# A Hybrid Optimization Approach for a Continuous and Efficient Pump Operation Scheduling in Water Supply Systems

Marlene Brás[1], Ana Moura[2], and António Andrade-Campos[3]

***Abstract*— Solving the pump scheduling problem in Water Supply Systems (WSS) is essential for reducing energy costs and ensuring operational efficiency. This study applies the Smart Dynamic Local Search (Smart-DLS), a hybrid optimization framework, as a decision support tool to reduce the costs of five days of continuous pumping operation in a real WSS. Smart-DLS integrates deterministic local search with an intelligent shaking process, enhancing solution robustness and scalability while efficiently exploring the solution space. The results demonstrate that Smart-DLS achieved an approximately 19% cost reduction while ensuring tank-level compliance and offering flexible solutions that balance cost efficiency and pump wear reduction. These outcomes demonstrate the Smart-DLS as an effective decision support tool for real-time WSS optimization.**

## I. INTRODUCTION

Water supply systems (WSS) face increasing challenges in balancing rising water demands with energy efficiency [1], as water utilities are among the largest energy consumers in the municipal sector, with energy costs accounting for up to 65% of their total operating expenses [2]. Since pump operations comprise most of this energy consumption, optimizing their scheduling is critical for reducing costs while ensuring a reliable water supply [3].

The complexity of pump scheduling problems (PSP) arises from the nonlinear hydraulic dynamics of water distribution networks, time-dependent energy tariffs, and multiple operational constraints, such as maintaining tank levels and avoiding excessive pump switching [4]. While traditional optimization methods offer mathematically rigorous solutions, these often face computational inefficiencies, particularly in large-scale WSS, where the high-dimensional, non-convex solution space makes real-time optimization challenging. Metaheuristic and hybrid optimization approaches have gained traction to address these limitations, as they efficiently navigate the solution space while avoiding local optima [5]. However, metaheuristics approaches are known to be computationally very demanding, requiring many function evaluations to achieve a solution [6].

Recent studies [7], [8] have shown that the optimization model based on duty-cycles combined with local search algorithms, such as Sequential Least Squares Quadratic Programming (SLSQP), can significantly improve computational efficiency and cost-effectiveness in pump scheduling. However, this method still faces challenges related to scalability and sensitivity to initial conditions, which may result in sub-optimal scheduling decisions. Overcoming these limitations CPU-efficiently is essential for water utilities to effectively implement this approach to optimize pump operations in real-world conditions. Considering this, [9] proposed an enhanced hybrid optimization approach, the Smart Dynamic Local Search (Smart-DLS), designed to overcome the limitations of this PSP optimization methodology. Smart-DLS integrates deterministic local search with an intelligent shaking process, allowing it to dynamically explore the solution space, escape local optima, and enhance robustness.

This study applies the Smart-DLS as a decision support tool in a real Portuguese WSS for five days, where its performance is compared with the actual operational data.

The paper is organized into four sections. Following this introduction, Section II provides an overview of the optimization methodology, detailing the duty-cycle formulation and Smart-DLS approach. Section III presents the multi-day optimization framework, the case study used to test this methodology as well as the results obtained. Finally, the article concludes by summarizing the results and potential areas for future research (Section IV).

## II. AN OVERVIEW OF THE OPTIMIZATION APPROACH

The optimization approach can be divided into two main components: the formulation of the optimization problem and the optimization algorithm. The Smart Dynamic Local Search, as proposed and validated in [9], is a hybrid methodology designed to optimize pump scheduling costs in WSS by solving the optimization model based on duty cycles. The subsequent sub-sections explain each of these components.

### *A. Pump Scheduling Problem: Duty-cycles Formulation*

The duty-cycles formulation is a time-position unrestricted explicit formulation, proposed and validated in [7], [8], in which pumps can start operating at any time during the total time horizon $T$. The set of decision variables $\mathbf{X}^{\mathrm{dc}} = [\mathbf{st}^{\mathrm{dc}} \boldsymbol{\Delta}\mathbf{t}^{\mathrm{dc}}]$ determines the starting time ($st_d^{\mathrm{dc}}$) and duration ($\Delta t_d^{\mathrm{dc}}$) of each pump operation, i.e. each duty-cycle, $d$. Consequently, two decision variables are associated with each duty-cycle, with values between 0 and $T$.

During the total time horizon, pumps may operate multiple times. However, this formulation implicitly constrained the maximum number of pump starts by defining the number

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of duty-cycles ($D$) per pump. For each pump, the first $D$ decision variables dictate the starting time, and the last $D$ define the duration of each duty-cycle for $P$ pumps. The optimization model can be formulated as:

$$\underset{\mathbf{X}^{\mathrm{dc}}\in\mathbb{R}_0^+}{\text{minimize}} \quad C(\mathbf{X}^{\mathrm{dc}}) = \sum_{p=1}^{P}\sum_{d=1}^{D}\int_{st_{d,p}^{\mathrm{dc}}}^{st_{d,p}^{\mathrm{dc}}+\Delta t_{d,p}^{\mathrm{dc}}} \$_p(t)\,\frac{\dot{W}_p(t)}{\eta_p}\,\mathrm{d}t \tag{1}$$

$$\text{subject to } \mathbf{AH} = \mathbf{q} \tag{2}$$

$$N_{wt}^{\min} \le L_{wt}(\mathbf{X}^{\mathrm{dc}}) \le N_{wt}^{\max},\ \forall wt \in \{1,...,WT\} \tag{3}$$

$$g_{d,p}^{\mathrm{dc}} = st_{d+1,p}^{\mathrm{dc}} - (st_{d,p}^{\mathrm{dc}} + \Delta t_{d,p}^{\mathrm{dc}}) \ge 0, \quad \forall d \in \{1,...,D-1\};\ \forall p \in \{1,...,P\} \tag{4}$$

$$g_{D,p}^{\mathrm{dc}} = T - (st_{D,p}^{\mathrm{dc}} + \Delta t_{D,p}^{\mathrm{dc}}) \ge 0, \quad \forall p \in \{1,...,P\} \tag{5}$$

$$X_{d,p}^{\mathrm{dc}} \in [0,T],\ \forall d \in \{1,...,D\}; \quad \forall p \in \{1,...,P\}. \tag{6}$$

The objective of this model is to minimize the energy consumption costs associated with a given set of pumps, represented as $C(\mathbf{X}^{\mathrm{dc}})$ in (1). These costs are calculated by considering the unit energy cost, $\$_p$, each pump's hydraulic power, $\dot{W}_p$, and efficiency, $\eta_p$. In addition to the standard constraints applied to this problem, such as mass and energy conservation (2) and the minimum ($N_{wt}^{\min}$) and maximum ($N_{wt}^{\max}$) water tank levels (3), it is essential to introduce an additional constraint related to the temporal logic of duty cycles. This new constraint, detailed in (4) and (5), ensures that the starting time of a given duty cycle is equal to or later than the stopping time of the previous duty cycle and that the last duty cycle does not exceed the total time horizon $T$. This prevents any overlap or conflict in pump operations.

Considering the non-linearity of this system constraints, especially the mass and energy conservation equation, these are typically extracted from the mathematical models and handled externally through hydraulic simulation software [10]. Therefore, as illustrated in Fig. 1, a framework based on the optimization module of the model predictive control (MPC) is adopted to solve this optimization model. Using this problem disaggregation structure, a vector of decision variables $\mathbf{X}$ is selected, which satisfies the explicit bound constraints (decision variables' domain). This vector $\mathbf{X}$ is converted into controls $\mathbf{s}(t)$. It is verified if these satisfy the systems' implicit constraints ($\mathbf{AH} = \mathbf{q}$) through the WSS simulator, obtaining the respective time-series vectors of hydraulic power ($\dot{\mathbf{W}}_p$) and water levels ($\mathbf{L}^{\mathrm{dc}}$). The optimization model verifies the constraints feasibility (i.e., water tank levels, number of pump switches, etc.) and calculates the value of the objective function associated with the decision variables vector $\mathbf{X}$. This cyclic process continues until the stopping criteria are achieved. As shown in Fig. 1, the optimization model blocks are based on a duty-cycle formulation, EPANET is used for hydraulic simulations, and the Smart-DLS algorithm is employed to solve the optimization model. The converter blocks translate the decision variables into controls according to the following equation:

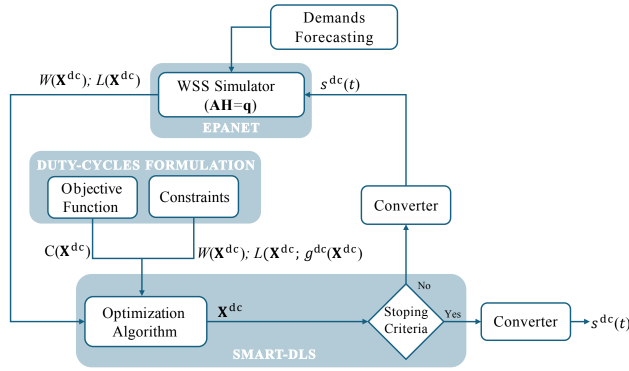



Fig. 1: Optimization framework based on the optimization module of the model predictive control theory.

$$s_p^{\mathrm{dc}}(t) = \begin{cases} 1, & \text{if } st_{d,p}^{\mathrm{dc}} \le t < st_{d,p}^{\mathrm{dc}} + \Delta t_{d,p}^{\mathrm{dc}},\ \forall d \in \{1,...,D\}; \\ & \forall p \in \{1,...,P\} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

### *B. Smart-Dynamic Local Search*

The Smart-DLS, proposed in [9], combines a deterministic local search with an intelligent shaking process, addressing key challenges such as sensitivity to initial solutions, local optima minimum, and scalability for large-real WSS. As illustrated in Fig. 2, the optimization process begins with a local search phase, where a gradient-based algorithm, the Sequential Least Squares Quadratic Programming (SLSQP), refine the solution by adjusting duty-cycle parameters to minimize energy costs while ensuring compliance with the operational constraints. The algorithm is available in the scipy.optimize python library [11].

Following the local search, an intelligent shaking process is applied to introduce modifications to the decision variables that prevent being trapped in local minima. Instead of relying on random perturbations, as in traditional Variable Neighborhood Search (VNS) [12], Smart-DLS systematically alters the solution of the previous iteration based on predefined rules to achieve a new initial one. Short duty-cycles are deleted, closely spaced duty-cycles are merged, and new duty-cycles are added if significant gaps exist in the schedule. Additionally, long duty-cycles are strategically split in the final iteration to optimize water storage utilization, further reducing operational costs. By dynamically adjusting the number of duty-cycles during the shaking process, the framework balances the trade-off between cost-efficiency and the wear and tear on pumping equipment, providing a more flexible and resilient approach to scheduling optimization. More details on the algorithm can be found in [9]. The optimization process continues iteratively until a predefined stopping criterion is met, which may include a maximum number of iterations, achieving a cost improvement tolerance over consecutive iterations, or exceeding a computational time limit.

Smart-DLS employs a multi-start strategy to enhance robustness, generating multiple initial solutions to explore different regions of the solution space. This mitigates the impact of initial conditions and increases the likelihood of achieving a globally optimal solution. This makes Smart-DLS a practical tool for water utilities seeking to reduce energy costs, improve operational efficiency, and ensure reliable water distribution.

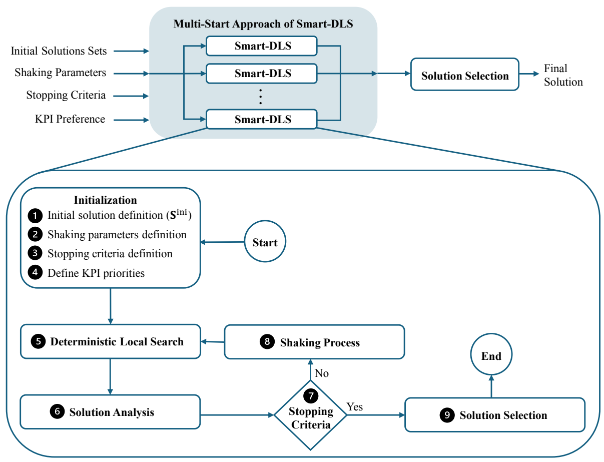



Fig. 2: Multi-starting approach of the Smart-DLS. This approach allows the algorithm to explore a larger part of the solution space from multiple entry points, reducing the risk of the algorithm finding a local optimum and increasing the chances of finding a global optimum (adapted from [9]).

## III. Multi-Day Optimization Approach

The main objective of this study was to validate the real-world applicability of Smart-DLS as a decision-support tool. Therefore, data from a Portuguese water supply network were used over five days to evaluate the proposed methodology. Fig. 3 presents the framework implemented for this validation. In this initial study, daily pump scheduling optimization was conducted at the beginning of each day over a total time horizon of 24 hours using real demands as forecasted.

Regarding the Smart-DLS, the following settings were applied: five multi-starts with a stopping condition set to three Smart-DLS iterations. Three multi-starts began with the same number of duty-cycles for each pump ($D = 5$), while the other two started with different values for the duty-cycles ($D = 4$ and $D = 6$). Regarding the shaking parameters, it was defined $\Delta t^{\text{del}} = 0.15$h, $\Delta t^{\text{merg}} = 0.1$h, $\Delta t^{\text{add}} = 3$h, and $\Delta t^{\text{div}} = 6$h. During the solution selection phase, the key performance indicators (KPIs) were the total cost as the primary priority, followed by the number of duty-cycles as the secondary priority. The network description and its results are discussed in the following sub-sections. The optimization framework was run on an 11th-gen Intel(R) CORE(TM) i7-1185G7 CPU @ 3.00 GHz with 16 GB RAM.

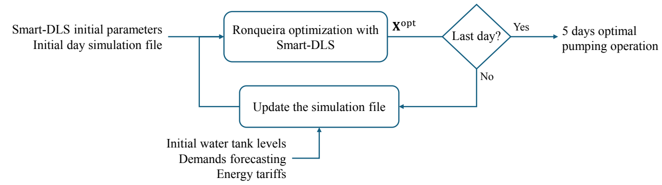



Fig. 3: Methodology conducted to test the Smart-DLS as a decision support tool.

### A. Case Study: Ronqueira Network

The Ronqueira network is a real-world water supply system managed by the Portuguese utility Águas do Centro Litoral (AdCL). As depicted in Fig. 4, this system serves eight localities: Aveleira, Albarqueira, Espinheira, Travanca, São Pedro Dias, Entroncamento, Outeiro Castro, and Casais. The infrastructure includes one water source, six fixed-speed pumps (p_Al1, p_Al2, p_Av, p_SPD, p_Trav, p_OC), six storage tanks (t_Al, t_Av, t_SPD, t_Trav, t_OC, t_Esp), and one automated valve (v_Esp). The valve at tank t_Esp operates based on water level thresholds, opening at 2.21 m and closing at 2.69 m.

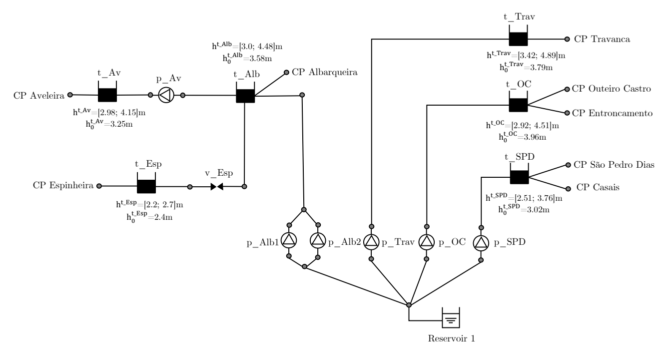



Fig. 4: Ronqueira network, a real Portuguese network (source: [9]).

The optimization approach was applied to this network from September 25 (Wednesday) to 29 (Sunday), 2024. The data was selected to include weekdays and weekends so that the tariffs would differ. The energy tariffs are detailed in Fig. 7, and the water demand patterns for each consumption point are provided in Fig. 8.

The optimization approach must be adapted for multi-day implementation to maintain continuity across successive days, ensuring water level stability while dynamically adjusting pump schedules in response to daily demand fluctuations. Therefore, a water tank level continuity constraint was incorporated into the mathematical formulation, requiring that the water level at the end of each 24 hours be equal to or greater than the minimum operational level, plus an additional 10% of the available storage range. This constraint guarantees a smooth transition between optimization cycles and prevents unplanned overflow of water tanks.

An analysis of the available data revealed that the initial tank levels were below the minimum operational limit on certain days. To address this, a modified tank level constraint was implemented: for days when the initial level falls below the operational minimum, the constraint was adjusted until 12:00, and the required minimum level remains equal to the lowest level recorded in the first half of the day. After 12:00, the standard operational minimum level is reinstated as a constraint. This adjustment ensures a gradual recovery of storage levels, preventing excessive pump usage in the early hours while still meeting operational safety requirements.

### B. Results

As previously mentioned, the optimization of each day was performed using three multi-start executions of Smart-DLS, each undergoing three iterations of local search, resulting in nine distinct solutions per day. Aligning with the KPIs priorities previously defined, for each day, only the solutions with the lowest cost and number of pump starts were selected for this analysis. Table I summarizes the results of the Smart-DLS approach alongside the network's real operation for each day. Figs. 5 and 6 show the solutions obtained regarding pump operation and tank levels on each day, considering the total cost of KPI and the number of pump starts, respectively. Additionally, it is also possible to see the real level of each tank.

TABLE I: Results of the Smart-DLS for each day alongside the real network operation.

| | Day | Cost (€) | No of pump starts | Objective function evaluations |
|---|---|---|---|---|
| Real operation | | 2900.67 | 21 | - |
| Smart-DLS (Best cost) | 25/Sept | 2432.88 | 22 | 1619 |
| Smart-DLS (No of pumps starts) | | 2453.84 | 19 | |
| Real operation | | 3061.73 | 17 | - |
| Smart-DLS (Best cost) | 26/Sept | 2502.88 | 22 | 1863 |
| Smart-DLS (No of pumps starts) | | 2528.25 | 15 | |
| Real operation | | 2752.11 | 19 | - |
| Smart-DLS (Best cost) | 27/Sept | 2260.12 | 26 | 1364 |
| Smart-DLS (No of pumps starts) | | 2263.84 | 19 | |
| Real operation | | 2759.65 | 16 | - |
| Smart-DLS (Best cost) | 28/Sept | 2242.19 | 19 | 2045 |
| Smart-DLS (No of pumps starts) | | 2251.94 | 18 | |
| Real operation | | 2688.19 | 26 | - |
| Smart-DLS (Best cost) | 29/Sept | 2043.22 | 21 | 2759 |
| Smart-DLS (No of pumps starts) | | 2043.22 | 21 | |

A comparison of the Smart-DLS solutions and the company's operational strategy indicates that Smart-DLS consistently outperforms the existing approach in terms of cost efficiency. The cost savings achieved by Smart-DLS range from 467.79€ (-16%) on the first day to 644.97€ (-24%) on the final day, demonstrating its effectiveness in optimizing pump scheduling. In terms of pump starts, Smart-DLS not only reduces costs but also achieves fewer pump starts compared to the company's approach in most cases. The only exception occurs on day 28, where the company's solution recorded two fewer pump starts than Smart-DLS. However, despite this slight increase in pump activations, the Smart-DLS solution still resulted in a significant cost reduction of €571.71, demonstrating its superior ability to balance operational efficiency and energy cost savings. Conducting additional multi-starts with a lower number of duty-cycles in the initial solution could further enhance the optimization results regarding the number of pump starts. Obtaining results in a short time is essential when applying an optimization methodology as a decision-support tool. By examining the number of objective function evaluations in Table I, it becomes evident that Smart-DLS is well-suited for this purpose, as it requires significantly fewer evaluations compared to many optimization methods proposed in the literature [13], [14], [15]. Given the characteristics of the workstation, an objective function evaluation takes 0.2 seconds. Therefore, the maximum number of iterations recorded was 2759, which would mean a maximum of 9 minutes to achieve a solution. This highlights its efficiency and practicality for near-real-time operational decision-making in water supply systems.

TABLE II: Results of the Smart-DLS for each day alongside the real network operation.

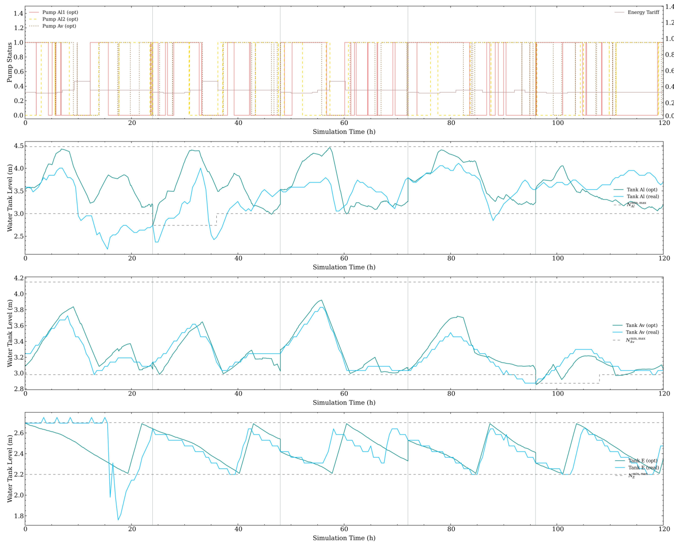
| KPIs | | Results for 5 days | Improvement (%) |
|---|---|---|---|
| | Real operation | 14162.35 | - |
| Cost (€) | Smart-DLS (Best cost) | 11481.30 | -18.93 (-2681.05 €) |
| | Smart-DLS (No of pumps starts) | 11541.09 | -18.51 (-2621.26 €) |
| | Real operation | 99 | - |
| No of pump starts | Smart-DLS (Best cost) | 110 | +0.1 (+11 pumps starts) |
| | Smart-DLS (No of pumps starts) | 92 | -0.07 (-7 pumps starts) |

Table II summarizes the optimization results over a five-day period, highlighting the effectiveness of Smart-DLS in cost reduction. Throughout these five days, Smart-DLS achieved approximately a 19% reduction in operational costs, resulting in savings of 2681.05€. However, this cost-optimal solution required 11 additional pump starts compared to the company's current operations. Despite this increase in pump starts, Smart-DLS fully complied with tank level constraints, which the company's current solution failed to meet. This is evident from the water levels shown in Figure 5. Moreover, when the Smart-DLS was selected to minimize pump starts through the KPIs priorities, it still achieved a cost reduction of 2621.26€ while reducing pump activations by seven compared to the company's operation. Ultimately, the decision between these solutions would depend on the company's operational priorities — whether to prioritize cost savings or minimize pump wear. In conclusion, the Smart-DLS offers flexibility by making both options readily available, allowing decision-makers to select the most suitable strategy for each day.

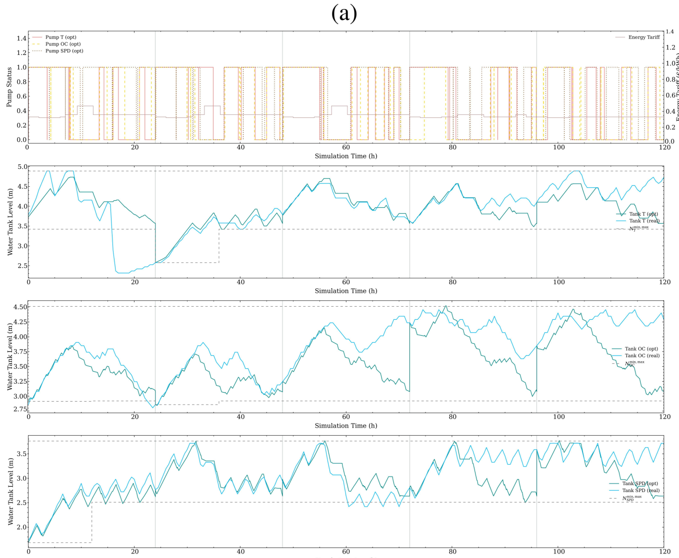
## IV. Conclusions and Future Work

Efficient pump scheduling is critical for WSS, as it directly impacts energy consumption, operational costs, and infrastructure longevity. Since pumps account for most energy usage in WSS, optimizing their operation is essential for cost reduction and system reliability. Effective scheduling strategies must balance minimizing energy expenses with maintaining water storage levels and reducing pump wear, making optimization a key factor in improving overall system performance.

This study applied the Smart-DLS methodology to optimize the pump scheduling in a real-world WSS over five days, evaluating its effectiveness as a decision support tool.

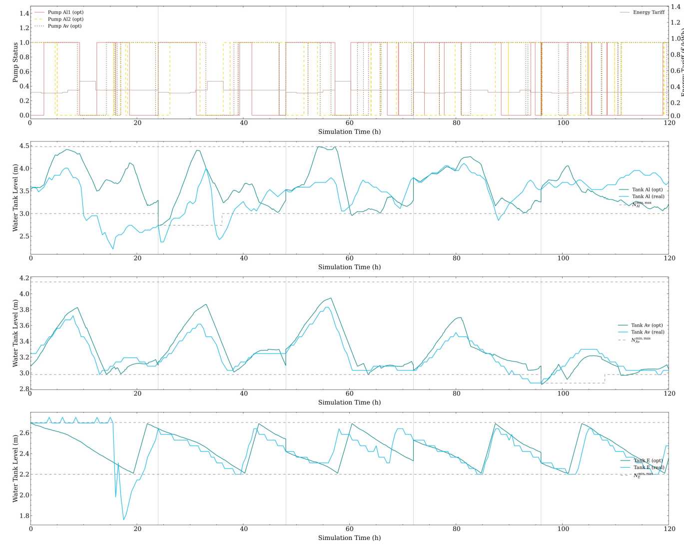


(a)

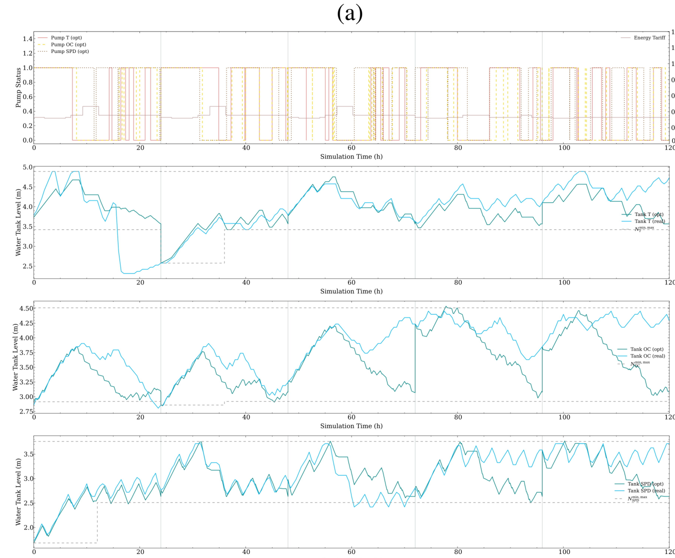


(b)

Fig. 5: Smart-DLS results for the Ronqueira network considering as priority the total cost: optimized pump operations, real and optimized water tank levels of the (a) left and (b) right sides of the network.



(a)



(b)

Fig. 6: Smart-DLS results for the Ronqueira network considering as priority the number of pump starts: optimized pump operations, real and optimized water tank levels of the (a) left and (b) right sides of the network

By integrating deterministic local search with an intelligent shaking process, Smart-DLS successfully optimized five days of pump operation, achieving a cost reduction of approximately 19% while maintaining tank level constraints and offering flexible scheduling solutions. The results demonstrated that Smart-DLS consistently outperformed the existing operational strategy, highlighting its potential for improving energy efficiency and operational cost savings.

Beyond cost savings, Smart-DLS provided alternative optimization scenarios, allowing operators to balance cost efficiency and pump wear. This flexibility makes the methodology particularly valuable for real-time WSS management, enabling informed decision-making based on operational priorities. Furthermore, the computational efficiency of the approach, requiring relatively few objective function evaluations, highlights its feasibility for practical implementation.

To further enhance the applicability of Smart-DLS to the real world, future research will focus on integrating demand forecasting into the optimization framework. Additionally, further studies should explore its scalability to even larger and more complex WSS networks, ensuring its robustness across diverse operational scenarios.

## APPENDIX

The following appendix presents the information on the Ronqueira network regarding the energy tariffs and consumption demands from September 25 to 29, 2024.

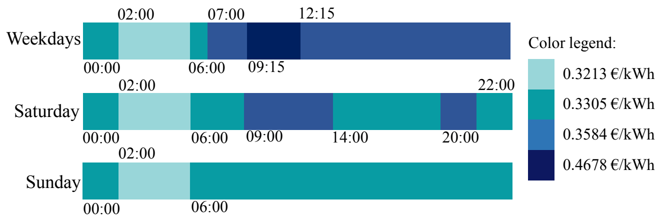



Fig. 7: Energy tariffs of the Ronqueira network.

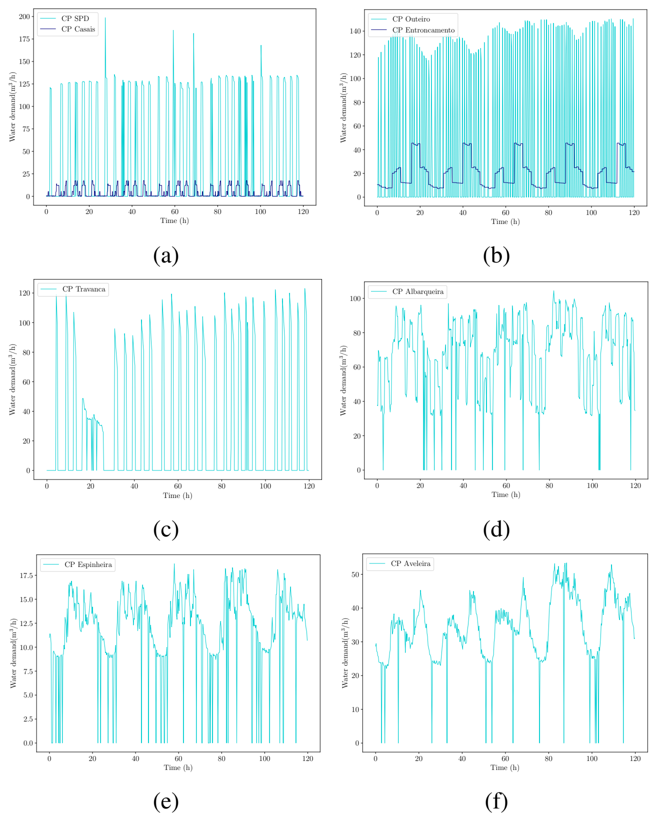



Fig. 8: Water demands of each consumption point from September 25 to 29, 2024: (a) São Pedro Dias and Casais, (b) Outeiro Castro and Entroncamento, (c) Travanca, (d) Albarqueira, (e) Espinheira, and (f) Aveleira.

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