SDRE-Based Estimation and Control: A Comparative Study of Kalman and H-infinity Filters in Nonlinear Systems

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***Abstract*— This paper presents a comparative analysis of state estimation techniques, SDRE-based Kalman and H-infinity filters, $\mathcal{H}_\infty$, within the context of nonlinear systems controlled via the State-Dependent Riccati Equation (SDRE) framework. While the SDRE-based Kalman filter is effective under Gaussian noise assumptions, it can underperform in environments with non-Gaussian disturbances. The SDRE-based $\mathcal{H}_\infty$ filter, on the other hand, offers a robust alternative by minimizing the worst-case estimation error without assuming any noise statistics. To evaluate both filters, we conduct simulation studies on two real-world scenarios: temperature control in a data center and interference mitigation in a wireless cellular network. Results demonstrate that the $\mathcal{H}_\infty$ filter outperforms its Kalman counterpart in the presence of non-Gaussian noise, validating its suitability for robust estimation in practical applications.**

I. INTRODUCTION

Among various nonlinear control methodologies, the SDRE technique has attracted considerable interest due to its conceptual alignment with the Linear Quadratic Regulator (LQR), offering both intuitive implementation and computational efficiency (e.g., [1], [2]). By restructuring the nonlinear system into a linear-like form using state-dependent coefficient (SDC) matrices, the SDRE approach enables pointwise solution of Riccati equations, facilitating the design of feedback controllers that preserve the nonlinear characteristics of the system. This makes SDRE particularly effective for complex, time-varying systems where traditional linearization may be inadequate. Furthermore, recent enhancements, such as the incorporation of policy iteration techniques, have improved its capacity for optimal control in nonlinear contexts [3], [4]. These developments have also paved the way for robust extensions, including its integration with integral sliding mode control [5], drawing inspiration from analogous efforts to strengthen classical LQR strategies for linear systems [6].

State estimation has an important role in various fields, which refers to reconstructing states of the system based on the available measurements. In many practical applications, not all state variables are measurable, which is where state estimation becomes crucial in determining all the states. Many real-world systems have nonlinear dynamics, where conventional control and estimation tehniques such as LQR and Kalman Filter (KF) fail to deliver satisfactory results. Therefore, more emphasis is placed on researching techniques that enable operation with nonlinear systems. Standard estimation methods often rely on linearizing the nonlinear system, which can lead to reduced stability and performance. The SDRE approach avoids this by representing the nonlinear system in a linear-like form using SDC matrices. One limitation of the KF is that it assumes the process and measurement noise are Gaussian. When this assumption does not hold, such as in the presence of non-Gaussian noise, the KF may provide inaccurate results. In such cases, the $\mathcal{H}_\infty$ filter offers a more robust alternative, as it does not require any statistical assumptions about the noise. Instead, it aims to minimize the worst-case amplification of the estimation error, which is why it is often referred to as the robust version of the Kalman filter.

A distributed SDRE control method for nonlinear interconnected systems was tested using two different techniques, both supported by a stability analysis. The results showed good tracking performance for both methods, with differences in oscillations and control effort required [7]. A decentralized version of the SDRE method, applied to a nonlinear interconnected system and also supported by stability analysis, showed good efficiency and confirmed that the overall system is asymptotically stable [8]. In hydrological modeling, a comparison between the KF and the $\mathcal{H}_\infty$ filter showed that the KF performs better when the noise is Gaussian, while the $\mathcal{H}_\infty$ filter gives better results in the presence of non-Gaussian noise or uncertain initial conditions. It also converges faster in these situations [9]. Another comparison was conducted for a twin rotor multiple-input multiple-output (MIMO) system, where the $\mathcal{H}_\infty$ filter outperformed the KF under worst-case noise conditions [10]. Both filters have also been used for vehicle state and parameter identification [11]. The $\mathcal{H}_\infty$ filter has also been applied in feedback control of asynchronous motors in electric trains, demonstrating accurate reference tracking [12]. An improved version of the $\mathcal{H}_\infty$ filter was proposed in [13] for estimating the state of charge in lithium-ion batteries. Additionally, the SDRE-based $\mathcal{H}_\infty$ filter has shown promising performance in terms of estimation accuracy and convergence [14]. The use of $\mathcal{H}_\infty$ control via the SDRE approach has also provided efficient and reliable results [15], [16]. In contrast to previous studies that focus on theoretical development of SDRE-based $\mathcal{H}_\infty$ control [15], [16], or robust $\mathcal{H}_\infty$ filtering under system uncertainty [14], our work uniquely combines SDRE-based Kalman and $\mathcal{H}_\infty$ filters with SDRE control and performs a comparative evaluation under both Gaussian and non-

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Gaussian noise using two applied nonlinear scenarios.

The main contributions of this paper are twofold. First, we investigate the use of the $\mathcal{H}_\infty$ filter within the SDRE-based control framework for state estimation in continuous-time nonlinear systems. Second, we conduct a detailed comparative analysis between the SDRE-based $\mathcal{H}_\infty$ filter and the SDRE-based Kalman filter (SDRE-KF) when used in conjunction with SDRE control. The performance of both approaches is evaluated through two case studies: temperature regulation in data centers and interference modeling in wireless networks.

The rest of the paper is organized as follows: Section II introduces the SDRE-based control approach for continuous-time nonlinear systems. In Section III, the SDRE-based estimation framework, including both the KF and the $\mathcal{H}_\infty$ filter is presented. Section IV provides a detailed performance comparison of the filters through simulation results. Finally, Section V summarizes the key findings of the paper.

II. SDRE BASED CONTROL FOR CONTINUOUS-TIME SYSTEMS

The SDRE method offers a structured and computationally tractable approach for designing feedback controllers for nonlinear systems (see, e.g., [2]–[4], [17]-[18]). Its fundamental principle lies in transforming the nonlinear system dynamics into a linear-like representation by employing SDC matrices:

$$\dot{x} = A(x)x + B(x)u \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $A(x) \in R^{n\times n}$ is the state-dependent system matrix, and $B(x) \in R^{n\times m}$ is the state-dependent input matrix.

This form enables the application of Riccati-based optimization to nonlinear systems by solving a modified version of the standard LQR problem. The performance objective typically involves minimizing an infinite-horizon cost function defined as:

$$J = \frac{1}{2}\int_0^\infty \left(x^T Q(x)x + u^T R(x)u\right) dt, \tag{2}$$

where $Q(x) = D^T(x)D(x) \geq 0$ and $R(x) > 0$ ensure positive semidefiniteness and positive definiteness, respectively.

The resulting feedback control law takes the form:

$$u(x) = -K(x)x = -R^{-1}(x)B^T(x)P(x)x, \tag{3}$$

where $P(x)$ solves the following state-dependent algebraic Riccati equation [1]:

$$\begin{aligned} &P(x)A(x) + A^T(x)P(x) \\ &\quad - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \end{aligned} \tag{4}$$

To ensure the feasibility of the solution, controllability of the pair $(A(x), B(x))$ must be verified. This is typically assessed through the rank condition of the controllability matrix:

$$\operatorname{rank}\left[B(x)\ A(x)B(x)\ \cdots\ A(x)^{n-1}B(x)\right] = n, \tag{5}$$

where n is the number of states in the system.

An important feature of the SDRE method is the flexibility in SDC matrix selection. For a given nonlinear vector field $f(x)$, multiple factorizations may exist, satisfying $f(x) = A_1(x)x = A_2(x)x$. This non-uniqueness enables convex combinations such as

$$A(x, \alpha) = \alpha A_1(x) + (1-\alpha)A_2(x) \tag{6}$$

to be valid factorizations as well, providing additional tuning possibilities [18].

While SDRE resembles the LQR framework in its use of Riccati-based control design [18], it should be noted that it is not derived from the Hamilton–Jacobi–Bellman equation. As such, it does not guarantee global optimality for nonlinear systems [3], [4], but nonetheless provides an effective and intuitive control strategy in many practical applications.

III. SDRE BASED STATE ESTIMATION FOR CONTINUOUS-TIME SYSTEMS

A. SDRE based Kalman filter

The SDRE framework has also been effectively adapted for nonlinear state estimation. In this context, the SDRE-KF emerges as a natural generalization of the classical KF for nonlinear systems. While the standard KF is optimal for linear systems with Gaussian noise, it requires system linearization when applied to nonlinear models, which can compromise estimation performance. In contrast, the SDRE-KF directly incorporates the nonlinear nature of the system through SDC matrices, allowing more accurate and robust estimation without resorting to local linearization.

The general form of the nonlinear system is:

$$\begin{aligned} \dot{x}(t) &= A(x)x(t) + B(x)u(t) + w(t), \\ y(t) &= C(x)x(t) + v(t), \end{aligned} \tag{7}$$

where $y(t) \in R^p$ is the output measurement vector, and $C(x) \in R^{p\times n}$ is the state-dependent output matrix that relates the state vector to the measurable outputs. The terms $w(t)$ and $v(t)$ denote zero-mean Gaussian process and measurement noise with state-dependent covariance matrices $Q(x)$ and $R(x)$, respectively:

$$w(t) \sim \mathcal{N}(0, Q(x)), \quad v(t) \sim \mathcal{N}(0, R(x)).$$

In the special case where system matrices are constant and independent of the state, the SDRE-KF reduces to the conventional linear KF. For general nonlinear dynamics, the SDRE-KF is governed by the following equation

$$\dot{\hat{x}}(t) = A(x)\hat{x}(t) + B(x)u(t) + K_f(x)\left[y(t) - C(x)\hat{x}(t)\right], \tag{8}$$

where the filter gain is expressed as

$$\begin{aligned} K_f(x) &= P(x)C^T(x)R^{-1}(x), \\ 0 &= P(x)A^T(x) + A(x)P(x) \\ &\quad - P(x)C^T(x)R^{-1}(x)C(x)P(x) + Q(x). \end{aligned} \tag{9}$$

To ensure filter effectiveness, the nonlinear system must be observable at every operating point. This is verified by checking the rank of the state-dependent observability matrix:

$$\text{rank}\left[C^T(x)\ (C(x)A(x))^T \dots (C(x)A^{n-1}(x))^T\right]^T = n, \tag{10}$$

where n is the number of state variables [19].

B. SDRE-Based $\mathcal{H}_\infty$ Filter

The KF is a widely used tool for state estimation. However, its performance can degrade in the presence of non-Gaussian noise, as it relies on the assumption of Gaussian statistics. In contrast, the $\mathcal{H}_\infty$ filter is a robust estimation approach that minimizes the worst-case estimation error without relying on any statistical assumptions about the process and measurement noise [20].

Consider the general continuous-time nonlinear system:

$$\begin{aligned} \dot{x}(t) &= A(x)x(t) + B(x)u(t) + w(t), \\ y(t) &= C(x)x(t) + v(t), \\ z(t) &= L(x)x(t) \end{aligned} \tag{11}$$

where $w(t)$ and $v(t)$ are the process and measurement noise, respectively, with associated covariance matrices $Q(x)$ and $R(x)$. The matrix $L(x)$ defines the quantity $z(t)$ to be estimated.

In the game-theoretic formulation of $\mathcal{H}_\infty$ filtering, the objective is to minimize the following cost function [21]:

$$J_1 = \frac{\int_0^T \|z(t) - \hat{z}(t)\|_S^2\, dt}{\|x(0) - \hat{x}(0)\|^2_{P_0^{-1}} + \int_0^T \left(\|w(t)\|^2_{Q^{-1}} + \|v(t)\|^2_{R^{-1}}\right) dt}, \tag{12}$$

where P_0, Q, R, and S are positive definite weighting matrices. The design goal is to ensure that:

$$J_1 < \frac{1}{\theta}, \tag{13}$$

where θ is a user-defined performance bound.

The SDRE-based $\mathcal{H}_\infty$ filter equations are:

$$\begin{aligned} \dot{\hat{x}}(t) &= A(x)\hat{x}(t) + B(x)u(t) + K_f(x)\left[y(t) - C(x)\hat{x}(t)\right], \\ \hat{z}(t) &= L(x)\hat{x}(t), \\ K_f(x) &= P(x)C^T(x)R^{-1}(x), \\ 0 &= P(x)A^T(x) + A(x)P(x) - K_f(x)C(x)P(x) \\ &\quad + \theta P(x)L^T(x)S(x)L(x)P(x) + Q(x). \end{aligned} \tag{14}$$

The inclusion of the θ term introduces robustness against system uncertainties. As θ increases, the solution $P(x)$ typically becomes larger, leading to a higher filter gain $K_f(x)$, and making the filter more responsive to measurement updates. When θ is set to zero, the $\mathcal{H}_\infty$ filter reduces to the KF, highlighting its interpretation as a minimax estimator under infinite performance bounds. A necessary condition for convergence is that the matrix $P(x)$ remains positive definite [20]. Although a fixed value of $\theta = 0.1$ was used in our simulations, an adaptive mechanism that dynamically adjusts θ based on real-time noise characteristics could further improve estimation robustness. This represents a promising direction for future research and practical implementation.

IV. SIMULATION RESULTS

A. Temperature Model of the Data Center Room

This subsection considers the problem of temperature regulation in a data center room, where accurate knowledge of thermal states is crucial. The system is modeled with two state variables: server temperature (T_{server}) and room temperature (T_{room}). The thermal dynamics are governed by the following set of differential equations [22]:

$$\begin{aligned} k_{server}\dot{T}_{server} &= W - kA(T_{server} - T_{room}) \\ k_{room}\dot{T}_{room} &= kA(T_{server} - T_{room}) \\ &\quad + \dot{m}_{in}c_{zr}T_{in} - \dot{m}_{out}c_{zr}T_{room} \end{aligned} \tag{15}$$

Here, k_{server} and k_{room} represent the thermal capacities of air in the server and room, respectively. The term k denotes the thermal conductivity of aluminum (assuming the servers are enclosed in aluminum cases), and A is the surface area of the server. Airflow into and out of the room is represented by $\dot{m}_{in}$ and $\dot{m}_{out}$, respectively, and c_{zr} is the specific heat capacity of air. The variable W denotes the thermal power generated by active computer equipment.

The control input U corresponds to the rotational speed of a fan (in RPM), which regulates airflow into the room. Since the room is enclosed, it is assumed that the same amount of air exits the room, i.e., $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. The airflow rate is modeled as:

$$\dot{m} = k_{vent}U \tag{16}$$

where k_{vent} is a constant relating fan speed to airflow. Substituting this into (15) yields the simplified system:

$$\begin{aligned} k_{server}\dot{T}_{server} &= W - kA(T_{server} - T_{room}) \\ k_{room}\dot{T}_{room} &= kA(T_{server} - T_{room}) \\ &\quad + k_{vent}c_{zr}(T_{in} - T_{room})U \end{aligned} \tag{17}$$

The model can be factorized into a state-dependent linear-like representation suitable for SDRE control and estimation:

$$\begin{aligned} \begin{bmatrix} k_{server}\dot{T}_{server} \\ k_{room}\dot{T}_{room} \end{bmatrix} &= \begin{bmatrix} -kA & kA \\ kA & -kA \end{bmatrix} \begin{bmatrix} T_{server} \\ T_{room} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ k_{vent}c_{zr}(T_{in} - T_{room}) \end{bmatrix} U + \begin{bmatrix} W \\ 0 \end{bmatrix} \end{aligned} \tag{18}$$

Simulation parameters are: $k = 237\ \frac{W}{mK}$, $A = 7\ m^2$, $k_{vent} = 6\ \frac{kg}{s \cdot rpm}$, $c_{zr} = 1.005\ \frac{J}{kgK}$, $T_{in} = 20\ °C$, $k_{server} = 10050\ \frac{J}{kgK}$, and $k_{room} = 351750\ \frac{J}{kgK}$. SDRE control uses weighting matrices $Q = \begin{bmatrix} 100000 & 0 \\ 0 & 100000 \end{bmatrix}$ and $R = 1$, with process and measurement noise covariances $Q_{\text{disturbance}} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}$, and $R_{\text{noise}} = 0.1$. The $\mathcal{H}_\infty$ filter employs $L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $S = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$, and $\theta = 0.1$.

Measurement noise is modeled as zero-mean Gaussian, while process noise W is modeled as a non-Gaussian step function with values ranging from 50 kW to 200 kW. This choice allows for evaluating the robustness of the $\mathcal{H}_\infty$ filter under realistic conditions where noise is not purely Gaussian. The sampling time is set to 0.01 s. In practice, only room temperature is assumed to be measurable, which leads to the output matrix $C = [0 \quad 1]$. The initial state is $[30; 20]$, and the target final state is $[50; 35]$.

Figs 1–2 show the true and estimated states obtained using SDRE-based Kalman and $\mathcal{H}_\infty$ filters, including the influence of process and measurement noise. These figures illustrate that both filters produce similar estimation results under the given conditions. However, to further investigate the robustness of each approach, particularly in the presence of non-Gaussian noise, a comparative analysis based on maximum estimation error was performed using 30 independent simulations. The metric e_{max} is defined as:

$$e_{\max} = e_{\mathrm{KF,max}} - e_{\mathcal{H}_\infty,\mathrm{max}} \tag{19}$$

where $e_{\mathrm{KF,max}}$ and $e_{\mathcal{H}_\infty,\mathrm{max}}$ represent the respective maximum estimation errors for the Kalman and $\mathcal{H}_\infty$ filters. The histogram in Fig. 3 shows that e_{max} is mostly positive, confirming that the SDRE-based $\mathcal{H}_\infty$ filter outperforms the SDRE-KF in scenarios involving non-Gaussian noise. This is expected, as the $\mathcal{H}_\infty$ filter is inherently designed to minimize the worst-case estimation error and does not rely on Gaussian noise assumptions, making it more robust to such disturbances.

Finally, Figs. 4 and 5 illustrate the impact of increasing the performance bound parameter θ on mean-squared error (MSE) and mean-absolute error (MAE), demonstrating how tuning θ can enhance the estimator's robustness and performance. As previously explained, larger values of θ typically lead to higher gains $K_f(x)$, making the filter more responsive to measurements and thus improving estimation under uncertainty.

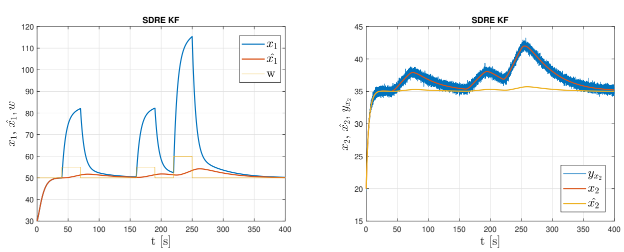

Fig. 1. True and estimated states with process and measurement noise - SDRE KF

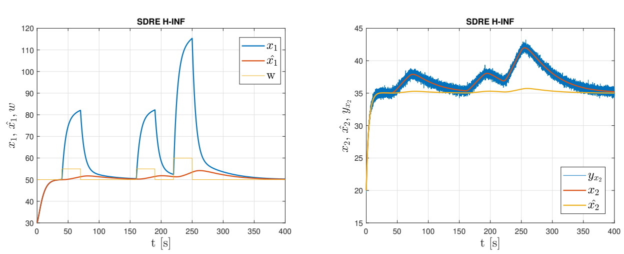

Fig. 2. True and estimated states with process and measurement noise - $\mathcal{H}_\infty$ filter

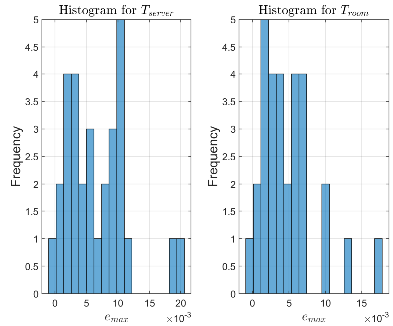

Fig. 3. Histogram of the maximum error difference between the KF and the $\mathcal{H}_\infty$ filter with non-Gaussian process noise for T_{server} and T_{room}

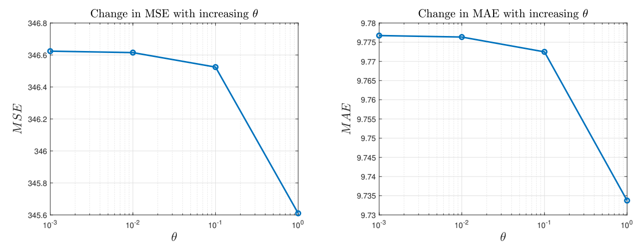

Fig. 4. Change in MSE and MAE with increasing θ for the T_{server} state

B. Interference in a Wireless Cellular Network

Consider the interference experienced by a mobile user device from two base stations (BS_1 and BS_2) in a wireless cellular network. The state variables, I_1 and I_2, represent the received interference power levels from each base station.

The state-dependent matrix $A(x)$ is defined as:

$$A(x) = \begin{bmatrix} -\alpha - \gamma \cdot I_1 & \beta_{12} \cdot I_2 \\ \beta_{21} \cdot I_1 & -\alpha - \gamma_2 \cdot I_2 \end{bmatrix} \tag{20}$$

where α is the basic wireless signal attenuation factor, γ_1 and γ_2 are quantified nonlinear self-interference, and β_{12} and β_{21} represent cross-interference effects between base stations.

The matrix $B(x)$ is given by:

$$B(x) = \begin{bmatrix} \sigma_1 \cdot \sqrt{I_1} & 0 \\ 0 & \sigma_2 \cdot \sqrt{I_2} \end{bmatrix} \tag{21}$$

where σ_1 and σ_2 are sensitivity coefficients indicating how transmission power adjustments affect interference at the user device.

The standard 5G Urban Macro wireless signal propagation model is used for the path-loss model. Given a base station

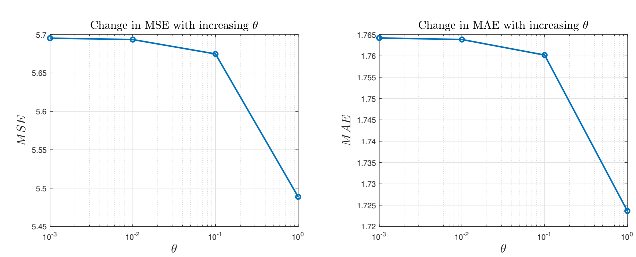

Fig. 5. Change in MSE and MAE with increasing θ for the T_{room} state

transmit power of 10 W (40 dBm), a carrier frequency (f_{GHz}) of 3.5 GHz, and a shadowing of 8 dB, the distances (d_{m}) from BS_1 and BS_2 to the user device are assumed to be 250 m and 500 m, respectively.

The path-loss is calculated as:

$$PL(\mathrm{dB}) = 28 + 22\log_{10}(f_{\mathrm{GHz}}) + 35\log_{10}(d_{\mathrm{m}}) \qquad (22)$$

which results in values of approximately 132.17 dB for BS1 and 142.4 dB for BS2. The initial state vector is assumed to be:

$$x_0 = \begin{bmatrix} 6.05 \cdot 10^{-13} \\ 5.75 \cdot 10^{-14} \end{bmatrix} \qquad (23)$$

with parameters: $\alpha = 0.05$, $\gamma = 10^{12}$, $\beta_{12} = 2 \cdot 10^{11}$, $\beta_{21} = 3 \cdot 10^{11}$, $\gamma_2 = 5 \cdot 10^{12}$, $\sigma_1 = 2 \cdot 10^{-14}$, and $\sigma_2 = 1 \cdot 10^{-14}$, leading to the following state-dependent matrices:

$$\begin{aligned} A(x) &= \begin{bmatrix} -0.05 - 10^{12} \cdot I_1 & 2 \cdot 10^{11} \cdot I_2 \\ 3 \cdot 10^{11} \cdot I_1 & -0.05 - 5 \cdot 10^{12} \cdot I_2 \end{bmatrix}, \\ B(x) &= \begin{bmatrix} 2 \cdot 10^{-14} \cdot \sqrt{I_1} & 0 \\ 0 & 1 \cdot 10^{-14} \cdot \sqrt{I_2} \end{bmatrix} \end{aligned} \qquad (24)$$

The SDRE control weights are $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$. For estimation, $Q_{\mathrm{disturbance}} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$ and $R_{\mathrm{noise}} = 0.1$. The $\mathcal{H}_\infty$ filter uses $L = I$ in eq. (11), $S = 0.01 \cdot I$ in eq. (12), and $\theta = 0.1$ in eq. (13). Simulations use both Gaussian and non-Gaussian step noise, with sampling time 0.01 s. Only I_1 is measured, so $C = [1 \;\; 0]$. Final state is $[1.06 \cdot 10^{-13}, 1.1 \cdot 10^{-14}]^T$.

Figs. 6–9 show the true and estimated states for both state variables using the SDRE-KF and SDRE-based $\mathcal{H}_\infty$ filter. The results indicate that both filters yield comparable estimation performance under the examined conditions. The resulting histograms of $e_{\max}$ for each state variable are presented in Figs. 10–11. It can be observed that there are more positive errors in the case of non-Gaussian noise, especially for the variable I_1, which confirms the assumption regarding the $\mathcal{H}_\infty$ filter. This is expected, as the $\mathcal{H}_\infty$ filter is designed for worst-case errors and is more robust to non-Gaussian noise, especially for I_1.

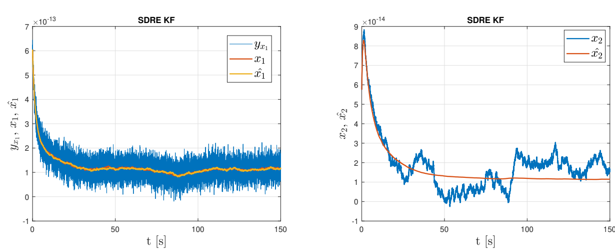

Fig. 6. True and estimated states with process and Gaussian measurement noise - SDRE KF

Both the SDRE-KF and SDRE-$\mathcal{H}_\infty$ filters involve solving a Riccati equation at each time step, with a computational complexity of $\mathcal{O}(n^3)$, where n is the number of system states. This complexity arises from matrix multiplications

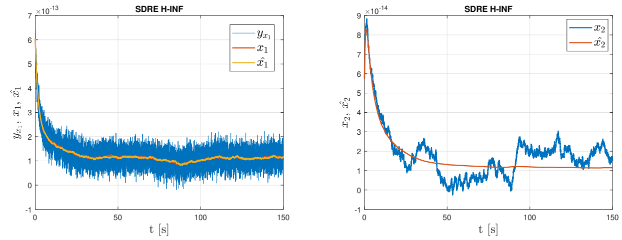

Fig. 7. True and estimated states with process and Gaussian measurement noise - $\mathcal{H}_\infty$

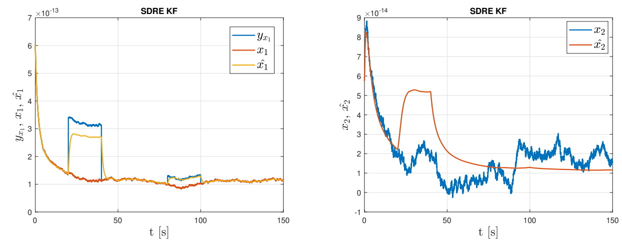

Fig. 8. True and estimated states with process and non-Gaussian measurement noise - SDRE KF

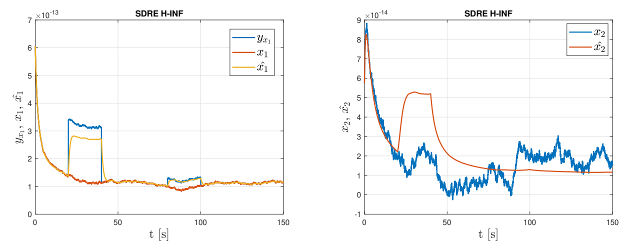

Fig. 9. True and estimated states with process and non-Gaussian measurement noise - $\mathcal{H}_\infty$

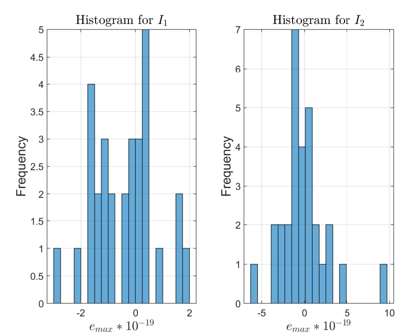

Fig. 10. Histogram of the maximum error difference between the KF and the $\mathcal{H}_\infty$ filter with Gaussian noise for state variables I_1 and I_2

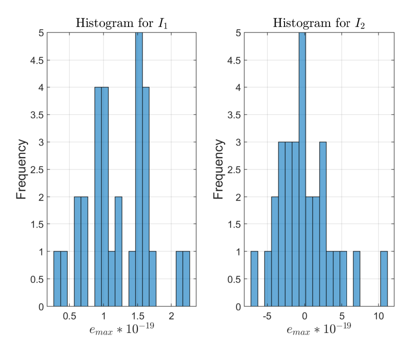

Fig. 11. Histogram of the maximum error difference between the KF and the $\mathcal{H}_\infty$ filter with non-Gaussian noise for state variables I_1 and I_2

and inversions required in solving the state-dependent Riccati equations. In practice, the computations remain tractable for systems of moderate dimension, making both estimation methods suitable for real-time implementation in many practical nonlinear control applications.

V. CONCLUSIONS

This paper presented a comparative study of state estimation strategies for continuous-time nonlinear systems under the SDRE framework. Both the SDRE-KF and the SDRE-based $\mathcal{H}_\infty$ filter were applied alongside SDRE-based control to assess their performance under different noise characteristics.

Through two real-world examples, temperature regulation in a data center and interference management in a wireless cellular network, the SDRE-based $\mathcal{H}_\infty$ filter demonstrated improved robustness, particularly in scenarios involving non-Gaussian disturbances. These results confirm its value as a complementary tool to the SDRE-KF in nonlinear estimation tasks.

These findings indicate that the SDRE-KF is well suited for systems with accurately known dynamics and Gaussian noise, providing efficient and precise state estimation. However, from a practical standpoint, in scenarios where robustness is essential, such as in the presence of model uncertainty or unreliable sensor data, the SDRE-based $\mathcal{H}_\infty$ filter emerges as a more reliable alternative. This observation aligns with findings in the literature, which suggest that $\mathcal{H}_\infty$ filter is better equipped to handle non-Gaussian disturbances and bounded uncertainties.

This study assumes perfect knowledge of the SDC matrices $A(x)$, $B(x)$, and $C(x)$ used in both SDRE-based Kalman and $\mathcal{H}_\infty$ filters. In real-world settings, however, modeling inaccuracies and parameter uncertainties are common. The SDRE-KF is particularly sensitive to such uncertainties due to its reliance on accurate system dynamics and noise statistics. By contrast, the SDRE-based $\mathcal{H}_\infty$ filter is inherently more robust, as it does not depend on precise noise models and aims to minimize the worst-case estimation error. Nonetheless, large deviations in the SDC matrices may still impact estimation performance. Future research could address this limitation by developing robust or adaptive extensions of the SDRE framework that account for model uncertainty explicitly.

Future work may include advancing optimal $\mathcal{H}_\infty$ estimation within the SDRE framework [4], developing adaptive θ tuning based on real-time noise, and optimizing SDC matrix design with observability constraints.

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