Gain-Switching UIO against Disturbance with Lossy Multiple Measurement

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***Abstract*— This paper studies a switching state estimation technique to address temporary measurment signal loss. Under certain conditions on system matrices and a signal loss pattern, we propose a UIO (unknown input observer) with gain switching against unknown disturbance input and lossy multiple output, provided that the signal loss is detected in the local side in real time. It turns out that by the proposed method a weighted square norm of the estimation error converges to zero for any such disturbance and observation signal loss. The switching gains are designed by solving LMIs (linear matrix inequalities) simultaneously. Numerical simulation has been carried out to evaluate the proposed method in comparison with a DO(disturbance observer) with gain switching.**

I. INTRODUCTION

Recent progress in information/communication technology has made it possible to acquire meaurement data via networks from various sensors in a remote place [1]. In the real world environment, measurement may sometimes fail temporarily and signal transmission may be lost in best-effort-type communication channels. Such a case arises, for example, due to occlusion or environmental lighting conditions when reading coordinates out of a camera image, or packet loss through wireless transmission. Though it is enough to re-send the lost data in many applications, such resending is undesirable in real-time processes like networked control, since it causes irregular delay.

One of the authors has proposed a gain-switching state observer to maintain state estimation even when observation signal is lost irregularly [2], [3]. This technique has to use the input signal sent to the remote system without loss (Fig. 1), which is restrictive and may not be applicable in bi-directional communications where signal loss occurs in both input and output channels. Furthermore, other than command input, some control systems are subject to disturbance input, which is *always unknown*; see Fig. 2 for an image of disturbance with lossy multiple measurement.

The objective of the present paper is to propose yet another technique to address this issue by introducing gain-switching to what is called an unknown input observer (UIO) [4], [5]. The UIO is also effective for an attack detection in security issues [6] but gain switching has not been considered there. Related works on the gain-switching observer are also found in [7], [8], [9], [10], [11].

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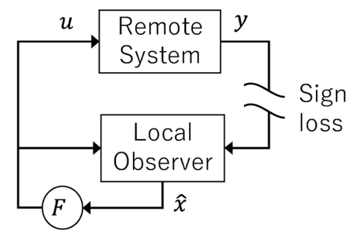

Fig. 1. Conventional observer in networked control

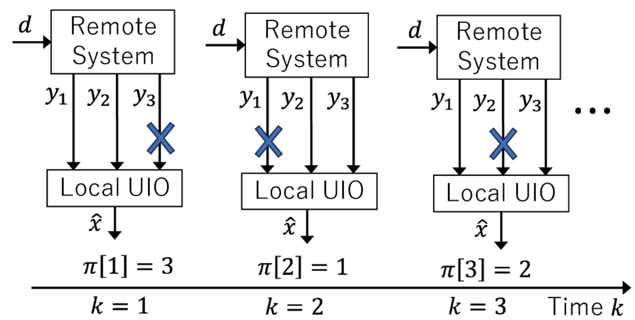

Fig. 2. Image of disturbance and lossy multiple measurement

A disturbance observer (DO) (Fig. 3) is also known as a technique for state estimation under unknown disturbance, when a generation model of the disburbance is assumed to be known [12], [13], [14]. We use a model augmented from the plant and the generation model, and the DO is effective when the disturbance is constant, for example, or at least slow-varying. By introducing gain switching, we can also address output signal loss by this method [3]. We will compare the DO with the proposed method in IV.

Fault tolerance issues have a long history of study which addresses the case when some communication channels cease permanently due to a fault. On the other hand, the present paper deals with a more general problem where lost channels change with time.

In the remainder of the paper, we first formulate our problem in II and discuss some related works. We then present our proposed method in III and evaluate the method with numerical simulation in IV.

Mathematical Notations

If $p \geq m$, we denote $X^\dagger = (X^\top X)^{-1} X^\top$ for matrix $X \in \mathbb{R}^{p\times m}$ such that $\operatorname{rank} X = m$. We denote $Y = X^\perp$ for matrx $Y \in \mathbb{R}^{(p-m)\times p}$ such that $YX = O,\ \operatorname{rank} Y = p - m$. ($Y$ is the empty matrix if $p = m$). If P is a symmetric and positive definite matrix, we denote $P \succ 0$ and the weighted square

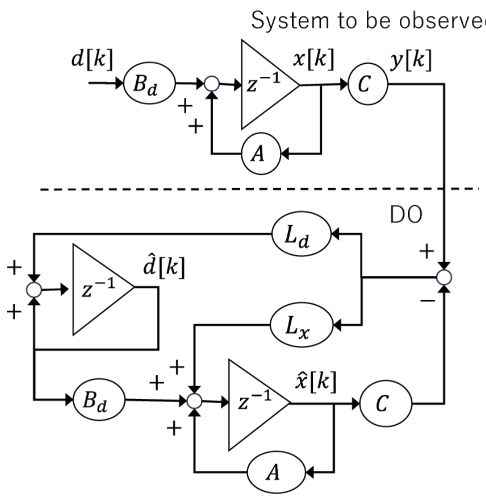

Fig. 3. Standard DO for constant disturbances

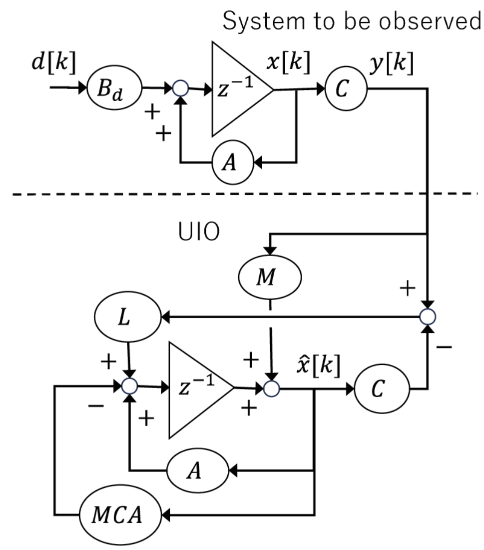

Fig. 4. Standard UIO

norm $\|x\|_P^2 = x^\top P x$ of a vector x. $\mathbb{R}, \mathbb{C}$, and $\mathbb{Z}_+$ denote the set of real numbers, complex numbers, and nonnegative integers, respectively.

For matrix $C \in \mathbb{R}^{p\times n}$, define $\tilde{C}_i \in \mathbb{R}^{(p-1)\times n}, i = 1, \ldots, p$ as the matrix made by removing the i-th row of C. For example, if $p = 3$ and $C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, we have

$$\tilde{C}_1 = \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \tilde{C}_2 = \begin{pmatrix} c_1 \\ c_3 \end{pmatrix}, \tilde{C}_3 = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}.$$

We denote $\tilde{C}_0 = C$ for convenience.

II. Problem Formulation

Consider a linear discrete-time plant

$$x[k+1] = Ax[k] + B_d d[k], \tag{1}$$

$$y[k] = Cx[k], \quad k = 0, 1, \ldots, \tag{2}$$

which is in the remote side of a network system. We assume that vectors $d[k] \in \mathbb{R}^m$, $x[k] \in \mathbb{R}^n$, and $y[k] \in \mathbb{R}^p$ are respectively the unknown disturbance, the state, and the sensor output. A, B_d, C are constant matrices whose sizes are compatible with the signal vectors. Throughout the paper we assume $p \geq m+1$. Namely, the number of the sensors is greater than that of the disturbances. We omit the command input term $B_u u[k]$ for brevity in (1) but it is straightforward to introduce this term as is the case in the standard observer design.

Our objective is to design an observer for state estimation of the plant under the following environment. When $y[k]$ is transmitted to the local side through the network, signal loss occurs due to sensing failure or packet loss in communication channels, and that at most one of the components $y_1[k], \ldots, y_p[k]$ may be lost at each time step k. Usually the loss probability is low enough to rule out the case where multiple signals are lost at the same time. On the other hand, we do not exclude the case where $y_i[k], \ldots, y_i[\ell], k < \ell$ are lost successively in the same channel i .

We also assume that the loss of signal $y_i[k]$ can be detected at time step k in the local side and denote the detected number of the lost channel by $\pi[k] = i$. If no loss, then we put $\pi[k] = 0$. Note that the index $\pi[\cdot]$ is unknown in the design stage but is available in operation, in real time.

In (2), we define

$$\tilde{y}_i[k] = \tilde{C}_i x[k], \quad i = 0, 1, \ldots, p. \tag{3}$$

See "Mathematical Notations" for the definition of $\tilde{C}_i$. We have $\tilde{y}_0[k] = y[k]$ as a special case.

As is assumed in the standard UIO [4], [5], we assume the following conditions for $i = 0, 1, \ldots, p$.

$$\text{A1: } (\tilde{C}_i, A) \text{ is observable;} \tag{4}$$

$$\text{A2: rank } \tilde{C}_i B_d = m; \tag{5}$$

$$\text{A3: rank} \begin{pmatrix} A - \lambda I_n & B_d \\ \tilde{C}_i & 0 \end{pmatrix} = n + m \tag{6}$$

for all $\lambda \in \mathbb{C}$ such that $|\lambda| \geq 1$.

Condition (5) results in $p \geq m$. Conditions (5) and (6) might look restrictive at a first glance but this is not the case in practice. Consider that

$$A = e^{A_c T}, \quad B_d = \int_0^T e^{A_c \tau} d\tau B_c,$$

for some continuous-time system (A_c, B_c) and sampling period T. Then (5) fails if and only if

$$\tilde{C}_i B_d u = \int_0^T \tilde{C}_i e^{A_c \tau} B_c u d\tau = 0 \quad \text{for some } u \in \mathbb{R}^m \backslash \{0\}.$$

This is true only if the step response of $(A_c, B_c, \tilde{C}_i)$ for the constant input u vanishes at the specific time T, which is a fairly rare case. Furthermore, if $m = 1$ and $p > 2$, (6) fails only if all zeros of the transfer functions $c_j(zI - A)^{-1} B_d, j \in \{1, \ldots, p\} \backslash \{i\}$ concentrate at a single point $\zeta \in \mathbb{C}$ such that $|\zeta| \geq 1$, which is also rare.

For (1) and (2), the UIO is represented as

$$\hat{x}[k+1] = (A - LC - MCA)\hat{x}[k] + Ly[k] + My[k+1], \tag{7}$$

for some gains L and M [4], [5]. Fig. 4 shows the configuration of (7). If no loss, the UIO works under the assumptions (4), (5), and (6). It also works if the loss channel is fixed permanently under the same conditions. However, we need a further technique to deal with the case where the loss channel changes in each step as we treat in the following section.

To this end, we switch observer gains according to detection of the loss channel number $\pi[k]$. Furthermore we design, in advance, the switching gains to maintain stability for any switching pattern [17]. A most difficult point is that the loss channels of $y[k]$ and $y[k+1]$ are not necessarily the same. It is by no means trivial in this sense to carry over the UIO to our problem.

III. Proposed method

Below is our main result in this paper.

Theorem: *Assume the conditions* (4), (5), *and* (6) *for plant* (1) *and* (2). *Define*

$$E_i = (\tilde{C}_i B_d)^\dagger \tilde{C}_i A, \quad D_i = (\tilde{C}_i B_d)^\perp \tilde{C}_i A, \quad \text{for } i = 0, 1, \ldots, p, \tag{8}$$

and assume that the LMIs (linear matrix inequalities)

$$\begin{pmatrix} P & P(A - B_d E_i) - \mathcal{M}_i D_i - \mathcal{L}_j \tilde{C}_j \\ * & P \end{pmatrix} \succ 0, \quad \text{for } i, j = 0, 1, \ldots, p \tag{9}$$

are solvable simultaneously with respect to the matrix variables

$$\begin{aligned} &P = P^\top \in \mathbb{R}^{n \times n}, \\ &\mathcal{M}_0 \in \mathbb{R}^{n \times (p-m)}, \mathcal{M}_i \in \mathbb{R}^{n \times (p-m-1)}, i = 1, \ldots, p, \\ &\mathcal{L}_0 \in \mathbb{R}^{n \times p}, \mathcal{L}_j \in \mathbb{R}^{n \times (p-1)}, j = 1, \ldots, p. \end{aligned}$$

Furthermore, using the solutions of (9)*, define the gains*

$$M_i = B_d(\tilde{C}_i B_d)^\dagger + P^{-1}\mathcal{M}_i(\tilde{C}_i B_d)^\perp, \quad i = 0, 1, \ldots, p, \tag{10}$$

$$L_j = P^{-1}\mathcal{L}_j, \qquad j = 0, 1, \ldots, p. \tag{11}$$

Then, for any initial value $x[0]$ and any signal loss $\pi[k], k \in \mathbb{Z}_+$, the gain-switching state UIO

$$\xi[k+1] = A\hat{x}[k] + L_{\pi[k]}\left(\tilde{y}_{\pi[k]}[k] - \tilde{C}_{\pi[k]}\hat{x}[k]\right) \tag{12}$$

$$\hat{x}[k] = \xi[k] + M_{\pi[k]}\left(\tilde{y}_{\pi[k]}[k] - \tilde{C}_{\pi[k]}A\hat{x}[k-1]\right) \tag{13}$$

$$k \in \mathbb{Z}_+, \ \xi[0] = 0, \ \hat{x}[-1] = 0$$

attains the property that $\|\varepsilon[k]\|_P^2 \to 0$ as $k \to \infty$, where

$$\varepsilon[k] = \hat{x}[k] - x[k] \tag{14}$$

is the estimation error.

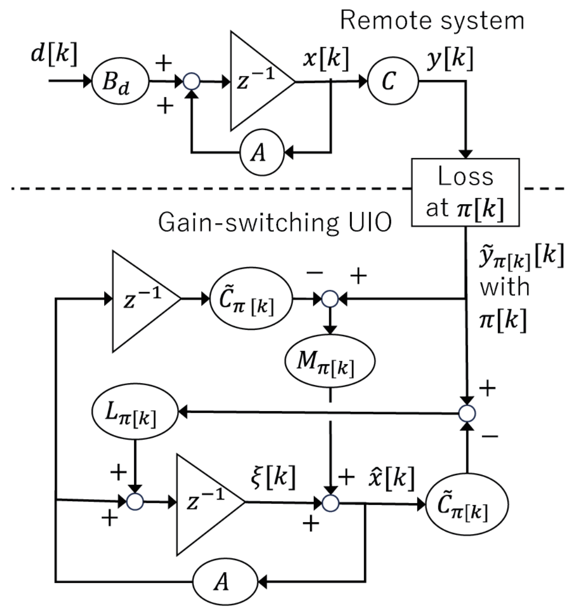

Fig. 5. Gain-switching UIO

Remark. Fig. 5 shows a block diagram of the structure (12) and (13). We do not use $d[k]$ in the proposed observer, similarly as in Fig. 4. A difference from the standard UIO is that:

1) we switch gains M_i and L_j according to the index $\pi[\cdot]$;
2) we introduce an additional delay element in (13), to overcome the difficulty when $\pi[k] \neq \pi[k+1]$.

Proof: We first write

$$\hat{x}[k+1] = \xi[k+1] + M_i\left(\tilde{y}_i[k+1] - \tilde{C}_i A\hat{x}[k]\right) \tag{15}$$

by replacing k with $k+1$ in (13) and denoting $i = \pi[k+1]$.

We then rewrite (12) as

$$\xi[k+1] = \left(A - L_j\tilde{C}_j\right)\hat{x}[k] + L_j\tilde{y}_j[k], \tag{16}$$

where $j = \pi[k]$.

Now we substitute (16) into (15) and rearrange the result as

$$\begin{aligned} \hat{x}[k+1] &= \left(A - L_j\tilde{C}_j\right)\hat{x}[k] + L_j\tilde{y}_j[k] \\ &\quad + M_i\left(\tilde{y}_i[k+1] - \tilde{C}_i A\hat{x}[k]\right) \\ &= \left(A - L_j\tilde{C}_j - M_i\tilde{C}_i A\right)\hat{x}[k] + L_j\tilde{y}_j[k] \\ &\quad + M_i\tilde{y}_i[k+1]. \end{aligned} \tag{17}$$

In the third term of the rightmost side we have, from (1) and (3),

$$\begin{aligned} M_i\tilde{y}_i[k+1] &= M_i\tilde{C}_i(Ax[k] + B_d d[k]) \\ &= M_i\tilde{C}_i Ax[k] + B_d d[k]. \end{aligned} \tag{18}$$

Here we have used the equality

$$\begin{aligned} M_i\tilde{C}_i B_d &= \left(B_d(\tilde{C}_i B_d)^\dagger + P^{-1}\mathcal{M}_i(\tilde{C}_i B_d)^\perp\right)\tilde{C}_i B_d \\ &= B_d \end{aligned}$$

by (10).

Hence we obtain

$$\hat{x}[k+1] = A\hat{x}[k] - (L_j\tilde{C}_j + M_i\tilde{C}_iA)(\hat{x}[k] - x[k]) \\ + B_d d[k] \qquad \text{by (17) and (18)}\ .$$

Subtracting the both sides of (1) from those of the above equation, we obtain by (14)

$$\begin{aligned}\varepsilon[k+1] &= \hat{x}[k+1] - x[k+1] \\ &= A\hat{x}[k] - (L_j\tilde{C}_j + M_i\tilde{C}_iA)(\hat{x}[k] - x[k]) \\ &\quad + B_d d[k] - (Ax[k] + B_d d[k]) \\ &= \left(A - L_j\tilde{C}_j - M_i\tilde{C}_iA\right)\varepsilon[k] \end{aligned} \tag{19}$$

Again using (10) and (11), we obtain

$$\begin{aligned} & A - L_j\tilde{C}_j - M_i\tilde{C}_iA \\ & = A - P^{-1}\mathcal{L}_j\tilde{C}_j \\ & \quad - \left(B_d(\tilde{C}_iB_d)^\dagger + P^{-1}\mathcal{M}_i(\tilde{C}_iB_d)^\perp\right)\tilde{C}_iA \\ & = A - B_d(\tilde{C}_iB_d)^\dagger\tilde{C}_iA \\ & \quad - P^{-1}\left(\mathcal{M}_i(\tilde{C}_iB_d)^\perp\tilde{C}_iA + \mathcal{L}_j\tilde{C}_j\right) \\ & = A - B_dE_i - P^{-1}(\mathcal{M}_iD_i + \mathcal{L}_j\tilde{C}_j) =: \mathcal{A}_{ij}, \end{aligned} \tag{20}$$

where we have used (8) in the last part. Now by LMIs (9) and Schur complement, we have

$$\mathcal{A}_{ij}^\top P\mathcal{A}_{ij} - P \prec 0.$$

This means that $V(\varepsilon) = \varepsilon^\top P\varepsilon$ gives a common Lyapunov function for the switched systems

$$\varepsilon[k+1] = \mathcal{A}_{ij}\varepsilon[k], \quad k \in \mathbb{Z}_+$$

for $i, j = 0, 1, \ldots, p$ [2], [17]. Since the number of these systems is finite (actually, $(p+1)^2$), there exists $\eta > 0$ such that

$$\mathcal{A}_{ij}^\top P\mathcal{A}_{ij} - P \prec -\eta I_n.$$

By (19), (20) and this inequality, we thus obtain

$$\|\varepsilon[k]\|_P^2 \to 0 \qquad (k \to \infty),$$

as expected. ■

IV. Numerical Simulation

A. Design Example

Let us take an example of (1) and (2) in the controllability canonical form as

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.37 & -3.57 & 3.2 \end{pmatrix}, \quad B_d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$
$$C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0.96 & -2 & 1 \\ 0.99 & -2 & 1 \\ 1.04 & -2 & 1 \end{pmatrix}, \tag{21}$$

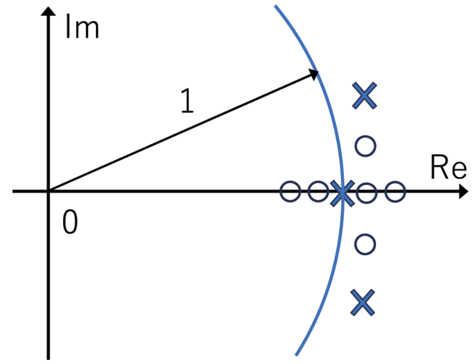

Fig. 6. Poles and zeros location in the complex plain

hence $n = 3, p = 3, m = 1$. This system is unstable since the poles are 1.0 and $1.1 \pm 0.4j$ (cross marks in Fig. 6). The transfer function matrix is

$$\begin{aligned} & C(zI_3 - A)^{-1}B_d \\ & = \begin{pmatrix} z^2 - 2z + 0.96 \\ z^2 - 2z + 0.99 \\ z^2 - 2z + 1.04 \end{pmatrix} \frac{1}{z^3 - 3.2z^2 + 3.57z - 1.37}. \end{aligned}$$

The zeros of $c_i(zI_n - A)^{-1}B$ are $1 \pm 0.2,\ 1 \pm 0.1$, and $1 \pm 0.2j$, respectively for $i = 1, 2$, and 3 (small circles in Fig. 6). (c_i, A) is observable for $i = 1, 2$, and 3, since there are no pole-zero cancellation in each of the above transfer functions. Hence $(\tilde{C}_i, A)$ is also observable for $i = 1, 2, 3$. Thus condition (4) is satisfied. Condition (5) is true since we have

$$\tilde{C}_iB_d = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \text{for } i = 1, 2, 3.$$

Condition (6) is also true since the above zeros are not concentrated at a single point.

The authors have solved LMIs (9) on a 64 bit PC equipped with Windows 11 and MATLAB R2023a, using LMI parser YALMIP and solver SDPT3. In order to reduce solvability of (9) to the semi-definite program, scalar η is maximized under the constraint that the left-hand side of (9) minus ηI_{2n} is positive semidefinite and we judge the LMIs are solvable when optimal η is positive. We further add conditions $\eta \le 20$ and $P \succeq I_n$ as consraints in order to maintain η to be finite and to avoid P tending to the zero matrix.

Under the above conditions the number of LMIs (9) becomes $4^2 = 16$. They are successfully solved simultaneously. Specifically we have

$$P = \begin{pmatrix} 19.7235 & -0.9942 & -1.6689 \\ -0.9942 & 16.4251 & -6.0007 \\ -1.6689 & -6.0007 & 9.9275 \end{pmatrix}.$$

We omit to write $\mathcal{M}_i, \mathcal{L}_i, i = 0, 1, 2, 3$. The observer gains

TABLE I

SIGNAL LOSS PATTERN

$y_1[k]$	S	S	F	S	S	F	S	S	F	S	S	F	S	S	F	...
$y_2[k]$	S	F	S	S	F	S	S	F	S	S	F	S	S	F	S	...
$y_3[k]$	F	S	S	F	S	S	F	S	S	F	S	S	F	S	S	...
$\pi[k]$	3	2	1	3	2	1	3	2	1	3	2	1	3	2	1	...

(10) and (11) are computed as

$$L_0 = \begin{pmatrix} -10.1051 & 0.3054 & 9.4849 \\ 10.2397 & 0.1634 & -9.2241 \\ 28.8797 & 0.5545 & -27.4887 \end{pmatrix}$$

$$L_1 = \begin{pmatrix} -16.0722 & 15.8480 \\ 17.0805 & -16.1324 \\ 46.4788 & -44.4108 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} -10.0451 & 9.8210 \\ 10.6753 & -9.7272 \\ 29.0493 & -26.9812 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} -26.4134 & 26.1892 \\ 26.8873 & -25.9391 \\ 74.0180 & -71.9500 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} -36.7915 & 43.4809 & -6.6893 \\ -93.0990 & 110.0260 & -16.9270 \\ -146.6534 & 174.0448 & -26.3914 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} -15.3856 & 15.3856 \\ -38.9324 & 38.9324 \\ -60.6006 & 61.6006 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} -9.6160 & 9.6160 \\ -24.3327 & 24.3327 \\ -37.8754 & 38.8754 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} -25.6427 & 25.6427 \\ -64.8873 & 64.8873 \\ -102.6677 & 103.6677 \end{pmatrix}$$

B. Simulation Result

Numerical simulation has been carried out to evaluate the proposed method.

We give the disturbance as

$$d[k] = \begin{cases} 1, & k \in [20\ell,\ 20\ell+10), & \ell \in \mathbb{Z}_+ \\ -1, & k \in [20\ell+10,\ 20(\ell+1)), & \ell \in \mathbb{Z}_+, \end{cases}$$

namely a rectangular wave with period 20 [step] and amplitude 1. We adopt the signal loss pattern shown by Table 1 where S means "successfully received" and F means "failure (signal loss)". This is a rather extreme case where signal loss takes place very often, but the loss pattern satisfies the assumption in II.

The authors have performed numerical simulation for the following three cases for comparison:

1) (Proposed method) the gain-switching UIO (12) and (13) with the gains obtained in IV-A.
2) (Conventional 1) the gain-switching DO. Instead of L_d and L_x in Fig. 3, we switch the gains according to the loss pattern, as

$$\begin{pmatrix} \hat{d}[k+1] \\ \hat{x}[k+1] \end{pmatrix} = \bar{A} \begin{pmatrix} \hat{d}[k] \\ \hat{x}[k] \end{pmatrix} + \bar{L}_{\pi[k]} \left(\tilde{y}_{\pi[k]} - \bar{C}_{\pi[k]} \begin{pmatrix} \hat{d}[k] \\ \hat{x}[k] \end{pmatrix} \right),$$

where

$$\bar{A} = \begin{pmatrix} 1 & 0_{1\times 3} \\ B_d & A \end{pmatrix}, \quad \bar{C}_0 = \begin{pmatrix} 0_{3\times 1} & \tilde{C}_0 \end{pmatrix},$$
$$\bar{C}_j = \begin{pmatrix} 0_{2\times 1} & \tilde{C}_j \end{pmatrix}, \quad j = 1,2,3,$$

with $0_{p\times m}$ denoting $p \times m$ zero matrix. The gains are computed as

$$\bar{L}_j = \bar{P}^{-1} \bar{\mathcal{L}}_j, \quad j = 0,1,2,3, \tag{22}$$

using solutions $\bar{\mathcal{L}}_j$ of the LMIs

$$\begin{pmatrix} \bar{P} & \bar{P}\bar{A} - \bar{\mathcal{L}}_j \bar{C}_j \\ * & \bar{P} \end{pmatrix} \succ 0, \ j = 0,1,\ldots,p.$$

3) (Conventional 2) the standard DO without switching gains. We use the augmented system $(\bar{A}, \bar{C}_0)$ in 2) and fix the observer gains to $\bar{L}_0$. When $\pi[k] = j \neq 0$, $y_j[k]$ is considered to be zero.

Fig. 7 shows time evolution of the disturbance $d[k]$ (upper) and three components of the state vector $x[k]$ (lower). The state diverges since the system is unstable.

Fig. 8 shows the estimation error $\varepsilon[k]$ by 1) the proposed switching UIO (upper) and 2) the switching DO (lower). (The latter has four components unlike the proposed method.) The error converges to zero rapidly in the former method. It also converges to zero in the latter method but the convergence is slow. This is because the DO assumes a constant disturbance but $d[k]$ is not constant in reality. The estimation error diverges by 3) the standard DO without switching, which we omit to show in Fig. 8.

Fig. 9 shows time evolution of the estimation error in log scale $\log_{10} \|\varepsilon[k]\|_P^2$, where blue, red, and yellow lines are respectively the results by 1) the switching UIO, 2) the switching DO, and 3) the standard DO without switching. It is observed that in the proposed method the error converges to zero exponentially. In the switching DO the error vibrates because of the rectangular disturbance. In the standard DO the error diverges.

V. CONCLUSION

A state estimation technique has been proposed in this paper, under unknown disturbance and lossy multiple measurement. The paper overcomes this challenging situation via gain switching UIO. Though we have assumed that signal loss occurs only at one channel, if any, generalization to the case of multiple channel loss is straightforward. In this case, however, the number of LMIs becomes large. If we allow loss of up to two channels simultaneously, the number of LMIs becomes $(1 + p +_p C_2)^2$ instead of $(p+1)^2$. As we consider more general loss pattern, the LMIs become more complex, which is a kind of trade-offs.

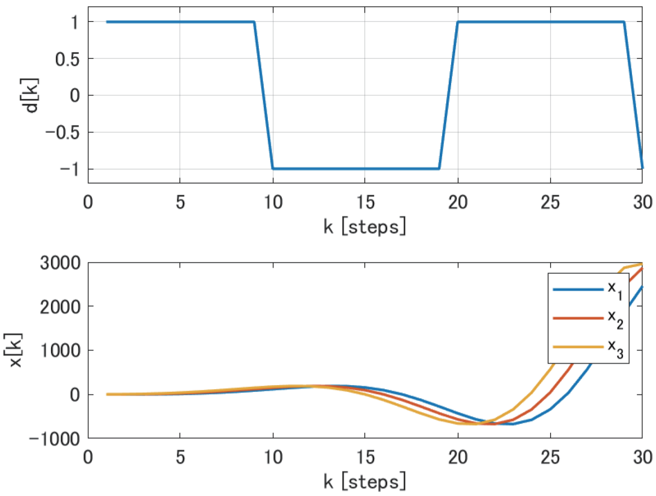

Fig. 7. Time evolution of disturbance (upper) and three components of the state vector (lower)

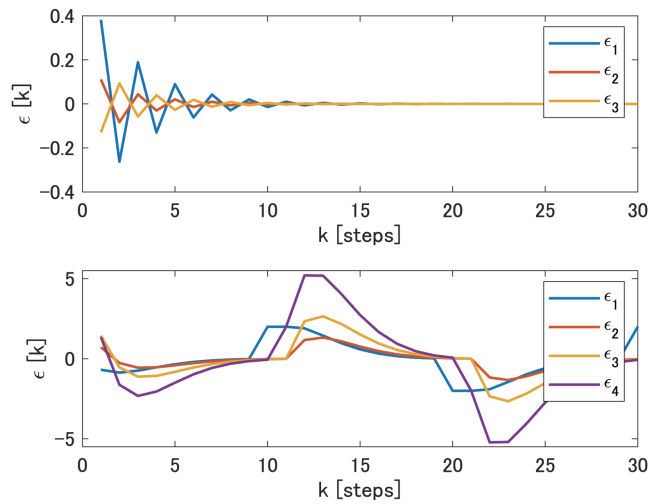

Fig. 8. Estimation error by proposed switching UIO (upper) and switching DO (lower)

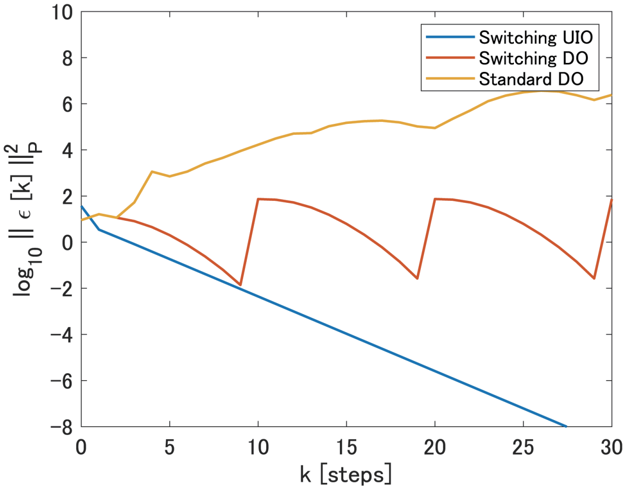

Fig. 9. Time evolution of estimation error in log scale: blue=Switching UIO (proposed), red=Switching DO, and yellow=Standard DO without switching

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