Adaptive Kinematic Control of Robot Manipulators: A Concurrent Learning Based Approach

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***Abstract*— This paper presents an adaptive kinematic control strategy for robotic manipulators, which takes advantage of a concurrent learning-based approach to address kinematic uncertainties. Departing from typical approaches that rely on position-level inverse kinematics, the developed framework operates directly in Cartesian space, thereby reducing computational complexity and mitigating singularity-related issues. The control framework incorporates a concurrent learning-based adaptive update law, providing precise end-effector trajectory tracking and real-time identification of uncertain kinematic parameters under interval excitation condition, which is less stringent than persistent excitation. Stability analysis is conducted using a Lyapunov-based framework which proves the global exponential convergence of both tracking and parameter estimation errors. Numerical simulations validate the effectiveness of the developed approach, accurately demonstrating trajectory tracking and identification of the uncertain kinematic terms.**

I. Introduction

The main control objective for robotic systems is generally specified in the robot's operational space, also known as Cartesian space. While the reference trajectory is typically formulated in Cartesian space, the dynamics of the system with the control input is defined in the joint space. As a result, the control input implementation necessitates transforming the tracking control objective to joint space and requiring the computation of inverse kinematics at each iteration which present commonly known challenges due to performing inverse kinematics calculations at the position level.

For kinematically redundant robotic manipulators, the redundancy problem arises from having an infinite set of inverse kinematic solutions at position level [1], [2], [3]. Despite complicating control design, redundancy can be advantageous by utilizing joint motion within the Jacobian matrix's null-space, thus allowing redundant degrees of freedom (DOF) to address secondary objectives such as optimizing manipulability, avoiding joint limits, preventing obstacles, minimizing impact forces, and reducing potential energy. In [4], a kinematic controller is proposed for kinematically redundant manipulators to address multi-task objectives with varying priorities. The work in [5] introduces a kinematic controller that utilizes prescribed performance to address constraints in joint positioning. In [6], a discrete-time kinematic control strategy is introduced to guarantee the avoidance of obstacles. [7] presents an approach that utilizes a filtered inverse Jacobian matrix designed to overcome kinematic singularities in redundant robotic manipulators, facilitating its implementation for the kinematic controller. The kinematic controllers referenced in [4], [5], [6], [7] require precise knowledge of the robot manipulators' kinematic configuration, which might not be feasible for some real-time applications.

Moreover, parametric uncertainties in the robot's kinematic and/or dynamic models can further complicate the control design process. Various control methods have been proposed to address uncertainties, reduce computational demands, and overcome numerical singularities, thereby enhancing the system's stability and improving performance of the robotic system [8], [9]. Adaptive control is commonly used to compensate for the effects of uncertainties in the dynamics and kinematics of robot manipulators and to ensure trajectory tracking [10]. The work in [11] introduces an accelerated robust adaptive control design to ensure the convergence of the tracking error in a prescribed time and compensate the effects of the uncertainties in the kinematic configuration of the robotic manipulators. In numerous robotic tasks, ensuring accurate tracking is critical, necessitating precise pseudo-inverse Jacobian matrix calculations that are typically utilized in the formulation of kinematic control [12]. Therefore, compensating the effects of the kinematic uncertainties alone may not be adequate for enhanced accuracy; identifying these parametric uncertainties is crucial. Various adaptive control methodologies have concentrated on achieving trajectory tracking and concurrently identifying the precise values of the uncertainties of the system model [13]. Composite adaptive controllers are based on a single channel regressor that utilizes only instantaneous data, which requires the fulfillment of the persistent excitation (PE) condition [14]. Recent studies introduced adaptive controllers for concurrent learning [15] and composite learning [16] that use multi-channel regressors, which require an interval excitation rather

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than a PE condition. By integrating historical data with real-time adaptation, concurrent learning and composite learning-based adaptive controllers improve parameter estimation accuracy, accelerate convergence, and lead to improved tracking performance for uncertain systems [17]. An integral-based concurrent learning adaptive controller has been developed to incorporate recorded input-output data into the update law to estimate uncertain dynamical terms [18] that require the interval condition milder than the PE condition. The work in [19] presents an integral adaptive controller at the torque (dynamic) level that ensures trajectory tracking for the end-effector in Cartesian space and simultaneously addresses the estimation of uncertainties in the system's dynamics, assuming known kinematic parameters, without necessitating the persistent excitation condition.

In this paper, a novel integral concurrent-learning-based kinematic controller at the velocity level is developed to ensure global exponential stability for both the trajectory tracking error and the parameter estimation error in the kinematic model of robot manipulators (e.g., link lengths, link twist angles, joint offsets, etc.), without requiring any dynamic model information or the PE condition. The developed concurrent-learning framework employs multi-channel regressors that incorporate the most informative historical and online data. The most informative historical data are derived from the singular value maximization algorithm, similar to [20]. The presented method enables accurate trajectory tracking by incorporating precise pseudo-inverse Jacobian matrix calculations, commonly used in kinematic control formulations. A Lyapunov-based stability analysis is used to ensure that both the task-space tracking error and the parameter estimation error converge exponentially. Subsequently, numerical simulations are conducted, and the results demonstrate that both the estimation error and the task tracking error converge exponentially, as expected from the Lyapunov stability analysis.

The rest of the paper is outlined as follows. In Section II, the kinematic formulation of the robot manipulator is presented. In Section III and in Section IV, the controller design process and the stability analysis are illustrated. In Section V, simulation results are provided. Finally, in Section VI, concluding remarks are given.

II. Kinematic Formulation

The forward kinematics of an n degree of freedom robot manipulator operating on an n dimensional workspace is expressed as

$$x \triangleq f\left(q\right), \tag{1}$$

where $x : [t_0, \infty) \to \mathbb{R}^n$ denotes the end-effector position vector in Cartesian space, $q : [t_0, \infty) \to \mathbb{R}^n$ represents the joint position vector in joint space, and the mapping $f : \mathbb{R}^n \to \mathbb{R}^n$ specifies the forward kinematic model[1].

[1]Since the number of degrees of freedom of the robot manipulator is equal to the dimension of its task space, it is non-redundant [21].

The velocity kinematics is represented as

$$\dot{x} = J\left(q\right)\dot{q}, \tag{2}$$

where $\dot{x} : [t_0, \infty) \to \mathbb{R}^n$ represents the end-effector velocity vector in Cartesian coordinates, $\dot{q} : [t_0, \infty) \to \mathbb{R}^n$ denotes the joint velocity vector in joint coordinates, and $J\left(q\right) : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ is the Jacobian matrix, expressed as $J\left(q\right) \triangleq \frac{\partial f(q)}{\partial q}$.

Property 1: The Jacobian matrix can be expressed in a linear parameterization form as [22]

$$J\left(q\right)\zeta = W\left(q, \zeta\right)\phi, \tag{3}$$

where $W : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times p}$ represents the regressor matrix formed by known/measurable quantities, while $\phi \in \mathbb{R}^p$ denotes the vector containing constant kinematic parameters, such as link lengths, link twist angles, and joint offsets [23].

Assumption 1: All the kinematic singularities are assumed to be avoided *a priori*. That is, inverse of the Jacobian matrix J is considered to exist for all possible q [24].

Assumption 2: The forward kinematics function f and the Jacobian matrix J remain bounded provided that the joint position vector q is bounded [25][2].

III. Control Development

The control objective is to design a kinematic controller to ensure that the end effector position vector x tracks a given desired trajectory $x_d : [t_0, \infty) \to \mathbb{R}^n$ despite the kinematic model parameter vector ϕ in (3) being uncertain. Additionally, identifying the uncertain kinematic model parameter vector is also aimed. Specifically, the joint velocity vector is considered as the control input $\tau(t) \triangleq \dot{q} \in \mathbb{R}^n$ and an adaptive control approach is to be pursued to ensure both the tracking of the end effector and the identification of the vector of parameters of the kinematic model. Subsequent control design is based on the availability of q and also x which is considered to be obtained through alternative sensing techniques, as the forward kinematics formulation in (1) cannot be used to construct x due to the presence of parametric uncertainties. The desired task space trajectory is considered to be sufficiently smooth in the sense that x_d and $\dot{x}_d$ are bounded functions of time.

To quantify the control goal, the task space tracking error signal $e(t) \in \mathbb{R}^n$ is defined as

$$e \triangleq x_d - x. \tag{4}$$

By setting $\tau = \dot{q}$ in equation (2), the time derivative of (4) is derived as follows

$$\dot{e} = \dot{x}_d - J\tau. \tag{5}$$

In accordance with the subsequent stability assessment, the control signal is designed as

$$\tau = \hat{J}^{-1}\left(\dot{x}_d + \kappa_e e\right), \tag{6}$$

[2]It is noted that Assumption 1 and Assumption 2 are standard assumptions in task space control of robot manipulators [25], [26].

where $\hat{J}(q) \in \mathbb{R}^{n\times n}$ represents the estimated Jacobian matrix, and $\kappa_e \in \mathbb{R}^{n\times n}$ is a tunable, constant, positive definite, diagonal gain matrix. By applying Property 1 and inserting the estimated kinematic parameters into the Jacobian matrix, following expression can be obtained:

$$\hat{J}(q)\,\tau = J(q)\,\tau \mid_{\phi=\hat{\phi}} = W(q,\tau)\,\hat{\phi}, \tag{7}$$

where $\hat{\phi} : [t_0,\infty) \to \mathbb{R}^p$ denotes the vector of estimated uncertain parameters and W was previously introduced in (3). Adding and subtracting $\hat{J}\tau$ in (5), and then substituting (6) into (5), yields the closed-loop dynamics for e as

$$\dot{e} = -\kappa_e e - \tilde{J}\tau, \tag{8}$$

with $\tilde{J}(q) \triangleq J - \hat{J} \in \mathbb{R}^{n\times n}$ that satisfies the subsequent expression

$$\tilde{J}\tau = W(q,\tau)\,\tilde{\phi}. \tag{9}$$

In (9), $\tilde{\phi}(t) \in \mathbb{R}^p$ is the mismatch between the uncertain parameter vector and its estimation and is defined as

$$\tilde{\phi} \triangleq \phi - \hat{\phi}. \tag{10}$$

The adaptive update law is designed as[3]

$$\dot{\hat{\phi}} = -\Gamma W^T e - \kappa_{cl}\Gamma \sum_{i=1}^{N} \mathcal{W}_i^T \left(\mathcal{X}_i - \mathcal{W}_i\hat{\phi}\right), \tag{11}$$

with $\Gamma \in \mathbb{R}^{p\times p}$ being a tunable, constant, positive definite, diagonal gain matrix, and $N \in \mathbb{Z}_{>0}$ denoting the quantity of historical data points in memory. Also in (11), the expressions $\mathcal{X}_i \triangleq \mathcal{X}(t_i) \in \mathbb{R}^n$ and $\mathcal{W}_i \triangleq \mathcal{W}(t_i) \in \mathbb{R}^{n\times p}$ represent the values of the following expressions at the time point $t_i \in [t_0, t]$

$$\mathcal{X} \triangleq \int_{\max(t-\Delta t,0)}^{t} \dot{x}(\sigma)\,d\sigma, \tag{12}$$

$$\mathcal{W} \triangleq \int_{\max(t-\Delta t,0)}^{t} W(q(\sigma),\tau(\sigma))\,d\sigma, \tag{13}$$

where $\Delta t \in \mathbb{R}$ is a constant time step that is used to adjust the window of integration. Using (12) and (13), the update law $\dot{\hat{\phi}}$ can be rewritten as follows

$$\dot{\hat{\phi}} = -\Gamma W^T e - \kappa_{cl}\Gamma \sum_{i=1}^{N} \mathcal{W}_i^T \mathcal{W}_i \tilde{\phi}. \tag{14}$$

IV. Stability Analysis

Theorem 1: The controller proposed in (6), in combination with the novel adaptation law designed in (11), guarantees the globally exponential stability (GES) of the end effector tracking and parameter estimation errors, such that

$$\|s(t)\| \leq \sqrt{\frac{\bar{\lambda}_V}{\underline{\lambda}_V}}\|s(T)\| \exp\left(-\frac{\alpha}{2\bar{\lambda}_V}(t-T)\right), \tag{15}$$

for all $t \in [T,+\infty)$, where $s(t) \triangleq \begin{bmatrix} e^T & \tilde{\phi}^T \end{bmatrix} \in \mathbb{R}^{n+p}$, and $\bar{\lambda}_V$, $\underline{\lambda}_V$, and α are positive constants.

Proof: Let $\mathcal{D} \subset \mathbb{R}^{n+p}$ be a domain containing the origin. Consider $V(s) : \mathcal{D} \to \mathbb{R}$ as a Lyapunov function candidate that is positive definite, continuously differentiable, and radially unbounded, expressed as

$$V \triangleq \frac{1}{2}e^T e + \frac{1}{2}\tilde{\phi}^T \Gamma^{-1}\tilde{\phi}. \tag{16}$$

Following inequalities can be established for the Lyapunov function defined above

$$\underline{\lambda}_V \|s\|^2 \leq V(s) \leq \bar{\lambda}_V \|s\|^2, \tag{17}$$

where $\underline{\lambda}_V \triangleq \frac{1}{2}\min\{1, \underline{\lambda}_\Gamma\}$ and $\bar{\lambda}_V \triangleq \frac{1}{2}\max\{1, \bar{\lambda}_\Gamma\}$. Here, $\underline{\lambda}_\Gamma$ and $\bar{\lambda}_\Gamma \in \mathbb{R}$ denote the minimum and maximum eigenvalues of Γ^{-1}, respectively.

The time derivative of (16) is obtained as

$$\dot{V} = e^T\dot{e} + \tilde{\phi}^T\Gamma^{-1}\dot{\tilde{\phi}}. \tag{18}$$

By substituting (5) for the dynamics of e and (14) along with the time derivative of (10) for the dynamics of $\tilde{\phi}$, and then applying Property 1 along with straightforward simplifications, we obtain the following result

$$\dot{V} = -e^T\kappa_e e - \kappa_{cl}\tilde{\phi}^T\left(\sum_{i=1}^{N}\mathcal{W}_i^T\mathcal{W}_i\right)\tilde{\phi}. \tag{19}$$

Since the history stack is assumed to lack sufficient richness during the initial time period $t \in [t_0, T)$, the summation $\sum_{i=1}^{N}\mathcal{W}_i^T\mathcal{W}_i$ remains positive semi-definite, ensuring that the right hand side of (19) is upper bounded as

$$\dot{V} \leq -e^T\kappa_e e \leq -\underline{\lambda}_{\kappa_e}\|e\|^2, \tag{20}$$

where $\underline{\lambda}_{\kappa_e} \in \mathbb{R}$ denotes the minimum eigenvalue of κ_e. Furthermore, the right hand side of (20) can be upper bounded as

$$\dot{V} \leq -\gamma(s), \quad \forall t \in [t_0, T). \tag{21}$$

where $\gamma : \mathbb{R}^{n+p} \to \mathbb{R}_{\geq 0}$ is a continuous, positive, semi-definite function defined as $\gamma \triangleq \min \underline{\lambda}_{\kappa_e}\|e\|^2$. Using Theorem 8.4 from [27], the inequality in (21) can be employed to conclude that V is bounded and, consequently, s is uniformly bounded. Further analysis can be conducted to establish the global boundedness of the remaining closed-loop signals. Since the controller in (6) accompanied by the adaptive update law in (11) ensures that all signals remain bounded, the occurrence of finite escape phenomena during $\forall t \in [t_0, T)$ is prevented. Moreover, Barbalat's Lemma [27] can be applied to demonstrate the asymptotic stability of the task space tracking error e.

[3] It is noted that, the proposed adaptive controller in (6) requires the estimated Jacobian matrix $\hat{J}$ to be invertible, thus the adaptive update law may be considered to be utilized along with a projection algorithm to ensure that the estimates of the kinematic parameters remain within *a priori* known bounds.

The preceding analysis was conducted under the assumption that the history stack was not rich enough to satisfy the condition $\lambda_{\min}\left\{\sum_{i=1}^{N}\mathcal{W}_i^T\mathcal{W}_i\right\}\geq\lambda_{\mathcal{W}}$ during $t\in[t_0,T)$ for some positive constant $\lambda_{\mathcal{W}}$. However, after some time T, the history stack becomes sufficiently rich to meet the condition $\lambda_{\min}\left\{\sum_{i=1}^{N}\mathcal{W}_i^T\mathcal{W}_i\right\}\geq\lambda_{\mathcal{W}}$. Thus, for all $t\geq T$, $\sum_{i=1}^{N}\mathcal{W}_i^T\mathcal{W}_i$ is positive definite, and the right-hand side of (19) can now be upper bounded as

$$\dot{V}\leq-\underline{\lambda}_{\kappa_e}\|e\|^2-\kappa_{cl}\underline{\lambda}_{\mathcal{W}}\|\tilde{\phi}\|^2. \tag{22}$$

In lieu of (17), the right hand side of (22) can further be upper bounded as

$$\dot{V}\leq\frac{\alpha}{\overline{\lambda}_V}V,\forall t\in[T,\infty), \tag{23}$$

where $\alpha\triangleq\min\{\underline{\lambda}_{\kappa_e},\kappa_{cl}\underline{\lambda}_{\mathcal{W}}\}$. This differential inequality can be solved to obtain the following result

$$V(t)\leq V(T)\exp\left(-\frac{\alpha}{\overline{\lambda}_V}(t-T)\right),\quad\forall t\in[T,\infty), \tag{24}$$

which, in accordance with (17), leads to the upper bound specified in (15). ■

V. Numerical Simulations

A numerical simulation is conducted to demonstrate the effectiveness of the kinematic controller expressed in (6), in conjunction with the integral concurrent learning update law specified in (11), for a planar, two link revolute joint robot arm. The forward kinematic formulation for a two link revolute robot arm is expressed as

$$f(q)=\begin{bmatrix}L_1c_1+L_2c_{12}\\L_1s_1+L_2s_{12}\end{bmatrix},$$

where $L_1,\ L_2\in\mathbb{R}$ represent uncertain link lengths and $c_1\triangleq\cos(q_1)$, $s_1\triangleq\sin(q_1)$, $c_{12}\triangleq\cos(q_1+q_2)$, $s_{12}\triangleq\sin(q_1+q_2)$. For numerical simulation, the two link revolute robot arm is configured with link lengths of $L_1=0.6$ m and $L_2=0.4$ m. The Jacobian matrix $J=\frac{\partial f(q)}{\partial q}\in\mathbb{R}^{2\times2}$ is obtained as

$$J=\begin{bmatrix}-L_1s_1-L_2s_{12} & -L_2s_{12}\\L_1c_1+L_2c_{12} & L_2c_{12}\end{bmatrix}.$$

The expression $J(q)\tau$ can be reformulated into a linearly parameterized form as

$$J(q)\tau=W(q,\tau)\phi,$$

in which $\phi=[L_1\ L_2]^T\in\mathbb{R}^2$ and the regression matrix $W(q,\tau)\in\mathbb{R}^{2\times2}$ is derived as

$$W(q,\tau)=\begin{bmatrix}-s_1\tau_1 & -s_{12}(\tau_1+\tau_2)\\c_1\tau_1 & c_{12}(\tau_1+\tau_2)\end{bmatrix}.$$

The desired task space trajectory was selected as

$$x_d=\begin{bmatrix}0.45+0.1(1-\exp(-0.5t))\cos(\pi t)\\0.45+0.1(1-\exp(-0.5t))\sin(\pi t)\end{bmatrix}\text{m}.$$

The joint positions were initially set to $q(0)=[-0.5\ 0.5]^T$ rad, resulting in $x(0)=[0.9265\ \ -0.2877]^T$ m. Initially, the update law is set to $\hat{\phi}(0)=[0.2\ 0.2]^T$. The control gain matrix was adjusted as $\kappa_e=\text{diag}\{1.5,1.5\}$. The adaptation gain matrix is configured as $\Gamma=\text{diag}\{0.5,0.5\}$, with the learning gain adjusted to $\kappa_{cl}=1\times10^4$. The number of historical data points stored was chosen $N=20$.

The results of the numerical simulation are depicted in Figures 1-3. Specifically, Figure 1 illustrates the task space tracking error e. The results in Figure 1 confirm that the proposed concurrent learning-based adaptive kinematic controller ensures exponential tracking, even with parametric kinematic uncertainties. The designed control input is presented in Figure 2. In Figure 3, the estimations of the kinematic model parameters are depicted. Figure 4 illustrates the parameter estimate errors $\tilde{\phi}(t)$, highlighting their convergence towards their respective actual values.

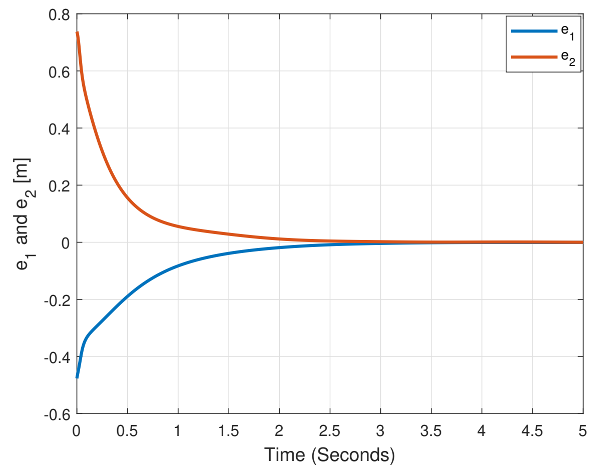

Fig. 1. The Cartesian space tracking error

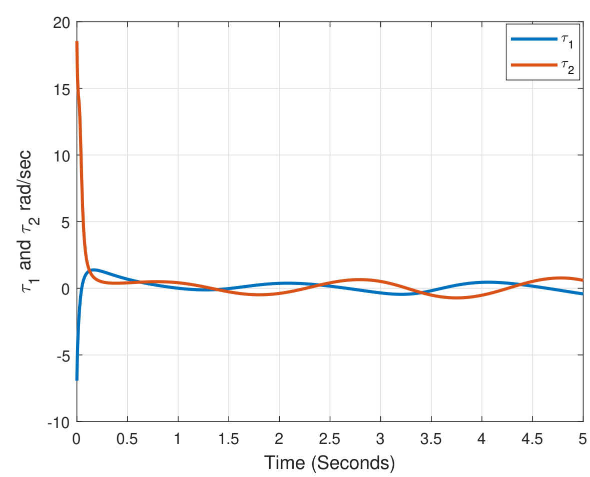

Fig. 2. The control input $\tau(t)$

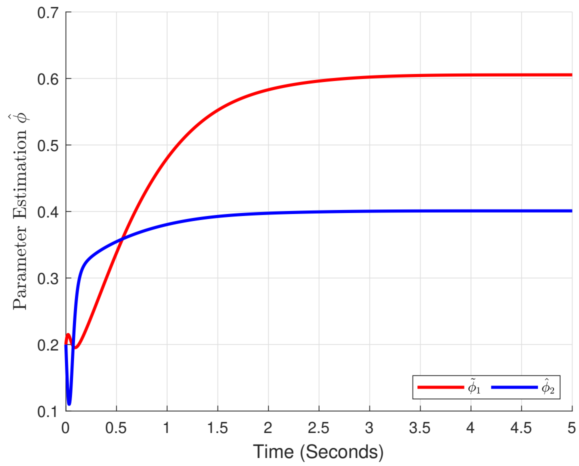

Fig. 3. Estimations of kinematic model parameters

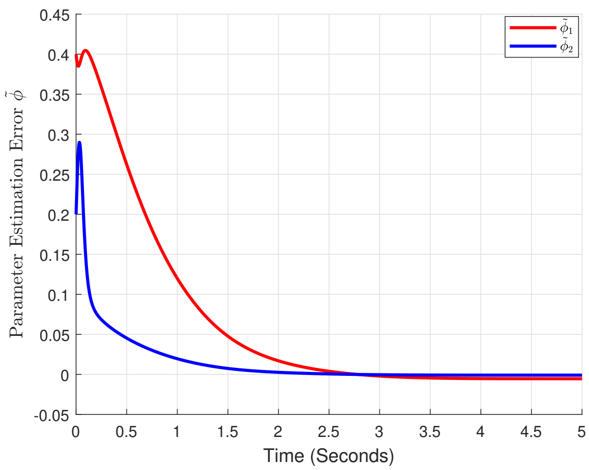

Fig. 4. The entries of the parameter estimation error

VI. Conclusions

This study introduced a concurrent learning-based adaptive kinematic control strategy for robotic manipulators, effectively eliminating the need for inverse kinematics computations in position control. The presented method ensures end-effector tracking while simultaneously identifying uncertain kinematic parameters to improve tracking accuracy. Lyapunov-based stability analysis is used to guarantee global exponential convergence of tracking and estimation errors, which was further validated through numerical simulations. The results demonstrate the efficacy of the proposed approach in improving tracking performance under kinematic uncertainties. Future research directions may include extending this framework for redundant manipulators with the addition of subtask objectives and experimental validation on physical robotic systems.

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