A Neural Network-Based Prescribed-Time Controller Formulation with Update Modularity for a Class of Nonlinear Systems

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***Abstract*— This work concentrates on a neural network-based, prescribed time controller formulation for a class of nonlinear systems having parametric uncertainty. The aim of the controller is to ensure that the tracking error converges to the origin within a user-defined prescribed time despite the presence of bounded disturbances and parametric uncertainties with controller/update law modularity. The stability of closed–loop error system has been ensured via Lyapunov-based arguments. Numerical simulations are conducted to illustrate the feasibility of the proposed method.**

I. Introduction

Prescribed time control aims to ensure that the tracking error converges to a small neighborhood of the origin within a user-defined time. Recently this type of controller formulations have gained the interest of researchers. To name some, in [1], authors employed time-varying scaling function that grows unbounded towards prescribed time and designed a controller that stabilizes the system to reach regulation at exactly pre-defined time. In [2], adaptive prescribed time controller designed for event-based nonlinear system contains time-varying uncertain parameters. In [3], a fuzzy reinforcement learning approach is presented to solve a prescribed-time optimal control problem for nonlinear systems while ensuring stability and convergence. In [4], prescribed-time mean-square stabilization for nonlinear systems with multiplicative noise is presented.

Significant research efforts have been dedicated, to developing prescribed time control strategies for robotic applications. In [5], a prescribed time controller designed for first- and n-order systems and numerical simulations for the two-link robot manipulator has been made. In [6], robust prescribed time controller is presented for uncertain Euler-Lagrange systems with time-varying disturbances. In [7], a safety filter is presented for a 7-DOF robot manipulator based on prescribed-time control, capable of obstacle avoidance. In [8], prescribed time controller with velocity observer is introduced for 3-dof dual-arm robot manipulator. In [9], prescribed time control combined with fuzzy logic control for robot manipulators with modeling uncertainties and unknown control input direction. In [10], a robust nonlinear model predictive controller was presented for mobile robots ensuring navigation within the prescribed time limits despite constraints and disturbances.

Controller/update law modularity refers to the control law ensures closed-loop stability independently of the update law, provided the update law ensures boundedness. In [11], adaptive position tracking controller for robot manipulators is presented which achieves input-to-state stability with controller/update law modularity. Similarly, in [12], the authors achieve adaptive tracking and regulation control for a wheeled mobile robot with update law modularity. In [13], adaptive control of Euler–Lagrange system with robust integral of the sign error based approach is presented to achieve modularity in the update law. In [14], a modular neural network-based modular adaptive control is presented for trajectory tracking tasks.

In this study, neural network-based prescribed time controller for a class of nonlinear systems contains additive disturbances and modeling uncertainties with controller/update law modularity presented. A time-varying scaling function and single layer neural network structure are defined for control design and the controller is designed independent from update law to ensure modularity. Boundedness of the proposed controller and error signals have been proven with Lyapunov-based arguments. Finally, simulations have been made to illustrate the efficacy of the suggested method using different update laws to observe the boundedness of the closed-loop system.

The rest of the paper is structured as follows: Section II introduces the system model used in this paper. In Section III, control design and in Section IV, stability analysis of the proposed controller is presented. In Section V, simulation results are shown and concluding remarks are given in Section VI.

II. System Model

We consider the class of nonlinear systems that are represented in the following form

$$\dot{y} = f\left(y, t\right) + d + u \tag{1}$$

where $y\left(t\right) \in \mathbb{R}^n$ is the state vector, $f\left(y, t\right) \in \mathbb{R}^n$ is the nonlinear function of system state, $d\left(t\right) \in \mathbb{R}^n$ models the additive disturbances and $u\left(t\right) \in \mathbb{R}^n$ is the control input. In the subsequent development, the assumption that the additive disturbances being a bounded function of time will be utilized.

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III. Error System Development & Control Design

The main objective of the control design is to ensure that the state vector y tracks the desired time-varying state vector within *a priori* given prescribed time despite the presence of modeling anomalies such as model uncertainties and additive disturbances.

To initiate the control design, we introduce the state tracking error $e\left(t\right)\in\mathbb{R}^{n}$ defined as follows

$$e\triangleq y-y_{d} \tag{2}$$

where $y_{d}\left(t\right)\in\mathbb{R}^{n}$ is the desired state vector. The objective of ensuring tracking exactly at prescribed time will be integrated into the controller design via the time-varying scaling function $\mu\left(t\right)\in\mathbb{R}$ and its inverse $\upsilon\left(t\right)\in\mathbb{R}$ which are defined as follows

$$\mu\triangleq\frac{T}{T-t},\quad \upsilon\triangleq\frac{1}{\mu}=\frac{T-t}{T},\quad \forall t\in[0,T) \tag{3}$$

where $T\in\mathbb{R}$ is positive user defined prescribed time. Using the defined scaling function, modified tracking error $e_{p}\left(t\right)\in\mathbb{R}^{n}$ is defined as

$$e_{p}\triangleq\mu^{2}e. \tag{4}$$

Taking the time derivative of (2) and substituting our system model (1) into yields

$$\dot{e}=f+d+u-\dot{y}_{d} \tag{5}$$

and after adding and subtracting f_{d} with $f_{d}\left(y_{d},t\right)\triangleq f\left(y,t\right)|_{y=y_{d}}$ to the right hand side of above equation yields

$$\dot{e}=f_{d}+\tilde{f}+d+u-y_{d} \tag{6}$$

where $\tilde{f}$ is estimation error and it is defined as

$$\tilde{f}=f-f_{d},\quad \tilde{f}\in\mathbb{R}^{n} \tag{7}$$

and following upper bound can be made using Mean Value Theorem [15]

$$\|\tilde{f}\|\leq\rho\left(\|e\|\right)\|e\| \tag{8}$$

where ρ is non-negative function. In the context of neural network-based synthesis, f_{d} is defined, corresponding to a single-layer neural network structure as follows [16].

$$f_{d}=W^{T}\sigma\left(y_{d}\right)+\epsilon\left(y_{d}\right) \tag{9}$$

where $W\in\mathbb{R}^{n\times n}$, $\sigma\left(y_{d}\right)\in\mathbb{R}^{n}$, $\epsilon\left(y_{d}\right)\in\mathbb{R}^{n}$ are constant weight matrix, activation function and functional reconstruction error, respectively. Substituting (9) into (6), we obtain the following expression

$$\dot{e}=W^{T}\sigma+\epsilon+\tilde{f}+d+u-\dot{y}_{d} \tag{10}$$

Taking the time derivative of (4) and substituting (10), open loop error term yields

$$\dot{e}_{p}=\frac{2}{T}\mu^{3}e+\mu^{2}\left(W^{T}\sigma+\epsilon+\tilde{f}+d+u-\dot{y}_{d}\right). \tag{11}$$

The control input u designed in the following form

$$u=-ke_{p}+\dot{y}_{d}-\hat{W}^{T}\sigma-k_{a}\|\sigma\|^{2}e_{p} \tag{12}$$

where $k\in\mathbb{R}$ is to be designed control gain, $k_{a}\in\mathbb{R}_{>0}$ is control gain and $\hat{W}\in\mathbb{R}^{n\times n}$ is estimate of the constant weight matrix W. The proposed controller is designed to be independent of the update law, enhancing its flexibility and applicability to various update laws. It is only required that the update law is bounded for $t\in[0,T)$ [11], [12]. Substituting control input into (11), the closed loop error term can be obtain as

$$\dot{e}_{p}=\frac{2}{T}\mu^{3}e+\mu^{2}\left(\tilde{f}+\tilde{W}^{T}\sigma-ke_{p}+\epsilon+d-k_{a}\|\sigma\|^{2}e_{p}\right) \tag{13}$$

where $\tilde{W}\in\mathbb{R}^{n\times n}$ is estimate error of weight matrix and it is defined as follows

$$\tilde{W}=W-\hat{W} \tag{14}$$

IV. Stability Analysis

Theorem 1: The proposed control input (12) ensures boundedness of all signals for $t\in[0,T)$ and converges the tracking error to small neighborhood of origin exactly at prescribed time for different bounded update laws provided that the control gain k designed as follows

$$k=\kappa+k_{n_{1}}+k_{n_{2}}+\frac{2}{T}+\rho \tag{15}$$

where $\kappa\in\mathbb{R}_{>0}$ is an auxiliary control gain, $k_{n_{1}}$, $k_{n_{2}}\in\mathbb{R}_{>0}$ are a damping coefficients and ρ was previously introduced in (8).

Proof: To proof the above theorem, we chose a non-negative Lyapunov function as

$$V=\frac{1}{2}e_{p}^{T}e_{p} \tag{16}$$

and taking the time derivative of (16) and substituting closed loop error term (13) yields

$$\begin{aligned}\dot{V} \quad = \quad & \frac{2}{T}\mu\|e_{p}\|^{2}+\mu^{2}e_{p}^{T}\tilde{f}+\mu^{2}e_{p}^{T}d+\mu^{2}e_{p}^{T}\epsilon-k\mu^{2}\|e_{p}\|^{2}\\ & +\mu^{2}e_{p}^{T}\tilde{W}^{T}\sigma-k_{a}\mu^{2}\|\sigma\|^{2}\|e_{p}\|^{2}.\end{aligned} \tag{17}$$

Following bounds can be made using Young's Inequality

$$e_{p}^{T}d \quad \leq \quad k_{n_{1}}\|e_{p}\|^{2}+\frac{1}{4k_{n_{1}}}\|d\|^{2} \tag{18}$$

$$e_{p}^{T}\epsilon \quad \leq \quad k_{n_{2}}\|e_{p}\|^{2}+\frac{1}{4k_{n_{2}}}\|\epsilon\|^{2} \tag{19}$$

and the second line of (17) can be upper bounded as

$$\begin{aligned}& e_{p}\tilde{W}^{T}\sigma-k_{a}\|\sigma\|^{2}\|e_{p}\|^{2}\\ & \leq\|e_{p}\|\|\tilde{W}\|_{i\infty}\|\sigma\|-k_{a}\|\sigma\|^{2}\|e_{p}\|^{2}\\ & \leq\frac{1}{4k_{a}}\|\tilde{W}\|_{i\infty}^{2}.\end{aligned} \tag{20}$$

Using (8), (18), (19), (20), (17) can be upper bounded as follows.

$$\begin{aligned}\dot{V}\leq & \frac{2}{T}\mu\|e_{p}\|^{2}+\rho\|e_{p}\|^{2}+k_{n_{1}}\mu^{2}\|e_{p}\|^{2}+\frac{1}{4k_{n_{1}}}\mu^{2}\|d\|^{2}\\ & +k_{n_{2}}\mu^{2}\|e_{p}\|^{2}+\frac{1}{4k_{n_{2}}}\mu^{2}\|\epsilon\|^{2}-k\mu^{2}\|e_{p}\|^{2}\\ & +\frac{1}{4k_{a}}\mu^{2}\|\tilde{W}\|_{i\infty}^{2}\end{aligned} \tag{21}$$

and using inverse scaling function defined in (3), (21) can be rewritten as follows

$$\begin{aligned}\dot{V} \quad &\le \quad -\left(k - k_{n_1} - k_{n_2} - \frac{2}{T}\upsilon - \rho\upsilon^2\right)\mu^2\|e_p\|^2 \\ &\quad + \frac{1}{4k_a}\mu^2\|\tilde{W}\|_{i\infty}^2 + \frac{1}{4k_{n_1}}\mu^2\|d\|^2 \\ &\quad + \frac{1}{4k_{n_2}}\mu^2\|\epsilon\|^2 \qquad (22)\end{aligned}$$

Observing from (3) that $|\upsilon(t)| \le 1$ for $t \in [0, T)$ and using the design of k in (15), the following upper bound can be made for time derivative of Lyapunov function

$$\begin{aligned}\dot{V} \quad &\le \quad -\kappa\mu^2\|e_p\|^2 + \frac{1}{4k_a}\mu^2\|\tilde{W}\|_{i\infty}^2 + \frac{1}{4k_{n_1}}\mu^2\|d\|^2 \\ &\quad + \frac{1}{4k_{n_2}}\mu^2\|\epsilon\|^2. \qquad (23)\end{aligned}$$

Using the Lemma 1 from [1], solution of the (23) can be obtained as follows

$$\begin{aligned}V(t) \quad &\le \quad \exp\{2\kappa T(1-\mu(t))\}V(0) + \frac{1}{8\kappa k_a}\|\tilde{W}\|_{[0,t]}^2 \\ &\quad + \frac{1}{8\kappa k_{n_1}}\|d\|_{[0,t]}^2 + \frac{1}{8\kappa k_{n_2}}\|\epsilon\|_{[0,t]}^2 \qquad (24)\end{aligned}$$

$\forall t \in [0, T)$ where $|\cdot|_{[0,t]} := \sup_{\tau\in[0,t]}|\cdot(\tau)|$. From (16), an upper bound for the modified tracking error $e_p(t)$ can be expressed as

$$\begin{aligned}\|e_p(t)\| \quad &\le \quad \exp\{\kappa T(1-\mu)\}\|e_p(0)\| + \frac{1}{\sqrt{4\kappa k_a}}\|\tilde{W}\|_{[0,t]} \\ &\quad + \frac{1}{\sqrt{4\kappa k_{n_1}}}\|d\|_{[0,t]} + \frac{1}{\sqrt{4\kappa k_{n_2}}}\|\epsilon\|_{[0,t]}. \qquad (25)\end{aligned}$$

Making use of (4), tracking error e can be upper bounded as follows

$$\begin{aligned}\|e(t)\| \le \upsilon^2\Bigg[&\exp\{\kappa T(1-\mu)\}\|e_p(0)\| + \frac{1}{\sqrt{4\kappa k_a}}\|\tilde{W}\|_{[0,t]} \\ &+ \frac{1}{\sqrt{4\kappa k_{n_1}}}\|d\|_{[0,t]} + \frac{1}{\sqrt{4\kappa k_{n_2}}}\|\epsilon\|_{[0,t]}\Bigg]. \qquad (26)\end{aligned}$$

$\forall\ t \in [0, T)$. From these bounds it is clear that as the prescribed time approaches tracking error e converges small neighborhood of zero and it can be seen that all signals remain bounded for $t \in [0, T)$. ■

V. Simulation Results

To test the effectiveness of the proposed neural network-based prescribed time controller, simulation studies were conducted in MATLAB/Simulink with the system model given in (1). The system characteristics and disturbances are chosen as follows

$$f = \begin{bmatrix} 2y_1^2 + 2 \\ y_1 y_2 sin(t) \end{bmatrix}, d = \begin{bmatrix} 0.25cos(5t) \\ 0.25cos(5t) \end{bmatrix}. \qquad (27)$$

where $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$ and initial condition of state vector y was set as $y(0) = \begin{bmatrix} 0.2 & 0.25 \end{bmatrix}^T$. The desired state vector y_d is chosen as follows for all simulation studies.

$$y_d = \begin{bmatrix} (0.1 + 0.25t)\, sin(10t) \\ 0.5(1 - exp(-5t)) \end{bmatrix}. \qquad (28)$$

The control gains were selected as $\kappa = 0.55$, $k_a = 5$, $k_{n_1} = 2.125$, $k_{n_2} = 1.5$ and non-negative function ρ was chosen as $\rho = \|e\| + 1$. Additive white Gaussian noise with an SNR value of 30 was added to y in all simulations studies.

For simulation studies, prescribed time was selected as $T = 1$ sec. and a modification has been made for μ in the form of $\mu = \frac{T}{T - \min\{t, \xi T\}}$ to avoid exceeding the limits of the simulation [6]. In this way μ will still remain after $t = \xi T$ where $\xi \in [0, 1)$. For the simulation studies $\xi = 0.99$ was chosen.

The activation function is selected in a smooth and differentiable form as follows

$$\sigma = \begin{bmatrix} \tanh(y_{d_1}) \\ \tanh(y_{d_2}) \end{bmatrix} \qquad (29)$$

where $y_d = \begin{bmatrix} y_{d_1} & y_{d_2} \end{bmatrix}^T$ and three different update laws used to evaluate the modularity of the proposed controller.

$$\dot{\hat{W}} = \Gamma\sigma e_p^T \qquad (30)$$

$$\dot{\hat{W}} = \Gamma\sigma e_p^T - k_w\hat{W} \qquad (31)$$

$$\dot{\hat{W}} = \Gamma\sigma e_p^T - k_w\|e_p\|\hat{W} \qquad (32)$$

where $\Gamma \in \mathbb{R}^{2\times2}$ identity matrix, $k_w \in \mathbb{R}_{>0}$ is a constant and it is selected as $k_w = 0.25$. The initial condition of $\hat{W}$ was chosen as $\hat{W}(0) = 0_{2\times2}$.

Figures 1-3 present the numerical simulation results corresponding to the update law in (30). Figures 1 and 2 show actual and desired state vector and tracking error signals respectively. It can be observed that tracking error converges the origin as the prescribed time approaches and the control objective is achieved exactly at $t = T$. Figure 3 shows the control input signal.

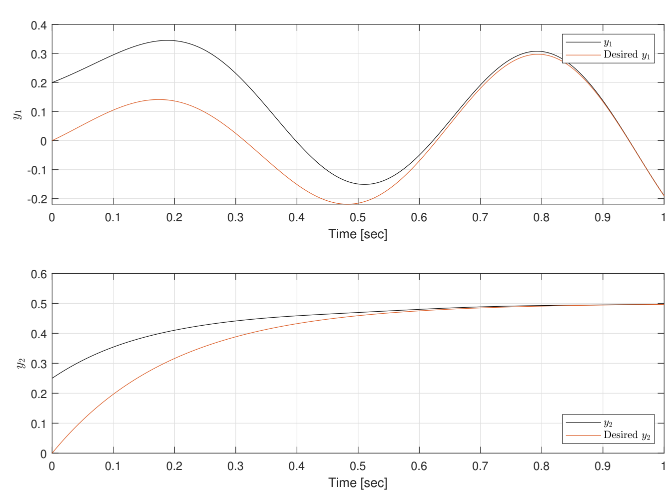

Fig. 1: y vs. y_d

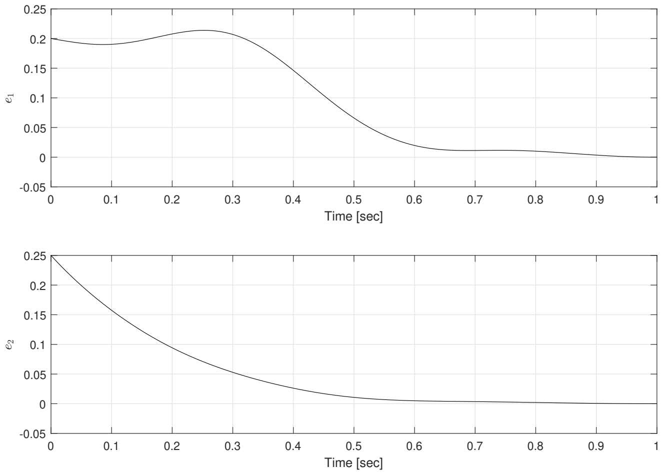

Fig. 2: Tracking error e

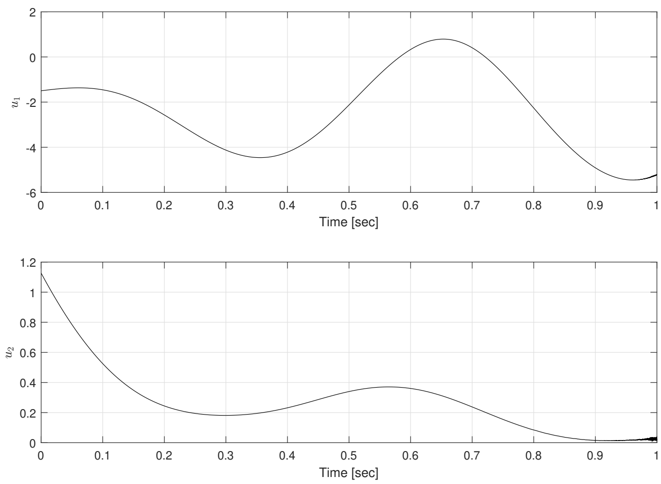

Fig. 3: Control input u

To test the proposed neural network-based prescribed time controller under different bounded update laws simulation was repeated. Table I shows the errors and control efforts for (12) using the update laws (30), (31) and (32). It can be observed that all update laws performed similarly and the control objective is achieved for different update laws.

TABLE I: Comparisons

Controller	$\int \|e_1\|$	$\int \|e_2\|$	$\int \|u_1\|$	$\int \|u_2\|$
(12) with (30)	0.096	0.0469	2.58	0.2731
(12) with (31)	0.0961	0.0469	2.5799	0.2731
(12) with (32)	0.096	0.0469	2.5799	0.2731

VI. Conclusion

In this paper, a neural network-based prescribed time controller with update modularity has been presented for a class of nonlinear systems. The prescribed time controller objective is achieved by defining a modified version of the tracking error signal that corresponds to a time-varying scaling prescribed time function. Boundedness of the close-loop error terms and control input have been ensured via Lyapunov-based argument. The control design and stability analysis are made independent of the update law, making the proposed controller modular for various bounded update laws. The feasibility of the proposed controller is numerically tested with different update laws, and it is observed that the system remains stable under bounded update laws. Future work will focus on the design of a multi-layer neural network architecture and its application to MIMO systems such as robotics, along with the experimental verification of the proposed controller formulation.

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