A Least Squares-Based Parameter Identification Methodology for Super Coiled Polymer Actuators

Cagri Hindistan[1], Erman Selim[1,2,⋆], Alper Bayrak[3], Enver Tatlicioglu[1], Erkan Zergeroglu[4]

***Abstract*— This work concentrates on dynamical parameter estimation problem for super-coiled polymer (SCP) actuator systems. Specifically, a filtered–based least squares estimator has been proposed. The stability of the estimator is ensured using Lyapunov-based arguments. Numerical studies are presented to illustrate the estimation performance.**

I. INTRODUCTION

Soft actuators, exemplified by shape memory alloys [1], [2], [3], and super-coiled polymer (SCP) actuators [4], [5], have garnered significant interest in recent years due to their potential for safer interaction with the surrounding environment [6], [7]. SCPs offer an effective solution, especially in applications where the actuator needs to elongate and contractions in addition to rotational and translational movements.

SCPs act as actuators by contracting and extending when heated and cooled. The scientific literature describes various methods for producing SCP actuators. SCP actuators can be obtained by winding different types of polymer yarns or filaments, such as Spandex yarn [8], nylon sewing threads and fishing lines [9].

In [10], the SCP actuator was produced by winding a conductive sewing thread around a helical mandrel. In [8], utilized a silver wire paste coating on the actuator's surface, which was then heated via the Joule effect. In this way, wrapping a conductive wire around or through the SCP allows the heat to be more evenly distributed. Using this method, each region of the SCP can be elongated and contracted uniformly, offering a measurement advantage. SCP actuators produced by different production techniques are used as actuators in different robotic systems such as artificial muscles [11], soft gripper [7], and robotic eye [12], [13], [14].

An alternative area of research on SCP actuators focuses on the control of their motion. However, the lack of reliable dynamic models becomes the control of SCP actuators is difficult [15]. Therefore, recent research has been devoted to obtaining mathematical models of SCP actuators. In [7], the authors made improvements on the mathematical model by presenting how different materials and winding methods change the spring index and stroke value. In light of the studies, one of the simplest and linear mathematical models of SCP actuators is given in [7], while a model based on the second-order temperature-force relationship for a spandex-nylon SCP actuator is proposed in [8] to represent the non-linear properties of SCP actuators. Most of the parameters in these presented mathematical models are not directly measurable or vary depending on environmental effects. Therefore, identification techniques are often required to accurately identify these parameters.

This study aims to design a least squares estimator to estimate the model parameters in the thermo-mechanical and thermo-electrical model of the SCP actuator. The main objective of the study is to eliminate the need for acceleration measurement. It is proven that the parameter estimation error goes to zero with the filtered regression matrix. To the best of the authors' knowledge, the model parameters of super coiled polymer actuators have not been determined by parameter estimation methods.

The rest of the paper is outlined as follows. In Section II, the mathematical model of the SCP actuator is presented. In Section III, the parameter estimation method with least squares is presented. In Section IV, simulation results are provided. Finally, in Section V, concluding remarks are given.

II. SYSTEM DESCRIPTION

The thermomechanical model (1) and the thermoelectrical model (2) of the SCP actuator are given as follows

$$m\ddot{x} + b\dot{x} + \alpha_1 \bar{T}\dot{x} + kx = -\alpha_2 \bar{T} \tag{1}$$

$$C_{th}\dot{\bar{T}} = -\lambda \bar{T} + \frac{1}{R}V^2 \tag{2}$$

where $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, respectively, represent the length, variation in length, and velocity of variation in length of the SCP actuator. $\bar{T}$ is defined as $\bar{T} \triangleq T - T_{amb}$, where $T(t)$ is the temperature of the actuator, T_{amb} is the ambient temperature and $V(t)$ is the voltage applied to the actuator.

In the thermo-mechanical model of (1), m is the total mass of the actuator including the load on it, b is the damping coefficient, k is the spring stiffness of actuator, α_1 is thermal damping coefficient and α_2 is thermal stiffness coefficient. In the thermo-electrical model of (2), C_{th}, λ and R represent thermal mass, thermal conductivity and resistance, respectively.

[1]C. Hindistan, E. Selim and E. Tatlicioglu are with Department of Electrical & Electronics Engineering, Ege University, 35040, Bornova, Izmir, Turkey cagri.hindistan@ege.edu.tr, erman.selim@ege.edu.tr, enver.tatlicioglu@ege.edu.tr

[2]E. Selim with the Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario, Canada erman.selim@uwaterloo.ca

[3]A. Bayrak with the Department of Electrical & Electronics Engineering, Bolu Abant Izzet Baysal University, 14030, Bolu, Turkey alperbayrak@ibu.edu.tr

[4]E. Zergeroglu with the Department of Computer Engineering, Gebze Technical University, 41400, Gebze, Kocaeli, Turkey e.zerger@gtu.edu.tr

The thermo-mechanical dynamical models (1) and thermo-electrical dynamical models (2) of SCP actuators can be written in the following form

$$\ddot{x} = -\frac{b}{m}\dot{x} - \frac{\alpha_1}{m}\bar{T}\dot{x} - \frac{k}{m}x - \frac{\alpha_2}{m}\bar{T} \tag{3}$$

$$\dot{\bar{T}} = -\frac{\lambda}{C_{th}}\bar{T} + \frac{1}{RC_{th}}V^2 \tag{4}$$

As a first step to obtain super coiled polymer, nickel wire is wound on the fishing line as in Figure 1a. A certain weight is hung on one end of this structure and then it is coiled by rotating it around itself and super coiled polymer is obtained as shown in Figure 1b.

(a) Nickel wire wound fishing line

(b) Super coiled polymer

Fig. 1: Super coiled polymer production steps

By applying voltage to the super coiled polymer in a controlled manner, it is heated by the Joule effect. While the heated structure shows a shortening movement, when it is cooled again, it elongates and returns to its previous state. In this way, linear/prismatic and revolute movements can be obtained.

III. LEAST SQUARES IDENTIFICATION

The main objective of this study is to estimate the parameters in the thermo-mechanical and thermo-electric model of the SCP actuator given in (1) and (2).

The unmeasurable auxiliary signal $a(t) \in \mathbb{R}^2$ is defined as follows

$$a(t) \triangleq \begin{bmatrix} \ddot{x} \\ \dot{\bar{T}} \end{bmatrix}. \tag{5}$$

The regression matrix $W(x, \dot{x}, \bar{T}, V) \in \mathbb{R}^{2x6}$ is defined in the following manner

$$W \triangleq \begin{bmatrix} -\dot{x} & -\bar{T}\dot{x} & -x & -\bar{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{T} & V^2 \end{bmatrix}. \tag{6}$$

The uncertain parameter vector θ is represented as follows

$$\theta \triangleq \begin{bmatrix} \frac{b}{m} & \frac{\alpha_1}{m} & \frac{k}{m} & \frac{\alpha_2}{m} & \frac{\lambda}{C_{th}} & \frac{1}{RC_{th}} \end{bmatrix}. \tag{7}$$

The unmeasurable auxiliary signal $a(t)$ is expressed as follows

$$a = W\theta \tag{8}$$

An auxiliary filter signal $a_f(t) \in \mathbb{R}^2$ is defined as follows

$$\dot{a}_f = -\beta a_f + \beta a \tag{9}$$

where $a_f(0) = 0_{2x1}$, and $\beta \in \mathbb{R}$ is a positive filter gain.

It is important to note that, since $a(t)$ is unmeasurable, an implementable form of $a_f(t)$ can be obtained as follows.

$$a_f = p + \beta v \tag{10}$$

where

$$v(t) \triangleq \begin{bmatrix} \dot{x} \\ \dot{\bar{T}} \end{bmatrix} \in \mathbb{R}^2, \tag{11}$$

and $p(t) \in \mathbb{R}^2$ is an auxiliary filter signal is defined as follows

$$\dot{p} = -\beta a_f;\ p(0) = 0_{2x1}. \tag{12}$$

The derivative of (10) with respect to time is as follows

$$\dot{a}_f = \dot{p} + \beta\dot{v}. \tag{13}$$

Substituting (10) into (12) rearranging them gives the following structure

$$\dot{p} = -\beta p - \beta^2 v. \tag{14}$$

Filtered regressor matrix $W_f(t) \in \mathbb{R}^{2x6}$ is formulated as follows

$$\dot{W}_f = -\beta W_f + \beta W \tag{15}$$

where $W_f(0) = 0_{2x6}$. Multiplying both sides of (15) by θ and utilizing (8) and (9), the following expression can be obtained

$$\dot{a}_f + \beta a_f = \dot{W}_f\theta + \beta W_f\theta. \tag{16}$$

since $a_f(0) = 0_{2x1}$, from (13), it is easy to see that

$$a_f = W_f\theta. \tag{17}$$

Utilizing (17), estimate form of $a_f(t)$, $\hat{a}_f(t)$ is defined as follows

$$\hat{a}_f = W_f\hat{\theta} \tag{18}$$

where $\hat{\theta} \in \mathbb{R}^6$ is the get to designed adaptive update law.

By taking the derivative of (18) with respect to time and using (15), the following expression can be written

$$\dot{\hat{a}}_f + \beta\hat{a}_f = \frac{dx}{dt}\left(W_f\hat{\theta}\right) + \beta W_f\hat{\theta}. \tag{19}$$

The prediction error $\varepsilon(t) \in \mathbb{R}^{2x1}$ is defined as follows

$$\varepsilon \triangleq a_f - \hat{a}_f \tag{20}$$

since $a_f(t)$ and $\hat{a}_f(t)$ are available signals then $\varepsilon(t)$ is also an available signal.

After subtracting (13) from (16), and using (20) the fallowing expression is obtained

$$\dot{\varepsilon} + \beta\varepsilon = \frac{d}{dt}\left(W_f\tilde{\theta}\right) + \beta W_f\theta. \tag{21}$$

The parameter estimation error $\tilde{\theta}(t) \in \mathbb{R}^6$ is defined as follows

$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta} \tag{22}$$

since $\varepsilon(0) = a_f(0) - \hat{a}_f(0) = 0_{2x1}$ and $\tilde{\theta}(0) = 0_{2x1}$, from (21) it is clear that an unmeasurable form of the prediction error $\varepsilon(t) \in \mathbb{R}^2$ can be written as follows

$$\varepsilon = W_f\tilde{\theta}. \tag{23}$$

A least-squares update law is chosen as the preferred method for estimating the unknown parameters, and it is formulated as follows [16]

$$\dot{\hat{\theta}} = P_{ls}W_f^T(I_2 + W_fP_{ls}W_f^T)^{-1}\varepsilon \tag{24}$$

where $k_{ls} \in \mathbb{R}^{6x6}$ is constant, diagonal, positive definite gain matrix and $P_{ls}(t) \in \mathbb{R}^{6x6}$ is covariance matrix. The covariance matrix $P_{ls}(t)$ is generated by the covariance propagation equation, which is described as follows

$$\dot{P}_{ls} = -\left(-\lambda_P P_{ls} + P_{ls}W_f^T W_f P_{ls}\right) \tag{25}$$

where $P_{ls}(0) = k_0I_6$ and $k_0, \lambda \in \mathbb{R}$ are positive constant gains.

Theorem 1: The least-squares update law, as defined in (24) and (25), guarantees that $||\tilde{\theta}(t)|| \to 0$ as $t \to \infty$, provided that the following sufficient conditions are satisfied: (a) the estimation plant is strictly proper, (b) the input is piecewise continuous and bounded, (c)the output of the estimation plant remains bounded, and the following persistence of excitation condition from [16] holds

$$\alpha_1 I_6 \leq \int_t^{t+T} W_f^T(\sigma)W_f(\sigma)d(\sigma) \leq \alpha_2 I_6 \tag{26}$$

where $\alpha_1, \alpha_2, T \in \mathbb{R}$ are positive constants.

Proof: To prove $||\tilde{\theta}(t)|| \to 0$ as $t \to \infty$, the proof of Theorem 2.5.3 from [16] is directly withdrawn. To prove that a sufficient condition (a) is valid; the estimation plant, as described in (10), can be expressed as follows

$$\frac{a_f(s)}{a(s)} = \frac{\beta}{s+\beta} \tag{27}$$

where $a(t)$ is the input of the plant of estimation. From (27), it is demonstrated that the plant is strictly proper. To establish the validity of sufficient condition (b); once the expression in (5) is accepted that acceleration and temperature are a continuous function of time, $a(t)$can be proved to be piecewise continuous and bounded. To prove that sufficient condition (c) is valid; from (10) and standard linear analysis methods are used to prove that $a_f(t)$ and $\dot{a}_f(t)$, and hence the plant estimation output is bounded. Following the proof in [16], the convergence result is presented as follows. ■

IV. SIMULATION RESULTS

The performance of the least squares-based parameter estimation method presented in Section III was evaluated through simulations in MATLAB/Simulink for the SCP actuator model. The model parameters, listed in Table I, are taken from [8] and [17], where they were identified through experimental modeling of an SCP actuator based on the thermal control of Spandex and nylon fibers.

TABLE I: Parameters for SCP Actuator Model

Parameter	**Value**
Actuator's mass m	100×10^{-3}[kg]
Resistance R	1 [Ω]
Thermal mass C_{th}	0.9090 [Ws/°C]
Heat conductivity λ	0.0540 [W/°C]
Damping coefficient b	124.7 [Ns/m]
Spring stiffness k	181.3 [N/m]
Thermal damping coef. α_1	0.08 [Ns/°C]
Thermal stiffness coef. α_2	0.01[N/°C]

The parameter vector θ to be estimated can be calculated as $\theta \triangleq [1.247 \times 10^3, 8 \times 10^{-4}, 1.813, 1 \times 10^{-4}, 5.94 \times 10^{-5}, 1.1 \times 10^{-3}]$ based on the simulation model parameters. Analyzing its nominal values reveals that some parameters are significantly large, while others are very small. Consequently, parameter estimation for the SCP system is a challenging problem due to its highly stiff dynamics.

The gains used in the parameter estimation algorithm were chosen as $\beta = 3$, $k_{ls} = I_6$, and $\lambda_P = 0.8$. When the forgetting factor λ is set to 1, all measurements are equally weighted. While $\lambda = 1$ may be suitable for short-term estimation, on the other hand, it is more appropriate to choose λ less than 1 for long-term systems, as the nominal values of the parameter vector are expected to change over time. Since no prior information is available about the parameters at the beginning, $\hat{\theta}(0) = 0_{1x6}$ is chosen, and the covariance matrix is set as $P_{ls}(0) = 100I_6$. To achieve persistency of excitation, we excited the SCP model with a combination of four periodic signals at different frequencies and phases, as shown in Figure 2. The total voltage applied to the system is $V = \{0.4 + 0.4\sin(2\pi \cdot 6.10^{-3}t - \pi/2)\} + \{0.4 + 0.4\sin(2\pi \cdot 6.10^{-3}t)\} + \{0.4 + 0.4\sin(2\pi \cdot 0.1t)\} + \{0.4 + 0.4\sin(2\pi \cdot 0.1t - \pi/2)\}$.

As a result of the simulation, $\hat{a}_f$ converges to a_f, as shown in Figure 3, and the prediction error ε approaches zero, as seen in Figure 4. The mechanical model parameters, $\hat{\theta}_1$ to $\hat{\theta}_4$, settle into the $\pm 5 \times 10^{-8}$ range within 80s, while the thermal model parameters, $\hat{\theta}_5$ and $\hat{\theta}_6$, settle into the $\pm 2 \times 10^{-5}$ range within 40s.

The nominal values of θ and its least squares-based estimate $\hat{\theta}$ are shown in Figure 5.

V. CONCLUSIONS

In this work, we have presented a least-squares-based estimator for the uncertainties in the thermomechanical and thermoelectric model parameters of super-coiled polymer actuators. The proposed methodology ensures that the parameter estimation error converges to zero provided that a

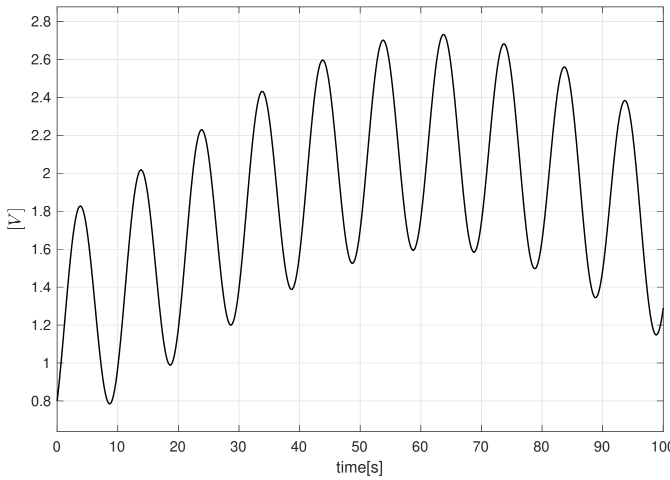

Fig. 2: Driven voltage of the SCP actuator model

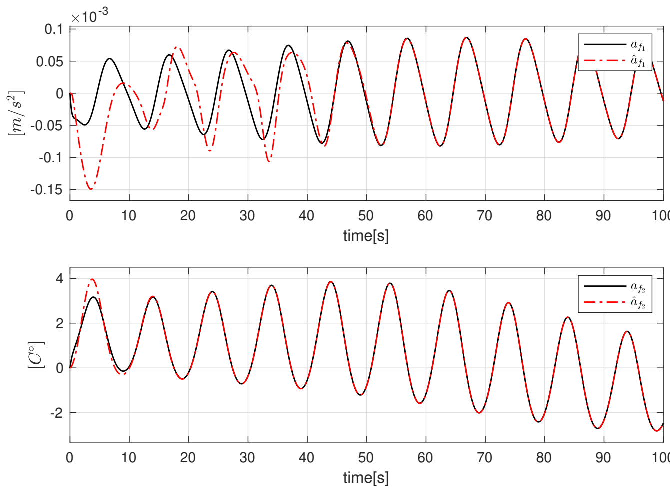

Fig. 3: Comparison of auxiliary filter signal a_f and $\hat{a}_f$

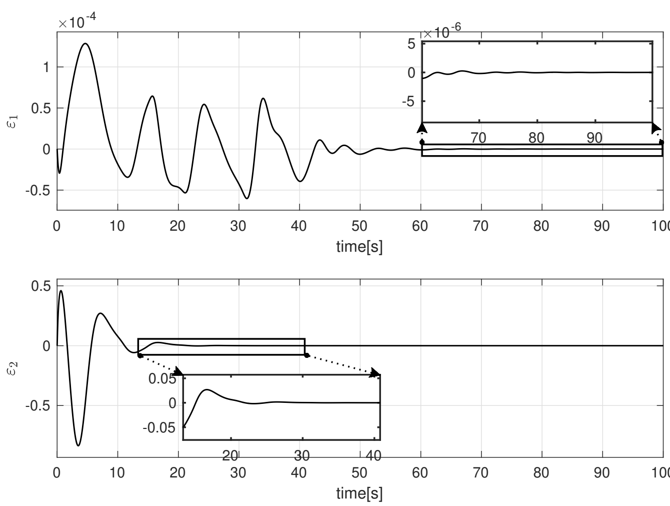

Fig. 4: The prediction error $\varepsilon(t)$

permanent excitation condition is met. The main advantage of the proposed work compared to the literature is the elimination of acceleration measurements. The results of numerical simulation are presented to demonstrate the proof of concept. In numerical simulations, parameter estimation errors are reduced to zero. Future research work will focus on experimental verification.

ACKNOWLEDGMENT

This study was supported by Scientific and Technological Research Council of Turkey (TÜBİTAK) under the Grant Number 124E256. The authors thank to TÜBİTAK for their supports.

REFERENCES

[1] D. Grant and V. Hayward, "Variable structure control of shape memory alloy actuators," *IEEE Control Systems Magazine*, vol. 17, no. 3, pp. 80–88, 1997.

[2] C.-A. Bojan-Dragos, R.-E. Precup, A.-I. Szedlak-Stinean, R.-C. Roman, E.-L. Hedrea, and E. M. Petriu, "Sliding mode and super-twisting sliding mode control structures for sma actuators," in *European Control Conference*, 2023, pp. 1–6.

[3] F. Simone, G. Rizzello, S. Seelecke, S. Borreggine, and D. Naso, "Modeling and identification of a shape memory alloy robotic finger actuator," in *European Control Conference*, 2019, pp. 1097–1102.

[4] C. S. Haines *et al.*, "Artificial muscles from fishing line and sewing thread," *Science*, vol. 343, no. 6173, pp. 868–872, 2014.

[5] Y. A. Tse, K. W. Wong, Y. Yang, and M. Y. Wang, "Novel design of a soft pump driven by super-coiled polymer artificial muscles," in *2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2020, pp. 8789–8794.

[6] D. Rus and M. T. Tolley, "Design, fabrication and control of soft robots," *Nature*, vol. 521, no. 7553, pp. 467–475, 2015.

[7] S. Wang, H. Huang, H. Huang, B. Li, and K. Huang, "A lightweight soft gripper driven by self-sensing super-coiled polymer actuator," *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 2775–2782, 2021.

[8] T. Luong, K. Kim, S. Seo, J. H. Park, Y. Kim, S. Y. Yang, K. H. Cho, J. C. Koo, H. R. Choi, and H. Moon, "Modeling and position control of a high performance twisted-coiled polymer actuator," in *International Conference on Ubiquitous Robots*, 2018, pp. 73–79.

[9] J. Zhang, "Modeling of a bending supercoiled polymer (scp) artificial muscle," *IEEE Robotics and Automation Letters*, vol. 5, no. 3, pp. 3822–3829, 2020.

[10] J. Sun, B. Tighe, Y. Liu, and J. Zhao, "Twisted-and-coiled actuators with free strokes enable soft robots with programmable motions," *Soft Robotics*, vol. 8, no. 2, pp. 213–225, 2021.

[11] M. C. Yip and G. Niemeyer, "On the control and properties of supercoiled polymer artificial muscles," *IEEE Transactions on Robotics*, vol. 33, no. 3, pp. 689–699, 2017.

[12] S. K. Rajendran, Q. Wei, N. Yao, and F. Zhang, "Design, implementation, and observer-based output control of a super-coiled polymer-driven two degree-of-freedom robotic eye," *IEEE Robotics and Automation Letters*, 2023.

[13] ——, "Observability analysis and reduced-order observer design for a super-coiled polymer-driven robotic eye," in *2022 IEEE 61st Conference on Decision and Control (CDC)*. IEEE, 2022, pp. 1385–1391.

[14] ——, "Modeling and learning-based control for super-coiled polymer-driven robotic eye," in *2022 American Control Conference (ACC)*. IEEE, 2022, pp. 3361–3361.

[15] J. Zhang, K. Iyer, A. Simeonov, and M. C. Yip, "Modeling and inverse compensation of hysteresis in supercoiled polymer artificial muscles," *IEEE Robotics and Automation Letters*, vol. 2, no. 2, pp. 773–780, 2017.

[16] S. Sastry and M. Bodson, *Adaptive control: stability, convergence and robustness*. Courier Corporation, 2011.

[17] T. A. Luong, K. H. Cho, M. G. Song, J. C. Koo, H. R. Choi, and H. Moon, "Nonlinear tracking control of a conductive supercoiled polymer actuator," *Soft robotics*, vol. 5, no. 2, pp. 190–203, 2018.

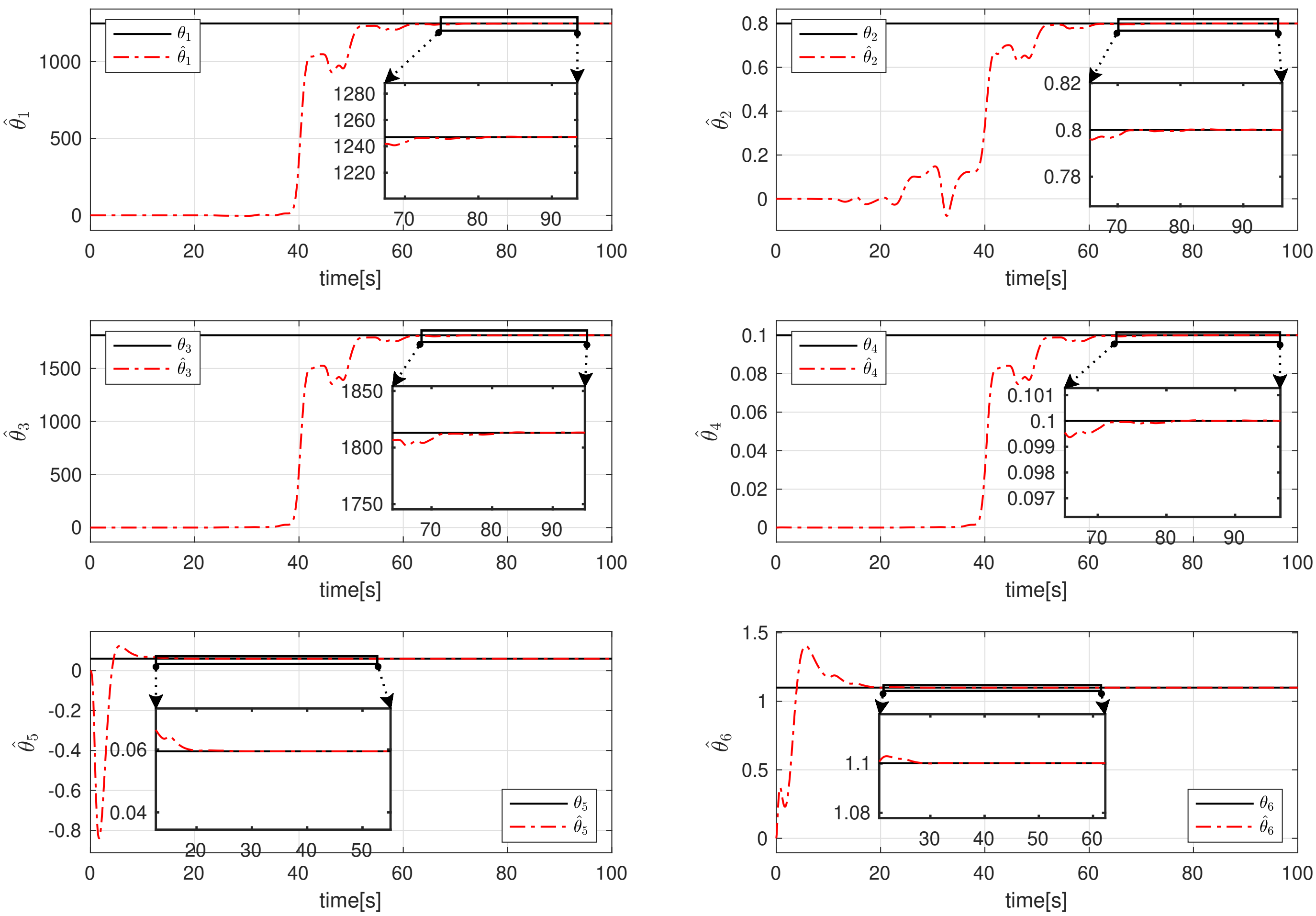

Fig. 5: Comparison of real θ and estimated $\hat{\theta}$