Thermal management of a school room via quadratic multi-objective optimization

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***Abstract*—This paper presents a multi-objective optimization framework to reduce energy costs while maintaining thermal comfort in a school classroom. The room is thermally controlled with an HVAC system. A first-order resistance-capacitance (RC) model is used to characterize the thermal behavior of the room. The multi-objective problem is solved using a scalarization method, which leads to a constrained quadratic programming (QP) problem. Simulations are carried out over a four-day period, considering dynamic electricity pricing and thermal inertia effects. The results demonstrate how increasing the thermal parameters of the room affects the thermal response and cost minimization. The study identifies optimal HVAC operation strategies that use the building's thermal storage. The proposed framework can be used for real-time control and multi-agent coordination in large-scale building networks.**

***Index Terms*—Multi-objective optimization, QP optimization, Thermal comfort, First-order RC method**

I. Introduction

The increasing energy demand in buildings, particularly in educational institutions, necessitates efficient energy management strategies to balance energy consumption and occupants' thermal comfort. Schools, in particular, require optimized HVAC operation due to their occupancy patterns and stringent comfort requirements [1], [2]. Energy efficiency measures, including improved insulation, renewable energy integration, and optimized control strategies, have been widely studied in recent years [3]. However, more research is needed to explore the interplay between thermal properties, energy consumption, and cost optimization in real-world scenarios [4]–[7].

To address cost reduction and occupant comfort in buildings, the thermal model of a building must be considered [8]. Among various approaches to model the thermal behavior of buildings, the RC method is widely used in the literature [9]. This method allows for the modeling of individual elements of a room, such as walls, windows, heaters, HVAC systems, indoor air, and solar irradiation. Moreover, it provides a clear physical interpretation of the system [10]. However, a major drawback of this model is the large number of state variables it generates, which can increase computational complexity [5]. Since optimization problems are solved computationally, reducing the number of state variables is essential to decrease computational time and improving real-time responsiveness [11].

Therefore, in this paper, a reduced first-order RC model of a single room is considered. This model is derived using first principles, offering a practical and applicable solution. The parameters of this model can be obtained directly from tables and data sheets of the building elements, but since buildings have numerous thermal characteristics and materials, calculating their exact properties is challenging, especially as these properties change over time. One solution is to use data-driven methods to estimate thermal parameters which can provide a sufficiently accurate model [12].

In this study, thermal room parameters, including thermal capacity and thermal resistance, are estimated using data provided by STAM Company for a real case study involving a single room in a building. The data is particularly valuable as it offers real-world insights into a building's thermal characteristics, helping to validate and improve the model's accuracy. Other parameters are calculated and derived from data sheets of the materials used in the building.

The remainder of this paper is organized as follows: Section II describes the problem formulation and system modeling. Section III outlines the optimization approach and constraints. Section IV presents the simulation results and discussion. Finally, Section V concludes the paper with future research directions.

II. Problem Formulation

In this section, the mathematical formulation of the optimization problem to be addressed is presented. Firstly, the notation used in this section will be explained. Then, the general definition of the multi-objective optimization problem will be given. Lastly, the statement of the problem will be formulated to fit the standard matrix form of a quadratic programming (QP) optimization problem.

A. Notation

Let $n \in \mathbb{N}$. $\mathbf{1}_{n\times 1}$ denotes the n-dimensional column vector of unit elements. Let $v = [v_1, v_2, ..., v_n]^T \in \mathbb{R}^n$ and $w = [w_1, w_2, ..., w_n]^T \in \mathbb{R}^n$ stand for generic real-valued n-dimensional column vectors. The notation $v \leq w$ ($v \geq w$) stands for the set of inequalities $v_i \leq w_i$ ($v_i \geq w_i$) $\forall i = 1, 2, \ldots, n$. Let $A > 0$ be a symmetric positive-definite matrix. The weighted A-norm of v is defined as

$$\|v\|_{2,A} = \sqrt{v^T A v}. \tag{1}$$

B. Multi-objective optimization problem

The considered discrete-time first-order dynamics of the room temperature is

$$\begin{aligned} T(k+1) = T(k) + \frac{\Delta T}{C_r R_w}(T_{\text{out}}(k) - T(k)) + \\ \frac{\Delta T}{C_r} \cdot [Q_{\text{sun}}(k) + Q_{\text{int}}(k) - P(k) \cdot \text{COP}] \\ T(0) = T_0, \end{aligned} \tag{2}$$

where $k = 0, 1, \ldots, N-1$ represents the discrete time variable, N is the number of sampling time intervals, ΔT $[hour]$ is the sampling time, $T(k)$ $[^\circ C]$ is the room temperature in the k-th time interval, C_r $[\frac{kWh}{K}]$ is the thermal heat capacity of the room, R_w $[\frac{K}{kW}]$ is the thermal resistance of the envelopes of the room, $T_{out}(k)$ $[^\circ C]$ is the external temperature, $Q_{int}(k)$ $[kW]$ is the internal heat power produced by the room occupants, $Q_{sun}(k)$ $[kW]$ is the solar power absorbed by the room envelopes, $P(k)$ $[kW]$ is the electrical power consumed by the HVAC system and COP is the coefficient of performance of the HVAC system.

Our goal is to reduce the energy cost while maintaining the thermal comfort in the room. The multi-objective optimization is tackled by employing the scalarization approach, thus the objective function incorporates energy-related expenses and deviations from the desired room temperature. The considered cost function J is given in the following equation

$$J = \sum_{k=0}^{N-1} \Delta T \cdot C(k) \cdot P(k) + \sum_{k=1}^{N} \alpha_c(k) \cdot (T(k) - T^*(k))^2, \tag{3}$$

where $C(k)$ $[\frac{€}{kWh}]$ is the cost of energy in the k-th time interval, $\alpha_c(k)$ is the weighting factor for the thermal comfort component term of the cost function, and $T^*(k)$ $[^\circ C]$ is the desired reference profile for the room temperature.

There are inequality constraints for this problem which impose limitations on the HVAC system power and enforce the desired comfort range for the room temperature, which are defined in the following equation

$$\begin{aligned} P_{min} \leq P(k) \leq P_{max}, \quad \forall k = 0, ..., N-1, \\ T_{min} \leq T(k) \leq T_{max}, \quad \forall k = 1, ..., N, \end{aligned} \tag{4}$$

where P_{min} and P_{max} are the minimum and maximum power consumed by the HVAC system, respectively. T_{min} and T_{max} are the minimum and maximum allowed room temperature, respectively.

The cost function (3) can be rewritten in the more compact matrix form as

$$J = \Delta T \mathbf{C}^T \mathbf{P} + \|\mathbf{T} - \mathbf{T}^*\|^2_{2,\mathbf{W}_\alpha}, \tag{5}$$

where

$$\mathbf{P} = [P(0), P(1), \ldots, P(N-1)]^T, \tag{6}$$
$$\mathbf{T} = [T(1), T(2), \ldots, T(N)]^T, \tag{7}$$
$$\mathbf{T}^* = [T^*(1), T^*(2), \ldots, T^*(N)]^T, \tag{8}$$
$$\mathbf{C} = [C(0), C(1), \ldots, C(N-1)]^T, \tag{9}$$
$$\mathbf{W}_\alpha = \text{diag}(\alpha_c(1), \alpha_c(2), \ldots, \alpha_c(N)), \tag{10}$$

stacking the values of the decision variables and the several signals involved in the present problem.

C. QP Formulation

The room temperature dynamics (2) is rewritten in the more compact form as

$$\begin{aligned} T(k+1) = aT(k) - bP(k) + E(k), \\ T(0) = T_0, \end{aligned} \tag{11}$$

where

$$\begin{aligned} a &= 1 - \frac{\Delta T}{C_r R_w}, \\ b &= \frac{\Delta T}{C_r} \cdot COP, \\ E(k) &= \frac{\Delta T}{C_r R_w} T_{\text{out}}(k) + \frac{\Delta T}{C_r} \cdot [Q_{\text{sun}}(k) + Q_{\text{int}}(k)]. \end{aligned} \tag{12}$$

The solution of (11) is

$$T(k) = a^k T(0) + \sum_{i=0}^{k-1} \left[a^{k-1-i}\left(-bP(i) + E(i)\right)\right], \tag{13}$$

in which for $k = 1, 2, ..., N$ can be rewritten in a matrix form as

$$\mathbf{T} = \mathbf{\Phi} T_0 - b\mathbf{M}\mathbf{P} + \mathbf{M}\mathbf{E}, \tag{14}$$

where

$$\mathbf{\Phi} = \left[a, a^2, \ldots, a^N\right]^T, \tag{15}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ a & 1 & 0 & \ldots & 0 \\ a^2 & a & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a^{N-1} & a^{N-2} & \ldots & a & 1 \end{bmatrix} \in \mathbb{R}^{N\times N}, \tag{16}$$

$$\mathbf{E} = [E(0), E(1), \ldots, E(N-1)]^T. \tag{17}$$

Substituting (14) into (5), the cost function becomes only dependent on the decision variables, that is, the elements of the vector $\mathbf{P}$. Therefore, the optimization problem can be written in a standard QP problem:

$$\min_{\mathbf{P}} \frac{1}{2}\mathbf{P}^\mathbf{T}\mathbf{Q}\mathbf{P} + \mathbf{f}^T\mathbf{P} + \mathbf{W}, \tag{18}$$

where

$$\mathbf{Q} = 2b^2 \mathbf{M}^T \mathbf{W}_\alpha \mathbf{M}, \tag{19}$$

$$\mathbf{f} = -2bT_0\mathbf{M}^T\mathbf{W}_\alpha\mathbf{\Phi}^T - 2b\mathbf{M}^T\mathbf{W}_\alpha\mathbf{ME} + 2b\mathbf{M}^T\mathbf{W}_\alpha T^* + \Delta T\mathbf{C}. \tag{20}$$

$\mathbf{W}$ is a constant scalar term that does not depend on the optimization variables. Note that $\mathbf{W}$ does not affect the solution of the minimization problem, hence its explicit form is not reported and it will be disregarded.

To have a convex QP optimization problem, $\mathbf{Q}$ must be a symmetric positive definite matrix. To ensure this, there must exist a lower triangular matrix $\mathbf{L}$ with positive diagonal entries such that $\mathbf{Q} = \mathbf{L}^T\mathbf{L}$. It is straightforwardly shown that the lower triangular matrix $\mathbf{L}$ is

$$\mathbf{L} = \sqrt{2}b\sqrt{\mathbf{W}_\alpha}\mathbf{M}, \tag{21}$$

where $\sqrt{\mathbf{W}_\alpha} = \text{diag}(\sqrt{\alpha_c(1)}, \sqrt{\alpha_c(2)}, \ldots, \sqrt{\alpha_c(N)})$ meets such a decomposition of $\mathbf{Q}$ and has positive diagonal entries.

The constraints in equation (4) can be represented as:

$$\mathbf{P} \leq \mathbf{P_{max}}, \quad -\mathbf{P} \leq -\mathbf{P_{min}}, \tag{22}$$

$$\mathbf{T} \leq \mathbf{T_{max}}, \quad -\mathbf{T} \leq -\mathbf{T_{min}}, \tag{23}$$

where

$$\mathbf{P_{min}} = P_{min}\mathbf{1}_{N\times1}, \quad \mathbf{P_{max}} = P_{max}\mathbf{1}_{N\times1}, \tag{24}$$

$$\mathbf{T_{min}} = T_{min}\mathbf{1}_{N\times1}, \quad \mathbf{T_{max}} = T_{max}\mathbf{1}_{N\times1}. \tag{25}$$

By substituting equation (14) in equation (23), the final form of the constraints of the optimization problem is obtained as

$$\mathbf{P} \leq \mathbf{P_{max}}, \quad -\mathbf{P} \leq -\mathbf{P_{min}}, \tag{26}$$

$$-\mathbf{P} \leq (b\mathbf{M})^{-1}(\mathbf{T_{max}} - \mathbf{\Phi}T_0 - \mathbf{ME}), \tag{27}$$

$$\mathbf{P} \leq (b\mathbf{M})^{-1}(-\mathbf{T_{min}} + \mathbf{\Phi}T_0 + \mathbf{ME}). \tag{28}$$

Therefore, the obtained QP optimization problem is defined as

$$\min_{\mathbf{P}} \frac{1}{2}\mathbf{P^T}\mathbf{QP} + \mathbf{f}^T\mathbf{P} \tag{29}$$

$$\textbf{s.t} : (26) - (28).$$

III. Optimization Approach

The aim of this research is to reduce energy costs while maintaining thermal comfort. Therefore, a multi-objective optimization problem is introduced as a constrained quadratic optimization, in which both the energy cost and the cumulative squared error between the actual and desired room temperature are minimized following the scalarization approach.

A weighting factor is introduced for the comfort objective. The comfort weight α_c varies throughout the day based on user preferences and the room specification, which is a school room. The room temperature should be sufficiently close to the reference temperature only during working hours, that is, the larger the weight α_c, the narrower the mismatch of the room temperature. For the rest of the day, the comfort weight is set to zero, and the indoor temperature is maintained within a larger comfort range, which is defined as a constraint in the optimization problem. Figure 1 shows time evolution of α_c with its maximum value α_c^*.

In this study, only summer period is considered. Therefore, thermal heat of the HVAC system which works in cooling mode is represented as a negative term in the thermal dynamics (2). For the future works, the optimization can be done also for winter and transition seasons.

Regarding the internal heat (Q_{int}) produced by occupants, it is considered that the classroom is only full of people during working hours. The heat produced by a person is equal to $100[W]$ [13]. The number of occupants is equal to 20. It is worth noting that the heat produced by electric devices inside the room is neglected. Figure 2 illustrates the internal heat produced and the price of consumed energy which has different values in off and peak periods during a day.

IV. Results and Discussion

The simulations have been done with MATLAB R2024a and a CPU with Core i7-1165G7, 2.80 GHz using 'quadprog' optimization tool.

Simulations are carried out using the following parameters: the optimization horizon (T_{sim}) is equal to four consecutive days, a single room in a school ($10\ m \times 5\ m \times 3\ m$) with large windows on the northern wall with a size of ($3\ m \times 1.5\ m$) is considered. The thickness of the envelopes is also equal to $30\ cm$. The minimum comfort range of the room temperature is between T_{min} and T_{max}. The reference value of the room temperature and its initial value are equal to T^* and T_0. The power consumed by the HVAC system is limited between P_{min} and P_{max}. Simulation parameters are given in table I.

The outside temperature (T_{out}) has been interpolated for four days, from data collected during a summer day, provided by STAM company which is given in Fig.3 [14].

To investigate the impact of thermal characteristics of the building on the cost of energy considering the building as a thermal storage, simulations are done with various values of room thermal parameters.

The total cost of energy consumed during optimization horizon (Energy Expense [€]) in purple color, the total energy consumed (Energy Consumption $[kWh]$) by the HVAC system in orange color, the power peak $[kW]$ during the period in blue color and the absolute value of maximum deviation between room temperature (T_{in}) and its reference value (Temperature

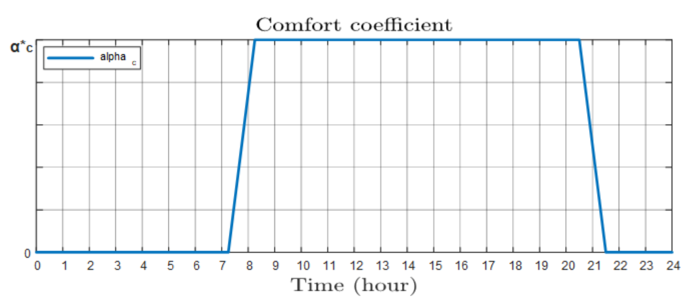

Fig. 1. Weighting factor of thermal comfort objective.

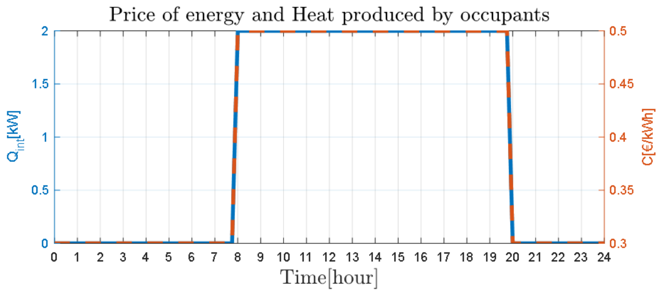

Fig. 2. Time evolution of the price of consumed energy (C [€/kWh]) and internal heat ($Q_{int}[kW]$) in a day.

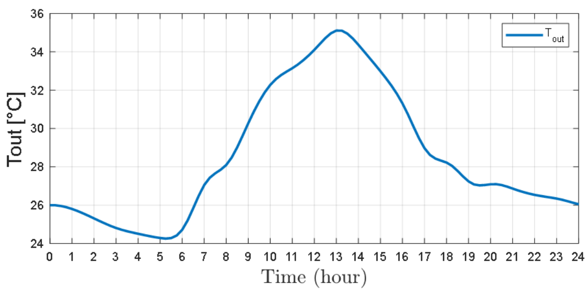

Fig. 3. Outside Temperature [$°C$].

Error) in green are obtained by simulations done with $\alpha_c^* = 1$. Table II shows the results.

The results indicate that increasing room thermal capacity generally allows the building structure to be considered as a thermal storage. However, it affects the increase in power peak. Moreover, the simulations with very high value of thermal resistance have been done to show that the increase of room thermal resistance does not have a significant effect on the optimization objectives. This means there is no need to invest in increasing room thermal resistance more than 390 [$\frac{K}{kW}$].

For investigating the effects of weighting factor of thermal comfort, the same simulations are done for $\alpha_c^* = 0.5$ and 10, which the results are given in tables III and IV, respectively. It is obtained that a larger weighting factor for thermal comfort results into a better tracking of the desired temperature. On the other hand, it causes to decrease the sensitivity of minimization of the cost of energy with respect to the room thermal capacity.

TABLE I
VALUES OF THE SIMULATION PARAMETERS

Parameters	Values	Unit
T_{sim}	96	$hour$
ΔT	0.25	$hour$
$N = \frac{T_{sim}}{\Delta T}$	384	–
T_{min}	20	$°C$
T_{max}	24	$°C$
T^*	22	$°C$
T_0	22	$°C$
P_{min}	0	kW
P_{max}	5	kW

TABLE II
MAXIMUM TEMPERATURE ERROR, ENERGY CONSUMPTION[kWh], ENERGY EXPENSE[€] AND POWER PEAK[kW],WITH $\alpha_c^* = 1$

Cr[kWh/K] \ Rw[K/kW]	39		117		390		500	
5.06	0.19	55.41	0.18	52.35	0.18	51.27	0.18	51.16
	27.10	1.43	25.74	1.36	25.26	1.34	25.21	1.34
15.18	0.19	54.84	0.19	51.76	0.19	50.67	0.19	50.57
	26.01	3.49	24.65	3.35	24.18	3.24	24.14	3.22
50.6	0.28	51.47	0.27	48.45	0.27	47.39	0.27	47.29
	21.35	5.00	20.07	5.00	19.62	5.00	19.58	5.00

TABLE III
MAXIMUM TEMPERATURE ERROR, ENERGY CONSUMPTION[kWh], ENERGY EXPENSE[€] AND POWER PEAK[kW],WITH $\alpha_c^* = 0.5$

Cr[kWh/K] \ Rw[K/kW]	39		117		390		500	
5.06	0.27	55.33	0.26	52.27	0.26	51.19	0.26	51.09
	26.89	1.64	25.53	1.56	25.05	1.50	25.01	1.47
15.18	0.30	54.27	0.28	51.21	0.27	50.14	0.27	50.04
	25.23	4.99	23.89	4.66	23.43	4.42	23.38	4.39
50.6	0.43	49.18	0.42	46.19	0.42	45.15	0.42	45.05
	18.27	5.00	17.04	5.00	16.61	5.00	16.57	5.00

Figures 4 and 5 depict the variation of cost of energy consumption [€] and total energy consumption [kWh], respectively, based on optimization done with various values of the room thermal parameters. It is seen that by increasing the thermal capacity and thermal resistance, the cost and consumption of energy reduced. However, from Figure 4, the cost does not change dramatically by increasing the thermal resistance of the room after a certain value. Figure 5 also shows the same trend for the energy consumption. In addition, increasing of the thermal capacitance has more constant effect on both the energy consumed and the cost of it. But it is seen that increasing the thermal capacitance of the room results in dramatic decrease in the cost. However, it has less effect on the energy consumption.

TABLE IV
MAXIMUM TEMPERATURE ERROR, ENERGY CONSUMPTION[kWh], ENERGY EXPENSE[€] AND POWER PEAK[kW],WITH $\alpha_c^* = 10$

Cr[kWh/K] \ Rw[K/kW]	39		117		390		500	
5.06	0.18	55.48	0.06	52.41	0.03	51.33	0.03	51.23
	27.44	1.43	26.07	1.36	25.59	1.34	25.54	1.34
15.18	0.06	55.43	0.06	52.35	0.06	51.27	0.06	51.17
	27.13	1.43	25.76	1.36	25.28	1.34	25.24	1.34
50.6	0.06	54.77	0.06	51.68	0.06	50.60	0.06	50.50
	25.91	3.66	24.56	3.49	24.09	3.34	24.04	3.33

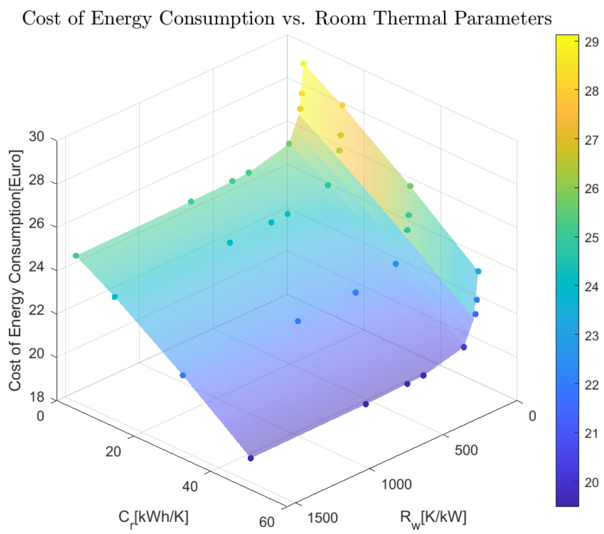

Fig. 4. The total cost of energy consumption variation with respect to the room thermal capacitance $C_r[kWh/K]$ and resistance $R_w[K/kW]$.

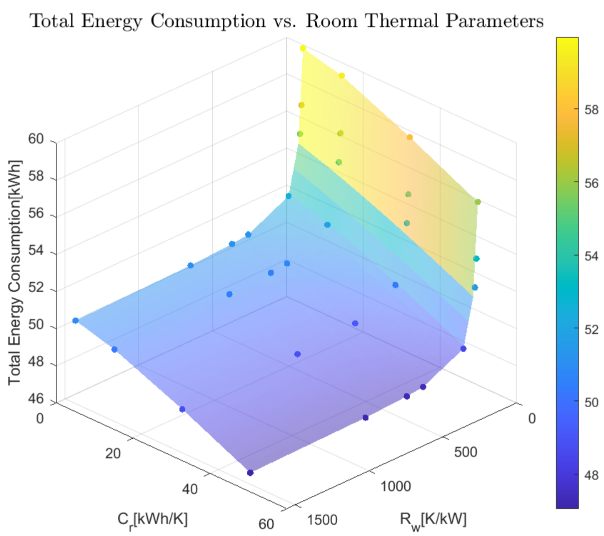

Fig. 5. The total energy consumption variation with respect to the room thermal capacitance $C_r[kWh/K]$ and resistance $R_w[K/kW]$.

V. Conclusion and Future Works

This paper presents a multi-objective optimization framework for a thermally controlled room integrating an HVAC system in a school building. The problem is formulated as a constrained quadratic programming (QP) problem, aiming to minimize the total energy cost while ensuring the occupant's comfort. The results suggest that integrating thermal storage strategies with optimal HVAC control can reduce energy costs while maintaining comfort in which the effect of increasing thermal resistance shows a saturation in both energy consumption and cost. Future work includes real-time optimization, integration with data-driven methods, using model predictive control, and expanding the approach to multi-room and multi-building configurations.

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