Order acceptance scheduling under Time-Of-Use and energy constraint

Imane Boukerrouis[1], H. Murat Afsar[1], Alice Yalaoui[1]

Abstract— **The present work addresses the issue of Order Acceptance Scheduling (OAS) problem on a single machine, incorporating periodic energy constraints and time-of-use (TOU) electricity pricing. The objective is to maximize net revenue by strategically selecting and scheduling orders while ensuring compliance with energy constraints and TOU pricing structures. This optimization process involves making informed decisions about order acceptance based on profitability, processing time, and energy requirements, while considering the cost fluctuations imposed by TOU pricing. To effectively address this problem, a time-indexed formulation is proposed that provides a structured approach to optimizing scheduling decisions under these constraints. In order to further enhance the performance of the proposed MILP model, two families of valid constraints have been developed. These developments have led to significant improvements in computational efficiency and solution quality.**

Keywords : Order Scceptance Scheduling, Energy Consumption, Time-Of-Use, Time Indexed

I. INTRODUCTION

The manufacturing industry has long been recognized as an energy-intensive sector. In recent years, many companies have sought to implement strategies to optimise energy consumption, motivated by both increasingly stringent government regulations and rising energy costs. Among these strategies, time-of-use (TOU) pricing has emerged as an essential tool for meeting these challenges and remains one of the most widely adopted policies in the world. TOU allows electricity prices to be dynamically adjusted at different times to reduce peak demand and encourage businesses to shift demand to off-peak hours.

Another key approach to optimize energy use in manufacturing is to actively manage the constraints on energy supply during the production process. In fact, in addition to adjusting prices through time-of-use pricing, some companies face direct energy constraints, whether due to supplier restrictions, stringent environmental regulations or the variability of renewable energy sources. These challenges require more intelligent, adaptive production planning to optimise the use of available energy while maintaining business competitiveness.

In this context, the acceptance or rejection of orders takes place in a wide range of manufacturing sectors. Faced with energy constraints and fluctuating electricity prices, it is essential to strategically select and prioritize the tasks to be performed according to the energy resources available. This involves integrating supply limitations, dynamic tariff variations and production targets to simultaneously optimise energy efficiency and production system profitability.

The present work addresses the problem of order acceptance scheduling that incorporates both periodic energy consumption constraints and time-of-use electricity pricing, with the aim of maximizing net profit. We present an improved time-indexed model based on the one proposed by [1].

The remainder of this paper is organized as follows. Section 2 presents a concise review of the existing literature on time-of-use pricing and energy efficiency in scheduling problems, with a particular focus on the OAS problem. Section 3 provides a description of the proposed problem. Section 4 outlines the improved solution approach. Section 5 presents computational results. The last section concludes the paper and draws perspectives.

II. STATE OF THE ART

The Order Acceptance Scheduling (OAS) problem has garnered increasing attention in the literature due to their relevance in modern industrial contexts. Since the early 90s, the OAS problem has been studied under different production settings and with diverse criteria. [2] provided a comprehensive review of the OAS literature, highlighting its applications in single- and multi-machine environments with various job attributes, such as preemption, release dates, and setup times. The OAS problem has been demonstrated to be NP-hard by [3]. The first study on the OAS problem was conducted by [4]. [5] broadened the production setting of the OAS problem by including sequence-dependent setup times and time-related penalties using a disjunctive Mixed Integer Linear Program (MILP). [6] introduced an efficient arc-time-indexed model and a branch-and-price algorithm. More recently [7] developed a matheuristic algorithm for the problem, integrating mathematical models with metaheuristic approaches.

Concurrently, the incorporation of time-of-use energy tariffs into scheduling models has gained significance, driven by economic and environmental concerns. [8] provided a comprehensive review of research on energy-efficient scheduling in manufacturing and categorized studies based on their historical evolution. However, few studies have addressed OAS under TOU tariffs. [9] studied an OAS problem with TOU tariffs and greenhouse gaz (GHG) emissions, proposing a benchmark and an exact solving approach with a disjunctive formulation. For the same problem, [10] introduced two time-indexed formulations and a metaheuristic based on the Dynamic Island Model

[1]Computer Science and Digital Society Laboratory, University of Technology of Troyes, 10000 troyes, France

(DIM) framework.

While substantial attention has been devoted to integrating TOU tariffs into scheduling problems, a critical challenge arises when energy availability itself is constrained. Typically, scheduling problems have focused on optimizing energy costs through the integration of consumption as an objective function. Yet, [11] highlighted that the literature on scheduling problems with energy constraints primarily focuses on parallel machines, job shop, and flow shop systems. In contrast, research on single-machine problems in this context remains very limited. In the study by [12], the simultaneous minimization of weighted completion times and weighted tardiness was addressed, while enforcing an energy consumption constraint for each interval. [13] proved the NP-hardness of the single-machine scheduling problem with non-renewable resources, aiming to minimize the makespan.

The literature on order acceptance scheduling problems under energy constraints is even more sparse. To date, only [14] and [1] have explored energy constraints within a production system that includes order acceptance. In [14], the authors impose a global energy consumption constraint on each parallel machine over the entire scheduling horizon, with the objective of maximizing total profits, and [1] introduces a formal framework to define five new variants of order acceptance scheduling problems under environmental constraints.

In this context, and building upon the work of [1], we enhance the MILP formulation by incorporating TOU tariffs to define our problem. To the best of our knowledge, the single-machine order acceptance scheduling problem with TOU tariffs and limited energy constraints has not yet been studied.

III. PROBLEM DESCRIPTION

This paper considers an order acceptance scheduling problem in presence of TOU-based energy tariffs and with energy consumption limits in each period.

This problem can be formally described as follows. There are n non-preemptive jobs to be processed by a single machine. Each job j is characterized by its processing time p_j, deadline d_j, revenue w_j and its unitary energy consumption e_j. The objective is to maximize the total profits of the accepted jobs. In this context, all orders are available at time zero. The energy consumption rate e_j per unit of time for the order j remains constant throughout the production process. The order is only accepted and the associated total revenue w_j is only obtained if the order is completed before its deadline d_j. The planning horizon is divided into discrete unit periods $t = 0, \dots, T$, where T represents the maximum deadline among all orders, calculated as $T = \max_{j=1,\dots,n}\{d_j + 1\}$. Each period t is defined by two key parameters: an energy limit E_t, which restricts the maximum allowable energy consumption, and a time-of-use energy cost c_t, reflecting the dynamic pricing of energy across the horizon.

Example

An example consisting of 4 jobs is shown in Table I. The energy limits and TOU cost parameters over the scheduling horizon are provided in Table II.

TABLE I: Job parameters

Job	p_j	d_j	w_j	e_j
1	2	5	30	2
2	3	5	40	3
3	1	6	15	1
4	2	4	25	4

TABLE II: Energy limits and costs per time period

t	0	1	2	3	4	5	6
E_t	4	5	3	2	4	3	5
c_t	1	2	3	2	1	1	1

The optimal solution found is the schedule $2 \to 1 \to 3$, with a total profit of 60. This solution is illustrated in Figure 1, where the red line represents the total energy consumption E_t over time, and the dashed blue line shows the energy demand e_j of each scheduled job j at each period t.

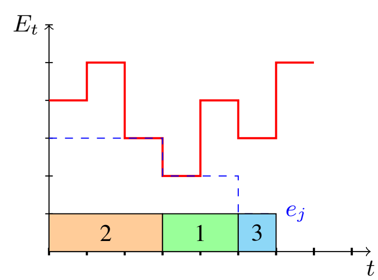

Fig. 1: Illustration of the schedule

IV. FORMULATION

In this section, the time-indexed formulation, denoted (P), is presented for the problem. Initially developed to maximize $\sum w_j a_j$, it serves as the foundation for the present research.

Decision variables are given in equation (1) and (2).

- A set of binary decision variables determining order acceptance:

$$a_j = \begin{cases} 1 & \text{if order } j \text{ is accepted,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

- A set of binary decision variables determining the production periods of the orders:

$$x_{jt} = \begin{cases} 1 & \text{if order } j \text{ is produced at time period } t, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The problem is formulated as a Mixed Integer Linear Program as follows.

$$\max \sum_{j=1}^{n} w_j a_j - \sum_{j=1}^{n} \sum_{t=0}^{d_j} x_{jt} c_t e_j \tag{3}$$

$$\sum_{j=1}^{n} x_{jt} \leq 1, \quad t = 0, \dots, T, \tag{4}$$

$$\sum_{t'=0}^{t-p_j} x_{jt'} + \sum_{t'=t+p_j}^{d_j} x_{jt'} \leq p_j(1 - x_{jt}),$$
$$j = 1, \dots, n, \quad t = 0, \dots, d_j, \tag{5}$$

$$\sum_{t=0}^{d_j} x_{jt} = p_j a_j, \quad j = 1, \dots, n, \tag{6}$$

$$\sum_{t=d_j+1}^{T} x_{jt} = 0, \quad j = 1, \dots, n, \tag{7}$$

$$\sum_{j=1}^{n} e_j x_{jt} \leq E_t, \quad t = 0, \dots, T. \tag{8}$$

$$x_{jt} = 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T, \quad e_j > E_t, \tag{9}$$

The objective (3) maximizes total profit, which is the revenue from accepted jobs minus the energy costs. Constraints (4) ensure that the machine is limited to processing at most one job during each period $t = 0, \dots, T$. Constraints (5) guarantees the non-preemptive scheduling of jobs by ensuring that the binary decision variables representing job processing over time are arranged in contiguous intervals. This prevents interruptions in the execution of each job. Constraints (6) impose that each accepted job $j = 1, \dots, n$ must be processed for exactly p_j time units and completed before its deadline. Constraints (7) ensure that each order cannot be processed after its deadline. Constraints (8) and (9) ensure that no job j can be processed during a period t if its energy consumption e_j exceeds the allowable limit E_t.

In order to enhance the original model and strengthen its ability to handle energy consumption limitations, two groups of additional constraints are introduced. These constraints, designated as (10) and (11), are intended to ensure the implementation of more stringent scheduling regulations, ensuring both the feasibility of the system under energy constraints and the optimised distribution of tasks across designated time periods.

If the energy requirement of job j, e_j, exceeds the available energy, E_t, during time interval $t \in [t', t'']$, then job j can only be scheduled entirely before t' or entirely after t'', but not both. This restriction prevents the job from being partially executed during the infeasible interval. Thus, it ensures compliance with energy limitations and maintains the non-preemptive scheduling requirement.

Proposition 1: The following constraint is valid for the order acceptance scheduling problem in presence of TOU-based energy tariffs and with energy consumption limits in each period.

$$\begin{aligned} x_{jt'-1} + x_{jt''+1} \leq 1, &\quad j = 1, \dots, n, \\ t' = 0, \dots, d_j, &\quad t'' = t', \dots, d_j, \\ e_j > E_{t'}, &\quad e_j > E_{t''}, \end{aligned} \tag{10}$$

Proof:

Suppose that there exists a job j with energy consumption e_j that simultaneously satisfies:

- $e_j > E_{t'}$ and $e_j > E_{t''}$, where the job *cannot* be executed within the interval $[t', t'']$ due to energy limitations.
- $x_{jt'-1} = 1$ and $x_{jt''+1} = 1$, which indicates that job j is executed at period $t' - 1$, then interrupted during the interval $[t', t'']$, before resuming execution at $t'' + 1$.

However, since preemption is not allowed, job j must be executed continuously for its entire duration p_j without interruption. This requirement implies that the job cannot be executed both at $t' - 1$ and $t'' + 1$ simultaneously.
Thus, we conclude that at most one of the conditions can hold:

$$x_{jt'-1} = 1 \quad \text{or} \quad x_{jt''+1} = 1.$$

Consequently, constraint (10) ensures energy feasibility and prevents fragmented scheduling. ■

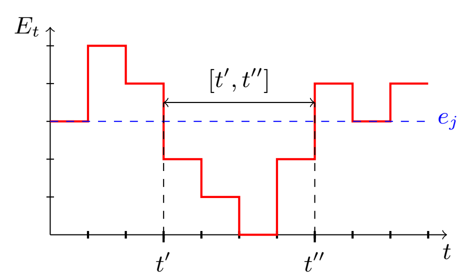

Fig. 2: Illustration of the energy constraints and the forbidden scheduling interval

The job j cannot be scheduled within any time interval $]t_0, t'[$ where the energy consumption limit E_t is satisfied for the job's energy requirement, and the length of this interval is shorter than the job's processing time p_j. This effectively prevents the scheduling of job j in any continuous interval that is too short complete when the energy limit is restrictive throughout that interval.

Proposition 2: The following constraint is valid for the order acceptance scheduling problem in presence of TOU-based energy tariffs and with energy consumption limits in each period.

$$\sum_{t=t_0}^{t'} x_{jt} = 0, \quad j = 1, \ldots, n, \quad t_0 = 0, \ldots, d_j,$$
$$t' = t_0, \ldots, d_j, \quad e_j > E_{t_0}, \quad e_j > E_{t'},$$
$$t^{'} - t_0 < p_j. \tag{11}$$

Proof:

Suppose that a job j is scheduled within the interval $]t_0, t'[$, where the available energy E_t for all $t \in]t_0, t'[$ is greater than or equal to e_j, meaning that the energy is sufficient to execute the job. Additionally, we have the duration of the interval:

$$t' - t_0 < p_j$$

Since preemption is not allowed, the accepted job must be executed continuously for its entire duration p_j. However, since the length of the interval is less than p_j, the job cannot be completed within this time frame. Given that job j cannot be split into multiple non-contiguous execution periods due to the non-preemption constraint, it follows that the scheduling assignment is infeasible.

In conclusion, a job j cannot be executed within a time interval where the available energy is sufficient, but the duration of the interval is smaller than p_j.

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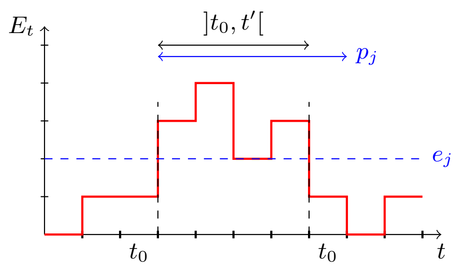

Fig. 3: Illustration of the energy constraints and the forbidden scheduling interval

V. EMPIRICAL RESULTS

The test instances are generated as explained in [15]. Similar TOU tariffs are applied, with a diverse range of job counts $n = 10, 25, 50, 80, 100, 120$, and five instances are generated for each value of n. Processing times (p_j) and revenues (w_j) are drawn independently from a uniform distribution, $p_j, w_j \sim \mathcal{U}(1, 10)$. Deadlines ($d_j$) are determined based on the processing times and are sampled from $d_j \sim \mathcal{U}(p_j, p_T)$, where $p_T = \sum_{j=1}^{n} p_j$ represents the total processing time across all jobs.

The unitary power consumption of each job (e_j) is generated as $e_j \sim \mathcal{U}(1, 90)$, ensuring variability in energy requirements. Energy consumption limits for each time period (E_t) are also randomly sampled from $E_t \sim \mathcal{U}(1, 100)$ for $t = 0, \ldots, T$.

Additionally, four different cases were tested to assess the impact of each constraint group on the overall model. The first case, P, represents the original model, while P_{10} and P_{11} include only constraint (10) and constraint (11), respectively. The final case, P_F, integrates both constraint groups. This experimental setup was designed to analyze the individual contributions of each constraint group as well as their combined influence on the model's overall performance.

The tests were conducted on a PC equipped with an Intel i5 2 GHz CPU processor and 4GB of RAM. The proposed model was implemented using the IBM CPLEX Solver v22.1, with a maximum solving time limited to 3600 seconds.

Table III summarizes the results of the tests on different instances. Each block of rows corresponds to five instances with the same number of jobs (n). For each instance, the table reports the computational time in seconds (cpu), the optimality gap reported by CPLEX in % and the objective function value (obj).

For small instances ($n = 10, 25, 50$), all formulations successfully find optimal solutions over all five instances. The computation time remains relatively small. As the problem size increases, significant differences in performance emerge between the formulations. Starting from $n = 80$, the average CPU time for the P formulation sharply increases, with some instances reaching the 3600-second time limit without finding an optimal solution. In contrast, the P_F formulation successfully solves all five instances with a much lower average CPU time of approximately 157 seconds. For problem sizes $n = 100$ and $n = 120$, both the P and P_{10} formulations failed to solve any instance optimally within the 3600-second time limit. Meanwhile, the P_F and P_{11} formulations succeeded in obtaining optimal solutions for 7 out of the 10 instances. Moreover, the average optimality gap over these 10 instances was 1.95% for P, compared to only 0.05% for P_F. The average CPU time for P_F was approximately 1949 seconds, while P consistently reached the full time limit of 3600 seconds.

Furthermore, the integration of constraints (11) appears to improve the performance of the solver. In fact, constraint (11) enhances the model more effectively than constraint (10), providing a tighter formulation and reducing solution times. Over all instances, the average CPU time is approximately 1573 seconds for P, 1507 seconds for P_{10}, and and notably lower at 735 seconds for P_{11}. This observation suggests that these constraints play a crucial role in speeding up the solution process while maintaining solution quality.

VI. CONCLUSIONS

In this paper, we propose an improved formulation of the time-indexed model for the Order Acceptance Scheduling (OAS) problem under time-of-use pricing and periodic energy consumption limits. Our empirical results show that this new formulation is significantly more efficient compared to the time-indexed formulation proposed in [1].

However, these formulations remain constrained by their spatial complexity, which may hinder their scalability for

TABLE III: Performance comparison

Tasks	Instance	P			P_{10}			P_{11}			P_F		
		cpu	gap	obj	cpu	gap	obj	cpu	gap	obj	cpu	gap	obj
10	1	0.25	0	20.13	0.11	0	20.13	0.14	0	20.13	0.17	0	20.13
	2	0.12	0	18.23	0.28	0	18.23	0.25	0	18.23	0.12	0	18.23
	3	0.46	0	35.84	0.34	0	35.84	0.61	0	35.84	0.37	0	35.84
	4	0.05	0	14.70	0.06	0	14.70	0.11	0	14.70	0.06	0	14.70
	5	0.17	0	24.76	0.17	0	24.76	0.23	0	24.76	0.14	0	24.76
25	1	0.61	0	55.60	0.95	0	55.60	0.90	0	55.60	0.65	0	55.60
	2	0.46	0	63.12	0.48	0	63.12	0.65	0	63.12	0.31	0	63.12
	3	1.25	0	49.92	1.22	0	49.92	0.43	0	49.92	0.18	0	49.92
	4	0.42	0	49.83	0.45	0	49.83	0.53	0	49.83	0.23	0	49.83
	5	0.29	0	51.85	0.33	0	51.85	0.31	0	51.85	0.23	0	51.85
50	1	6.76	0	135.95	6.86	0	135.95	4.68	0	135.95	3.89	0	135.95
	2	7.42	0	142.71	7.45	0	142.71	4.40	0	142.71	4.43	0	142.71
	3	14.79	0	109.30	13.61	0	109.30	10.37	0	109.30	8.40	0	109.30
	4	57.29	0	145.68	56.65	0	145.68	12.97	0	145.68	11.50	0	145.68
	5	18.11	0	149.04	14.98	0	149.04	4.50	0	149.04	3.70	0	149.04
80	1	775.70	0	233.08	427.58	0	233.08	45.10	0	233.08	37.76	0	233.08
	2	3600	0.04	230.03	1773.31	0	230.06	55.56	0	230.06	56.90	0	230.06
	3	2259.60	0	210.89	2554.44	0	210.89	637.01	0	210.89	552.20	0	210.89
	4	853.85	0	216.21	824.11	0	216.21	35.95	0	216.21	31.41	0	216.21
	5	3600	1.71	241.34	3556	0	241.84	113.41	0	241.84	107.65	0	241.84
100	1	3600	0.17	302.52	3600	0.11	302.61	586.94	0	302.64	510.10	0	302.64
	2	3600	4.33	309.39	3600	4.33	309.39	3124.8	0	318.85	3016.56	0	318.85
	3	3600	1.16	222.32	3600	0.45	223.76	475.86	0	223.79	459.03	0	223.79
	4	3600	0.05	291.10	3600	0.72	290.89	79.13	0	291.33	75.27	0	291.33
	5	3600	0.29	289.36	3600	0.27	289.36	443.92	0	289.48	485.50	0	289.48
120	1	3600	4.15	433.53	3600	2.40	441.94	3600	0.33	444.93	3600	0.27	446.27
	2	3600	1.64	307.63	3600	1.64	307.63	3600	0.27	310.26	3600	0.17	310.37
	3	3600	2.31	386.92	3600	2.32	386.91	3600	0.10	392.10	3600	0.09	392.12
	4	3600	3.64	363.38	3600	3.64	363.37	3273.11	0	371.93	2756.27	0	371.93
	5	3600	1.72	360.57	3600	1.69	360.7	2361.08	0	361.35	1388	0	361.35

large problem instances. Future research will focus on the development of special cutting planes and alternative exact approaches to improve computational efficiency and performance for large instances. In addition, a more comprehensive formalization of additional constraints relevant to energy-efficient scheduling is essential. In this context, an extension of the proposed formulation can be explored to integrate setup times, release dates, sequence-dependent setup times, and penalty costs, further increasing its applicability to real-world scheduling environments while ensuring a more precise and adaptable decision framework.

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