, J. Pebrianto Trinugroho, D. Sudigyo, A. Ahmad Hidayat, B. Pardamean, "Time-series Analysis of Correlation between Climatic Parameters and Dengue Fever in Indonesia", 2022 International Conference on Informatics, Multimedia, Cyber and Information System (ICIMCIS), pp. 161-165, 2022.

[4] C.Barcellos, V. Matos, R. Lana and R. Lowe, "Climate change, thermal anomalies, and the recent progression of Dengue in Brazil", Scientific Reports, Vol. 14, N. 5948, 2024.

[5] G.M. Vazquez-Prokopec, "Quantifying the spatial dimension of Dengue virus epidemic spread within a tropical urban environment", PLoS Neglected Tropical Diseases 4, 2010

[6] P. Di Giamberardino, D. Iacoviello, "Evaluation of the effect of different policies in the containment of epidemic spreads for the COVID-19 case", Biomedical Signal Processing and Control, 65, 2021

[7] P. Di Giamberardino, D. Iacoviello, F. Papa, C. Sinisgalli, "Dynamical evolution of COVID-19 in Italy with an evaluation of the size of the asymptomatic infective population", IEEE Journal of Biomedical and Health Informatics, 25(4), pp. 1326?1332, 2021.

[8] World Health Organization, Dengue - Global situation https://www.who.int/emergencies/disease-outbreak-news/item/2024-DON518

[9] European Centre for Disease Prevention and Control, https://www.ecdc.europa.eu

[10] C.Xu, J. Xu and L. Wang, "Long-term effects of climate factors on Dengue fever over a 40-year period", Public Health, Vol. 24, n. 1451, pp. 1-11, 2024.

[11] M.Arquam, A. Singh and H. Cherifi, "Impact of seasonal conditions on vector-borne epidemiological dynamics", IEEE Access, Vol. 8, pp. 94510-94525, 2020.

[12] L. C. Chien, H.L. Yu, Impact of meteorological factors on the spatiotemporal patterns of Dengue fever incidence. Environment International 73, 46?56, 2014

[13] A. Aswi, S. M. Cramb, P. Moraga, K. Mengersen, "Bayesian spatial and spatio-temporal approaches to modelling Dengue fever: a systematic review", Epidemiol Infect., 147, 2019.

[14] F. Rocha, M. Aguiar, M. Souza, N. Stollenwerk, "Understanding the effect of vector dynamics in epidemic models using center manifold", AIP Conf Proc, 1479:1319, 2012

[15] K. Hu, C. Thoens, S. Bianco, S. Edlund, M. Davis, J. Douglas J, "The effect of antibody-dependent enhancement, cross immunity, and vector population on the dynamics of Dengue fever". J Theor Biol pp. 62-74, 2013

[16] M. Aguiar, V. Anam, K. B. Blyuss, C. D. Estadilla, B. V. Guerrero, D. Knopoff, B. W. Kooi, A.K. Srivastav, V. Steindorf, N. Stollenwerk, "Mathematical models for Dengue fever epidemiology: A 10-year systematic review", Physics of life. Reviews, pp. 65-92, 2022.

[17] P. Di Giamberardino, D. Iacoviello, "Modelling and analysis of spread characteristics of arbovirus infections", Proceedings of the ICINCO 2024 Conference, 2024.

[18] A. Abidemi, Fatmawati and O. Peter, "An optimal control model for Dengue dynamics with asymptomatic, isolation, and vigilant compartments", Decision analytics Journal, n. 100413, pp.1-20, 2024.

[19] R. Ben-Shachar, K. Koelle, "Minimal within-host Dengue models highlight the specific roles of the immune response in primary and secondary Dengue infections". J R Soc Interface 2015

[20] C.Yi, L. W. Cohnstaedt and C. Scoglio, "SEIR-SEI-EnKF: a new model for estimating and forecasting Dengue outbreak dynamics", IEEE Access, Vol. 9, pp. 156758-156767, 2021.

[21] C.Yi, L. W. Cohnstaedt and C. Scoglio, "SEIR-SEI-EnKF: a new model for estimating and forecasting Dengue outbreak dynamics", IEEE Access, Vol. 9, pp. 156758-156767, 2021.

[22] Q. Zeng, X. Yu, H. Ni, L. Xiao, T. Xu, H. Wu, Y. Chen, H. Deng, Y. Zhang, S. Pei, J. Xiao, P. Guo, "Dengue transmission dynamics prediction by combining metapopulation networks and Kalman filter algorithm", PLOS Neglected tropical diseases, 2023

[23] A. Schaum, R. Bernal-Jaquez and G.Sanchez-Gonzales, "Model-based monitoring of Dengue spreading", IEEE Access, Vol. 10, pp. 126892-126898, 2022.

[24] H. Golami, M. Gachpazan, M. Erfanian, "Mathematical modeling and dynamic analysis of dengue fever: examining economic and psychological impacts and forecasting disease trends through 2030—a case study of Nepal", Scientific Reports, 15, 1-31, 2025.

[25] A. Sow, C. Diallo and H. Cherifi, "Interplay between vaccines and treatment for Dengue control: an epidemic model", PLOS ONE, pp.1-27, 2024.

[26] S.P. Wijayanti, "The importance of socio-economic versus environmental risk factors for reported Dengue cases in Java, Indonesia", PLoS Neglected Tropical Diseases 10, 2016