A Synergistic Approach to Velocity Control under Intentional and Inherent Time Delays

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***Abstract*— This paper explores optimizing Integral-Retarded (IR) controller strategies for first-order linear time-delayed processes. By employing a model for DC motor dynamics, we incorporate a design approach that considers an unintentional delay within the control loop. The IR controller is optimized using meta-heuristic algorithms, specifically genetic algorithms. The optimization framework integrates multi-objective cost functions, balancing system response speed and control efforts. Experimental verification is carried out on a physical test setup, where the controller's efficacy is validated against a square wave reference signal. The outcomes demonstrate that the IR controller, optimized for maximum exponential decay and control efficiency, significantly enhances the motor's velocity tracking and response smoothness.**

***Index Terms*— Delay-based controllers, Time-delay systems, optimization, Integral-retarded controller.**

I. Introduction

Proportional-Integral-Derivative (PID) controllers play a primary role in a diverse and expansive industrial automation and control systems field [1], [2]. They are essential in many processes, ranging from basic temperature regulation to the tough operation of complex machinery [3]. The ubiquity of PID controllers is not without reason; their simplicity, reliability, and effectiveness in a wide range of conditions have been shown over decades. Integrating meta-heuristic optimization methods in control system design has revolutionized the field, offering a dynamic approach to navigating the complex landscapes of parameter optimization ([4]). In this context, exploring and implementing advanced control strategies beyond traditional PID frameworks has gained traction.

Meta-heuristic algorithms [5], [6], inspired by naturally occurring processes and phenomena, such as genetic evolution, swarm intelligence, and annealing processes, provide a framework for exploring and exploiting the search space efficiently and effectively. These algorithms, characterized by their flexibility, simplicity, and adaptability, have been successfully applied to optimization problems in control systems. From tuning PID controllers to optimizing the structure and parameters of advanced control systems, such as Model Predictive Control (MPC) and Adaptive Control Systems, meta-heuristics have demonstrated their potential to enhance system performance, robustness, and efficiency [7].

As control systems grow increasingly complex and multidimensional, traditional optimization methods often fall short due to computational inefficiency or complexity. This is where meta-heuristic optimization techniques come into play, offering an alternative capable of navigating the intricate landscapes of control system parameters to find near-optimal solutions. The use of meta-heuristic optimization methods in control systems has enabled the management of multi-objective optimization issues, constraint handling, and uncertainty management, resulting in innovative solutions that are both achievable and effective. This synergy speeds up the design process and allows the creation of control systems that can adjust to changing conditions and requirements, guaranteeing optimal performance in a wide range of situations [8].

The pursuit of enhancing system performance while reducing susceptibility to disturbances and noise has prompted the research into alternative control strategies beyond the traditional PID framework. Delay-based controllers (DBC) [9]–[13] have become a promising alternative, particularly well-suited for linear time-invariant systems. These controllers utilize deliberate delays in the control algorithm to effectively harness the temporal aspect of system dynamics and accomplish improved control objectives.

In this sense, the Integral-Retarded (IR) controller is designed to address the challenge of zero steady-state error in set-point tracking, a key performance indicator for many control systems [14]. The IR controller ensures that cumulative errors are corrected over time through its integral component, driving the system towards the desired set-point without steady-state error. This feature is particularly valuable in scenarios where using a low-pass filter to reduce noise impact is undesirable due to its potential to introduce phase lag and affect system responsiveness. While the inclusion of a retarded gain and intentional delay is theorized to preserve the advantages of some DBC's, it may also contribute to the refinement of control signals, particularly when measurements are noisy. This potential benefit aligns with recent studies, such as those by [15], which explore the complexities of this problem to understand and quantify the impact of the controlled process.

While recent works address the complexities of control systems with inherent delays, applying a meta-heuristic al-

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gorithm to fine-tune the IR controller represents a novel contribution to the field. Furthermore, our case study focuses on the velocity control of a motor, employing the genetic algorithm to optimize the IR controller's parameters, thereby solving the issue of delayed responses in the control process.

The subsequent sections of this paper unfold as follows: Section 2 delves into the integral-retarded controller with unintentional delay and design procedure, offering a comprehensive overview of its theoretical foundations and design methodology. Section 3 explores the design and optimization, elucidating the implementation and optimization of the IR controller within various control systems. Section 4 presents experimental evaluations, where the performance of the IR controller is empirically assessed through simulations and real-world experiments. Finally, Section 5 concludes our findings and discusses avenues for future research in advanced control strategies.

II. Integral-Retarded Controller with Unintentional Delay and Design Procedure

The control and analysis of DC motors are common tasks in various engineering and automation applications. Managing the velocity of a DC motor based on voltage input is a classic control problem addressed using the type-0 transfer function. In this case study, we explore a direct current motor where the input $u(t)$ [V] represents the voltage, while the output $y(t)$ [rad/s] indicates the angular velocity of the motor (see Fig. 1). In contrast, the output $y(t)$ [rad/s] denotes the angular velocity of the motor (see Fig. 1). Hence, the connection between the output and the input is represented by the model

$$G_p(s) := \frac{Y(s)}{V(s)} = \frac{K}{s+a}, \quad K, a \in \mathbb{R}. \tag{1}$$

In this equation, K represents the motor's gain, and a represents the natural resistance to changes in speed. This

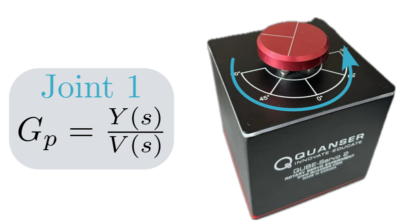

Fig. 1: Configuration of Joint 1 in the QUBE-Servo 2 DC Motor, illustrating the transfer function G_p.

study focuses on a system characterized by an inherent time-delay τ. This delay is constant and integrated into the system's dynamic model. Considering the known delay factor, the system can be described as transitioning to a state that includes the effects of this latency. The influence of the delay on the system's behavior can be explained through the modified transfer function

$$G(s) := G_p(s)e^{-\tau s} = \frac{K}{s+a}e^{-\tau s}, \quad \tau > 0. \tag{2}$$

Therefore, designing a controller is crucial in accounting for and counteracting the time-delayed effects within the system. This aspect of control design poses significant challenges as it entails striking a delicate balance between compensating for the delay while maintaining overall system performance.

A. IR + Delay Structure: Theoretical Framework

The design of the IR controller, originally posited by ([14]), omits unintentional delays within the control loop, an assumption permissible when such delays are negligible. Considering that the controller and the plant are well-known in the literature, this section and the rest of the paper employ a fixed unintentional measurement delay, $\tau > 0$, in the IR to manage the system $G(s)$. The IR controller is defined by

$$C(s) := \frac{k_i - k_r e^{-hs}}{s}, \quad k_i, k_r \in \mathbb{R},\ h > 0, \tag{3}$$

where k_i represents the integral action, k_r the delayed action and h the delay of the controller. The controller system, as depicted in Fig. 2, illustrates the IR controller's structure. It shows the standard integral component, which assures error correction over time, and the retarded component, which introduces a calculated delay, counterbalancing the delay inherent in the system's measurements. An advantage of

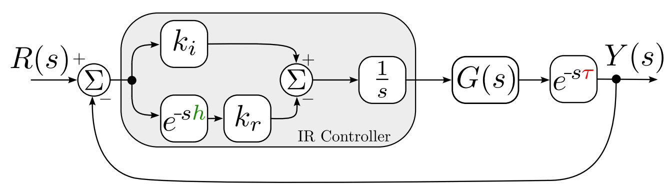

Fig. 2: Single-loop control scheme using the IR controller and a delay measurement at the output of the process.

employing this controller is its ability to ensure a consistent and steady error even when subjected to step-type inputs. Conversely, the integrator that influences both control actions (k_i, k_r) functions as a filter. As a result, this term filters out external disturbances or high-frequency noise in the data.

B. Systematic Design Procedures

The closed-loop system depicted in Fig. 2 is described by the transfer function

$$H_{yr} := \frac{Y(s)}{R(s)} = \frac{Kk_i - Kk_r e^{-hs}}{s^2 + as + Kk_i e^{-\tau s} - Kk_r e^{-(\tau+h)s}}. \tag{4}$$

To analyze the system's stability, one must examine the characteristic quasi-polynomial $\Delta_{yr}(s) : \mathbb{C} \times \mathbb{R}^2 \times \mathbb{R}_+ \to \mathbb{C}$

$$\Delta_{yr}(s; k_i, k_r, h) := s^2 + as + Kk_i e^{-\tau s} - Kk_r e^{-(\tau+h)s}. \tag{5}$$

Identifying the system's stability zones is the initial step in controller tuning. This step is vital to apply a meta-heuristic approach that efficiently searches within the range of stabilizing controller parameters.

Remark 1: In the absence of delay ($\tau = 0$), the tuning problem adheres to the solutions presented in [14], offering analytical expressions to define the parameters k_i, k_r, and h. This tuning-rule obtains a triple real root in $-\gamma$, resulting in the maximum exponential decay (MED). Consequently, any unintentional delay within the feedback loop presents a persistent challenge for these systems.

Proceeding with the assumption that $\tau > 0$. The system described in (4) is categorized as a retarded-type due to the higher-order variable coefficient not being multiplied by exponentials, i.e., the absence of s^2 in the exponential terms. This classification is significant, as retarded-type systems are known to demonstrate continuous root displacement in response to parameter variations ([16]). As such, the roots cannot cross from the left-half plane (LHP) to the right-half plane (RHP) without first encountering stability boundaries. Several foundational definitions are provided to address the stability of such systems.

Definition 1: The *frequency crossing set* $\Omega \subset \mathbb{R}$ is the set of all ω for which there exists at least one triplet (k_i, k_r, h) in $\mathbb{R}^2 \times \mathbb{R}_+$ such that

$$\Delta(\boldsymbol{i}\omega; k_i, k_r, h) = 0. \tag{6}$$

Remark 2: The numerator and denominator of (4) as polynomials with real coefficients satisfy the relation

$$\Delta(-\boldsymbol{i}\omega; k_i, k_r, h) = \overline{\Delta(\boldsymbol{i}\omega; k_i, k_r, h)}. \tag{7}$$

In other words, the above equality states that $\Omega \subset \mathbb{R}_+ \cup \{0\}$.

Definition 2: For a fixed $h > 0$, the *stability crossing curves (stability crossing boundaries)* will be denoted by $\mathcal{K}$, and is defined as the set of all parameters $(k_i, k_r) \in \mathbb{R}^2$ for which there exists at least one $\omega \in \Omega$ satisfying $\Delta_{yr}(\boldsymbol{i}\omega; k_i, k_r, h) = 0$. Moreover, any point $\boldsymbol{k} \in \mathcal{K}$ is known as a *crossing point*.

III. Design and Optimization of IR Controller

This section explains the control design process by illustrating the stability crossing curves, corresponding to the controller gains (3), while maintaining a constant intentional delay $h > 0$. Also, we describe the optimization strategy, proposing two different cost functions in order to reach both velocity and performance of the closed-loop stabilizing the system with reference step changes.

A. Design Requirements

Characterizing the stability crossing curves is essential for control design. Stability may be compromised or enhanced in two scenarios: a root crossing at the Complex Root Boundary (CRB) or the Real Root Boundary (RRB). In this approach, we maintain a fixed value of the intentional delay $h > 0$ to find the stability boundaries in the parameter plane (k_i, k_r).

Proposition 1 (CRB): Let $h > 0$ be a constant value, and define $\Omega := \bigcup_{\ell} \Omega_\ell$ for $\ell \in \mathbb{N} \cup \{0\}$ where

$$\Omega_\ell := \left\{ \omega \in \mathbb{R}_+ \middle| \omega \in \left(\frac{\pi}{h}\ell, \frac{\pi}{h}(\ell+1) \right) \right\}.$$

Then, $\omega \in \Omega$ is a crossing frequency iff $\boldsymbol{k}(\omega) := [k_i(\omega)\ k_r(\omega)]^T \in \mathcal{K}_\omega$, where

$$k_i(\omega) := \frac{\omega}{K \sin(h\omega)} \left(-a \cos(\delta\omega) + \omega \sin(\delta\omega) \right), \tag{8a}$$

$$k_r(\omega) := \frac{\omega}{K \sin(h\omega)} \left(-a \cos(\tau\omega) + \omega \sin(\tau\omega) \right), \tag{8b}$$

where $\delta = \tau + h$.

Sketch of proof: The proof follows straightforwardly by noting that if $\Delta_{yr}(\boldsymbol{i}\omega; k_i, k_r, h) = 0$ then

$$\Re\{\Delta_{yr}(\boldsymbol{i}\omega; k_i, k_r, h)\} = 0,$$
$$\Im\{\Delta_{yr}(\boldsymbol{i}\omega; k_i, k_r, h)\} = 0.$$

■

Proposition 2 (RRB): For a fixed delay $h > 0$, the line $\boldsymbol{k}_0 \in \mathcal{K}_0$ is described by

$$\boldsymbol{k}_0 = \left\{ [k_{r_0}\ k_{r_0}]^T \middle| k_{r_0} \in \mathbb{R} \right\}, \tag{9}$$

which represents a root crossing at the origin of the complex plane.

Sketch of proof: This proposition is supported analogously to the CRB, focusing on the condition where $\Delta_{yr}(0; k_i, k_r, h) = 0$. ■

Remark 3: The set $\mathcal{K}$ consists of locations that correspond to a complex crossing ($\mathcal{K}_\omega$) or a crossing by the origin ($\mathcal{K}_0$), as shown by the definitions above. Therefore, it can be concisely stated that the limit of these crossings is $\mathcal{K} = \mathcal{K}_\omega \cup \mathcal{K}_0$.

For γ-stability analysis, i.e. when $s = -\gamma + \boldsymbol{i}\omega$, $\gamma \in \mathbb{R}_+$ it is recommended to study of [15].

B. Identification and Calibration of Process Parameters

The system parameters must be accurately identified prior to the controller design. This can be achieved through classical or state-variable approaches, with the controller being either continuous or discrete in nature. In this instance, MATLAB's System Identification Toolbox was employed to derive a first-order delayed model from experimental data gathered within a physical test setup.

$$G_1(s) = \frac{179.93}{s + 7.6} e^{-0.0051s}. \tag{10}$$

Fig. 3 contrasts the experimental and simulated responses to a dynamic input over time. The acquired results yield a correlation factor of $R^2 = 0.9875$.

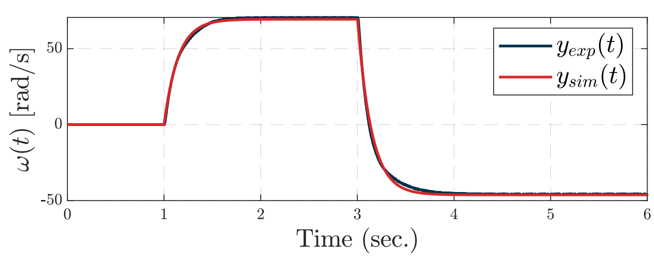

Fig. 3: Process parameter identification using a real-time hardware in the loop with a simple simulation testing.

C. Optimization Strategy and Implementation

As highlighted in [15], the delay is set at a fixed value while the other controller gains of the IR controller are tuned. Designing a controller based on performance criteria like maximum exponential decay (MED), control effort, or settling time can pose an analytical challenge.

A viable approach involves the construction of multi-objective cost functions that integrate various performance indicators into a singular design framework. The challenge of employing meta-heuristic methods in controlled systems is discerning an optimal search space for the controller gains, which traditionally relies on the designer's expertise and could lead to suboptimal results if improperly defined.

Thus, analytical approaches should be combined with meta-heuristics as a potential solution. The analytical component constrains the search space to ensure stability. At the same time, the meta-heuristic element will be utilized to identify the gains that optimize the predefined cost function in a local neighborhood. This integrative method ensures optimal gains are selected from a set that guarantees stability, thereby enhancing computational efficiency.

The optimization problem is formulated as follows:

$$\begin{aligned} &\text{minimize} \quad J \\ &\text{subject to} \quad \boldsymbol{\kappa} \in \mathcal{S}, \end{aligned} \tag{11}$$

where $\boldsymbol{\kappa} := [k_p^\star \; k_i^\star]^T$ with $h > 0$ or $\boldsymbol{\kappa} := [k_p^\star \; k_i^\star \; h^\star]^T$ according to the case, and $\mathcal{S}$ denotes the set of admissible control gains. Fig. 4 illustrates the geometric representation of the optimization problem within the multidimensional gain space. The shaded area, denoted $\mathcal{S}$, encompasses the feasible region of controller gains that satisfy the stability criteria. Within this domain, the point indicating $\min J$ marks the location of the optimal gain set, where the cost function J attains its lowest value, subject to the constraints of admissible control gains. This visual aid underpins the search strategy within the permissible gain space, guided by the objective of cost minimization. The exploration of two

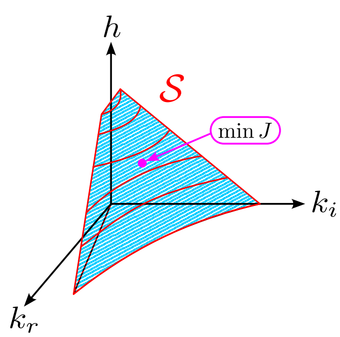

Fig. 4: Geometric Visualization of Gain Space Optimization. The shaded region represents the permissible gain space $\mathcal{S}$, with the highlighted point indicating the optimal solution where the cost function J is minimized

cost functions is presented herein. The first function, J_1, is focused on optimizing the MED by determining the optimal values for (k_i, k_r, h)

$$J_1 = -\gamma, \quad \gamma \in \mathbb{R}_+. \tag{12}$$

Its primary goal is to shift the dominant roots of the system as far left as possible in a local neighborhood, thereby expediting the decay rate and enhancing speed regulation. The second function, J_1, aims to balance the rapidity of response with the minimization of control efforts over the interval from the initial condition $(t_0 = 0)$ to the settling time t_s

$$J_2 = t_s(\alpha) + \int_0^{t_s} \left(\Psi u(\tau)^2 + \Upsilon \dot{u}(\tau)^2\right) \mathrm{d}\tau, \;\; \alpha, \Psi, \Upsilon \in \mathbb{R}_+. \tag{13}$$

This function seeks to engineer a controller that achieves a swift response while maintaining an optimal energy profile.

To solve the optimization problem, Algorithm 1 is recommended. It integrates the limits of stability boundaries, the formulation of cost functions, and the implementation of a principal optimization algorithm, such as the genetic algorithm, to navigate towards the optimal solution.

Algorithm 1 Synergetic optimization procedure for controller gains selection.

Input: $\mathcal{K}_\omega$,$\mathcal{K}_0$, opt, Ψ, Υ
Output: sol

```
K=D_METHOD(K_ω, K_0):
    K_ω ≜ [k_i(ω), k_r(ω)]^T
    K_0 ≜ [k_{r0}, k_{r0}]^T | k_{r0} ∈ ℝ
    K ≜ K_ω ∪ K_0
J=COST_FUNCTION(opt):
if opt == 1 then
    J ≜ −γ
else if opt == 2 then
    J ≜ t_s(α) + ∫_0^{t_s} (Ψu(τ)^2 + Υu̇(τ)^2) dτ
end if
sol=MAIN():
    ω = 0^+ : ω_s : ω_f
    K=D_METHOD(ω)
    [k_i^l, k_r^l]^⊤ ≜ min_ω(K)
    [k_i^u, k_r^u]^⊤ ≜ max_ω(K)
    sol=GA@(κ) COST_FUNCTION(opt),2,[],[],[],[],[k_i^l, k_r^l],[k_i^u, k_r^u]
return sol
```

Remark 4: This research applies the genetic algorithm paradigm, though it acknowledges the viability of alternative algorithms, including particle swarm optimization, least squares, and gradient descent, among others, for similar optimization tasks.

Remark 5: The compactness of the constrained domain and the continuity of the cost function concerning its parameters (given its convexity) ensure the presence of a global minimum, as established by the Bolzano–Weierstrass Theorem (see [17] for more details).

IV. Experimental Verification

This section presents the validation of the controller design through offline optimization using an experimental setup as shown in Fig. 5. The platform allows for the practical implementation and assessment of the controller's performance in a controlled environment. All experiments were executed within MATLAB/Simulink using the ode1 (Euler) solver with a $\Delta t = 2$ ms sampling time.

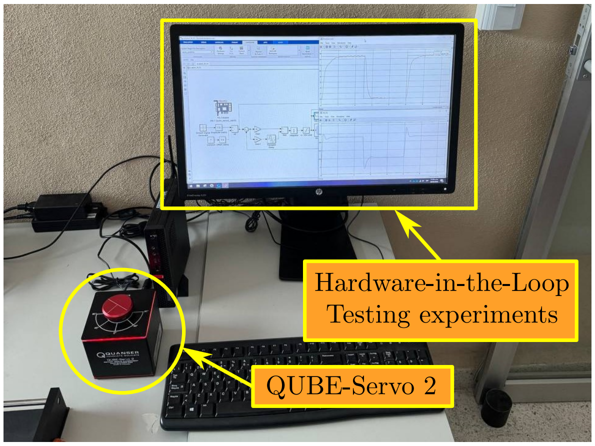

Fig. 5: Experimental setup employed for validating the controller design.

A. Laboratory-Based Validation Studies

Employing the stability conditions derived in the previous section, we obtain the stability region depicted in Fig. 6. The genetic algorithm aids in restricting the search space for controller gains to a well-defined and compact region, thereby ensuring that the selected gains confer stability to the system. The benchmark experiments were conducted using a

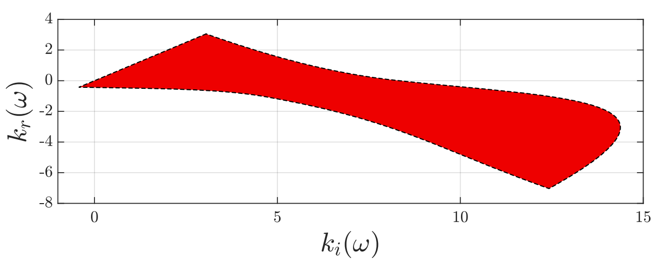

Fig. 6: Numerical stability chart boundary with $h = 0.1$ s. The illustrated region denotes the stability boundaries determined by the derived stability conditions.

square wave reference signal with an amplitude of 15 rad/s and a period of 5 seconds, enabling a consistent basis for comparison across different controller configurations.

Remark 6: When $h = 0$, the proposed IR controller looks like a PI controller, unlike the fact that no zero (in $-\frac{k_i}{k_p}$) is added. For our case study, a detailed analysis of how to solve it with the synergestic approach is presented in [8].

1) Experiment No. 1: The objective of the first experiment is to enhance the closed-loop response by maximizing γ—or, equivalently, minimizing $-\gamma$ to shift the dominant poles of the system as far to the left of the imaginary axis as possible, thereby expediting the exponential decay rate. The optimization involved fine-tuning the IR controller's (k_i, k_r, h) parameters. The optimized parameters were determined to be $k_i = 2.3394$, $k_r = 1.4666$ and $h = 0.0466$, positioning the dominant poles at $s_1 = -22.7660$, $s_{2,3} = -22.7616 \pm 4.3096\boldsymbol{i}$. This configuration confirms that the maximum value of $\gamma = 22.7616$ was attained, with the implemented delay exceeding the minimum system sampling time. Alternatively, we propose a classic strategy to tune a Proportional-Integral (PI) controller. The controller should be adjusted to achieve the maximum exponential decay rate of the solutions, with gains set as $k_p = 0.4931$ and $k_i = 18.5254$ for a dominating root group position of $s = -118.634$.

On Fig. 7 displays the experimental[1] outcomes achieved with the controllers. The experiment showcases Algorithm 1's capability to minimize oscillations when examining a servo-problem and vulnerability of the PI controller, even when it is analytically set to achieve maximum exponential decay. The settling times within the specified error range are $t_s \approx 0.4$ s for the PI controller, and $t_s \approx 1.2$ s for the IR controller. However, damped oscillations are present when the IR controller is in regulation. Upon evaluating the control signal, it becomes reasonable that voltage peaks occur when there is a change in the reference signal.

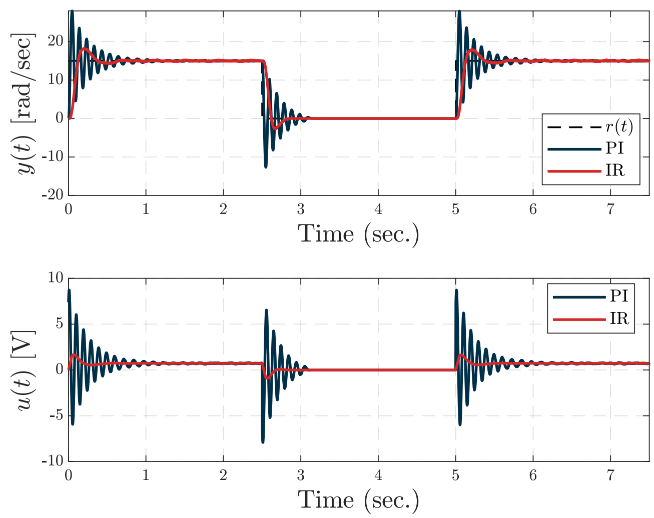

Fig. 7: System Response for Cost Function J_1. The temporal response of the system with the controller optimized for J_1, demonstrating the achieved settling time and control signal characteristics.

2) Experiment No. 2: In the second experiment, the focus was to optimize the settling time within a 1% tolerance band, analogous to optimizing MED, while also considering the reduction of control effort and smoothing the control signal. The cost function introduced two weighting factors, allowing for prioritization among the response time, control effort, and signal smoothness in each optimization iteration.

Table I details the controllers synthesized offline prior to executing the experiments visualized in Fig. 8. The experimental data demonstrate that increasing Ψ leads to a reduced control signal amplitude, albeit with a slower convergence to the reference signal. Conversely, an increase in Υ results in a smoother control signal, with observable trends consistent across different response profiles.

[1]For a detailed demonstration of the experiments, the authors refer to the video at `https://youtu.be/sTJPnixdvAY`.

TABLE I: Resulting parameters in various controllers for J_2, it is worth noting that Ψ and Υ weights parameters of the cost function are different starting points in the controller algorithm.

Controller			k_i	k_r	h [s]	γ
Name	Ψ	Υ				
IR_1	1	1	0.6532	0.3046	0.1	7.6825
IR_2	2	1	0.6116	0.2791	0.1	8.4531
IR_3	1	10	0.5860	0.2642	0.1	9.3234
IR_4	10	1	0.5415	0.2481	0.1	8.7143
IR_5	50	1	0.5243	0.3464	0.1	2.6643

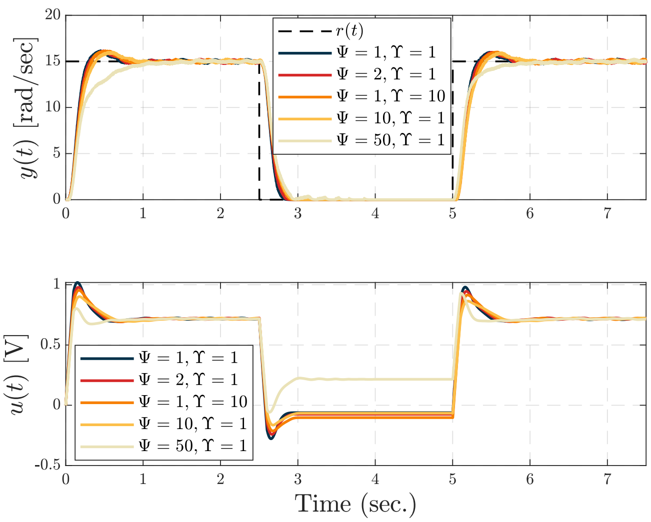

Fig. 8: System Response for Cost Function J_2. The graph showcases the system's performance when optimized for the multi-objective cost function J_2, reflecting the trade-offs between response speed and control signal smoothness.

V. Conclusions and Future work

The study demonstrates that integrating intentional delays in IR controllers can significantly enhance precision and stability in systems with inherent time delays. The use of genetic algorithms for parameter optimization has proven particularly effective, offering a robust approach for navigating complex tuning landscapes that are often beyond the reach of conventional methods. The resulting controllers exhibit strong theoretical alignment and improved practical performance, characterized by faster response, smoother control action, and overall greater efficiency.

While the present work focuses on a specific class of systems and relies on simulation-based evaluation, the results suggest promising potential for broader application. Further investigation into the robustness of the proposed approach under varying system conditions, as well as more extensive comparisons with traditional control strategies such as PID and MPC, would provide deeper insights into its relative strengths and practical viability. These aspects are left for future work, alongside the extension to more complex system models—such as second-order systems with zeros—and experimental validation on faster platforms, including power electronic converters.

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References

[1] P. Airikka, "The PID controller: algorithm and implementation," *Computing and Control Engineering*, vol. 14, no. 6, pp. 6–11, 2003.

[2] W. S. Levine, *The Control Handbook (three volume set)*. CRC Press, 2018.

[3] D. Y. Abramovitch, "Thoughts on Furthering the Control Education of Practicing Engineers [Focus on Education]," *IEEE Control Systems Magazine*, vol. 43, no. 1, pp. 64–88, 2023.

[4] Š. Kozák, "State-of-the-art in control engineering," *Journal of Electrical Systems and Information Technology*, vol. 1, no. 1, pp. 1–9, 2014.

[5] T. Dokeroglu, E. Sevinc, T. Kucukyilmaz, and A. Cosar, "A survey on new generation metaheuristic algorithms," *Computers & Industrial Engineering*, vol. 137, p. 106040, 2019.

[6] M. Nasir, M. Saloumi, and A. B. Nassif, "Review of Various Metaheuristics Techniques for Tuning Parameters of PID/FOPID Controllers," in *ITM Web of Conferences*, vol. 43, p. 01002, EDP Sciences, 2022.

[7] M. A. Boschetti, V. Maniezzo, M. Roffilli, and A. Bolufé Röhler, "Matheuristics: Optimization, simulation and control," in *International workshop on hybrid metaheuristics*, pp. 171–177, Springer, 2009.

[8] J.-A. Hernández-Gallardo, A.-J. Guel-Cortez, E.-J. González-Galván, J.-A. Cárdenas-Galindo, L. Félix, and C.-F. Méndez-Barrios, "Synergistic Design of Optimal PI Controllers for Linear Time-Delayed Systems," in *Advances in Automation and Robotics Research* (M. N. Cardona, J. Baca, C. Garcia, I. G. Carrera, and C. Martinez, eds.), (Cham), pp. 77–88, Springer Nature Switzerland, 2024.

[9] A. Ramirez, R. Sipahi, S. Mondié, and R. Garrido, "An analytical approach to tuning of delay-based controllers for LTI-SISO systems," *SIAM Journal on Control and Optimization*, vol. 55, no. 1, pp. 397–412, 2017.

[10] J.-E. Hernández-Díez, C.-F. Méndez-Barrios, S. Mondié, S.-I. Niculescu, and E. González-Galván, "Proportional-delayed controllers design for LTI-systems: a geometric approach," *International Journal of Control*, vol. 91, no. 4, pp. 907–925, 2018.

[11] J.-A. Hernández-Gallardo, A.-J. Guel-Cortez, E. J. Gonzalez-Galvan, and C.-F. Méndez-Barrios, "Designing Proportional Delayed Integral Control for Fast Regulation in Second-Order Systems: A Geometric Approach," in *2023 9th International Conference on Control, Decision and Information Technologies (CoDIT)*, 2023.

[12] R. Villafuerte-Segura, "Delayed controllers for time-delay systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 117, p. 106934, 2023.

[13] J.-A. Hernández-Gallardo, C.-F. Méndez-Barrios, J.-E. Escalante-Martínez, and E. J. Gonzalez-Galvan, "Stabilizing Second-Order Non-Minimum Phase Systems via PδI Controller," in *2024 10th International Conference on Control, Decision and Information Technologies (CoDIT)*, pp. 1849–1854, 2024.

[14] A. Ramírez, R. Garrido, and S. Mondié, "Velocity control of servo systems using an integral retarded algorithm," *ISA transactions*, vol. 58, pp. 357–366, 2015.

[15] H. Fan, A. Ramírez, S. Mondié, and R. Sipahi, "DC Motor Velocity Control With Integral Retarded Controller Under Unintentional Delay," *Journal of Computational and Nonlinear Dynamics*, pp. 1–7, 2024.

[16] L. El'sgol'ts and S. Norkin, *Introduction to Theory and Applications of Differential Equations with Deviating Arguments*. New York, NY: Academic Press, 1973.

[17] P. Fitzpatrick, *Advanced calculus*, vol. 5. American Mathematical Soc., 2009.