

# Fractional Order Lyapunov based Indirect Adaptive Backstepping Control Design for DELTA Robot

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**Abstract**—This paper presents a novel Fractional Order Lyapunov-based Indirect Adaptive Backstepping Control strategy for the precise trajectory tracking of a DELTA robot. The proposed approach integrates fractional calculus with Lyapunov stability theory to enhance the robustness and adaptability of the controller in the presence of system uncertainties and external disturbances. Unlike conventional backstepping controllers, the indirect adaptive mechanism estimates unknown system parameters online, improving control performance without requiring precise dynamic modeling. In robotic systems, accurately capturing the inherent viscoelasticity, actuator dynamics, and noninteger-order behaviors requires fractional-order modeling, which provides a more realistic and flexible representation of system dynamics. The necessity of fractional calculus in robotics motivates the design of the proposed controller, ensuring better adaptability and robustness. The effectiveness of the control strategy is validated through simulation results, demonstrating its feasibility and improved performance in robotic applications.

**Index Terms**—{redFractional-order modeling, Indirect adaptive control, Lyapunov theory, Backstepping approach, DELTA robot, Robustness to uncertainties.

## I. INTRODUCTION

Fractional calculus, which generalizes integer-order differentiation and integration to non-integer orders, has become increasingly important in control theory due to its ability to more accurately model complex dynamical systems [30]. Fractional-order systems (FOS) [22], [23] have demonstrated improved robustness and stability compared to classical controllers, making them suitable for systems with uncertainties and nonlinear behavior [1], [2].

Parallel robots, such as the {redDELTA robot, are widely employed in industrial applications for their high-speed capabilities, precision, and rigidity. However, their nonlinear dynamics, coupled kinematics, and parameter sensitivity pose significant control challenges [3], [4]. Conventional control methods, including PID, LQR, SMC, and optimization  $H_\infty$  [2] have been widely applied [5], but often face limitations.

Advanced strategies like adaptive and robust control have been proposed to enhance performance [6].

Recent studies have explored various control strategies for parallel robots [7], [8], [10], with fractional-order controllers emerging as promising tools. These controllers exploit the memory and hereditary characteristics of fractional derivatives to improve response and robustness [11]. Lyapunov-based stability analysis further supports their potential to ensure uniform stability in nonlinear systems [12].

This paper investigates the application of fractional-order backstepping control. {redDELTA parallel robots [26], renowned for their high-speed and high-precision capabilities, present complex nonlinear dynamics that pose significant challenges for control. Backstepping control has been effectively employed to address these challenges by decomposing the control problem into a series of sub-problems. This method facilitates the design of virtual control inputs that ensure the stability of each subsystem, ultimately leading to the stabilization of the entire system. This approach has proven beneficial in enhancing the trajectory tracking performance of {redDELTA robots, particularly in applications requiring precise end-effector positioning and rapid response times.

Building upon the foundational principles of backstepping, Adaptive Fractional-Order Backstepping Control (AFOBC) [20], [7] integrates fractional calculus and adaptive control strategies to further improve the robustness and accuracy of {redDELTA robot control systems. The incorporation of fractional-order dynamics allows for a more accurate representation of the system's memory and hereditary properties [31], which are often observed in mechanical systems like {redDELTA robots. Adaptive mechanisms within this framework enable real-time estimation and compensation for uncertainties and external disturbances, ensuring consistent performance even under varying operating conditions.

This paper is structured into 6 sections; Section 2 presents a dynamical modeling of {redDELTA robot, the third section

states the problem formulation. Section 4 d describes the design of the proposed AFOBC strategy. Section 5 discusses the simulation setup and results, highlighting the performance improvements achieved. Finally, Section 6 concludes the paper and suggests directions for future research.

## II. MODELING OF DELTA ROBOT MANIPULATOR

According to [13], [14] the dynamic model of the DELTA robot can be derived by the following equation:

$$M(\theta_i)\ddot{\theta}_i + C(\theta_i, \dot{\theta}_i)\dot{\theta}_i + G(\theta_i) + \Delta(t) = u_i \quad (1)$$

where:

- {red  $M(\theta_i)$  is the inertia  $3 \times 3$  matrix, representing the robot's resistance to acceleration.
- {red  $C(\theta_i, \dot{\theta}_i)$  is the Coriolis and centrifugal force  $3 \times 3$  matrix, accounting for velocity-dependent forces.
- {red  $G(\theta_i)$  a vector of of dimension  $3 \times 1$  representing the gravitational forces acting on the system.
- {red  $u_i$  is the torque applied at the joints, it is the control signal of dimension  $3 \times 1$ .
- {red  $\theta_i$  represents the joint position of the actuators, for  $i = 1, 2, 3$
- The term  $\Delta(t)$  denotes uncertainties due to external disturbances, resistive torques, and payload effects acting on the system.



Fig. 1. DELTA Robot

## III. PROBLEM FORMULATION

### A. Preliminaries

Fractional order calculus extends the concept of integer-order differentiation and integration to non-integer (fractional) orders, allowing for more precise modeling of dynamic systems with memory and hereditary properties [30] [16].

The integro-differential operator  $aDt^\alpha$  where  $a$  and  $t$  are the limits of the operation is defined as:

$$aDt^\alpha = \begin{cases} \frac{d^\alpha}{dx^\alpha}, & \Re(\alpha) > 0, \\ 1, & \Re(\alpha) = 0, \\ \int_a^t (d\tau)^{-\alpha}, & \Re(\alpha) < 0. \end{cases} \quad (2)$$

where  $\alpha$  is the order of the operation, generally  $\alpha \in R$ .

Several mathematical definitions exist for fractional-order integration and differentiation. These definitions do not always lead to identical results but are equivalent for a wide range of functions, let us consider the Caputo definition given by [17]:

$$D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (3)$$

where  $n-1 < \alpha < n$ ,  $n \in N$ , and  $\Gamma(\cdot)$  is the Gamma function.

*Definition 1* ([23], [22]): Let  $g : [0, b) \rightarrow \mathcal{D}$  be a continuous function. If  $g$  is strictly increasing and satisfies  $g(0) = 0$ , then  $g$  is said to belong to class- $\mathcal{K}$ .

The following lemma is important for constructing Lyapunov functions in order to analyze the stability of non linear fractional-order systems.

*Lemma 1* ([23], [22]): Assume that the origin is an equilibrium point of the following fractional-order nonlinear system:

$${}_0D_t^\alpha x(t) = f(t, x(t), u(t)) \quad (4)$$

where  $f : \mathcal{D} \rightarrow R^n$  is a Lipschitz continuous function. If there exists a Lyapunov function  $V(t, x(t))$  and class- $\mathcal{K}$  functions  $g_i$  ( $i = 1, 2, 3$ ) such that

$$g_1(\|x(t)\|) \leq V(t, x(t)) \leq g_2(\|x(t)\|), \quad (5)$$

$${}_0D_t^\alpha V(t, x(t)) \leq -g_3(\|x(t)\|), \quad (6)$$

then the system in (4) is asymptotically stable, i.e.,  $\lim_{t \rightarrow \infty} x(t) = 0$ .

*Lemma 2* ([22], [24]): If  $e(t)$  is a smooth function, then

$$\frac{1}{2} {}_0^C D_t^\alpha (e^T(t)e(t)) \leq e^T(t) {}_0D_t^\alpha e(t), \quad \forall t \in \mathcal{D}. \quad (7)$$

### B. Fractional Order state space model for delta robot

Fractional order modeling is essential to accurately capture the dynamic behavior of the Delta robot [30], as it accounts for the effects of memory and nonlocal properties inherent in robotic systems. Unlike integer-order models, fractional derivatives provide enhanced flexibility in representing system dynamics [18], [19]. This section proposes a fractional-order state-space representation of the Delta robot, offering a more precise framework for control design [31].

The state vector for the fractional-order model is defined as:

$$\mathbf{x} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad D^\alpha \theta_1 \quad D^\alpha \theta_2 \quad D^\alpha \theta_3]^T \quad (8)$$

$$\mathbf{x}_1 = [\theta_1 \quad \theta_2 \quad \theta_3]^T \quad (9)$$

$$\mathbf{x}_2 = [D^\alpha \theta_1 \quad D^\alpha \theta_2 \quad D^\alpha \theta_3]^T \quad (10)$$

The input vector is:

$$\mathbf{u} = [u_1 \quad u_2 \quad u_3]^T \quad (11)$$

Hence, the fractional-order state-space representation of the {redDELTA} robot is given by:

$$D^\alpha \mathbf{x}(t) = \begin{bmatrix} x_2 \\ M(x_1)^{-1}(\mathbf{u} - C(x_1, x_2)x_2 - G(x_1) + \Delta(t)) \end{bmatrix} \quad (12)$$

Where;

- The non-integer order  $\alpha$  the non integer-order to capture the intrinsic fractional dynamics of robots.

#### IV. ADAPTIVE BACKSTEPPING CONTROL LAW DESIGN

##### A. Backstepping control concept

We first define the tracking error vector:

$$e_i(t) = x_i(t) - x_{ir}(t), i = 1, 2 \quad (13)$$

The estimation error is given by,

$$\tilde{\Delta}(t) = \Delta(t) - \hat{\Delta}(t) \quad (14)$$

Assuming the slow dynamics of uncertainties ,

$$D^\alpha \Delta(t) \approx 0 \quad (15)$$

##### B. Adaptive FO Backstepping Control Law

**Theorem 1.** Consider the plant described by Eq. (12), with order  $0 < \alpha < 1$ , a positive gain  $\lambda > 0$ , and a symmetric positive definite matrix  $\Gamma > 0$ . Under the respectively virtual, adaptation, and global following control laws:

$$u_v = -\lambda e_1(t) + D^\alpha x_{1r} \quad (16)$$

$$D^\alpha \hat{\Delta}(t) = \Gamma^{-1} M^{-T} e_2 \quad (17)$$

$$\mathbf{u} = M(D^\alpha u_v - e_1 - \lambda e_2) + CD^\alpha x_1 + G - \hat{\Delta}(t) \quad (18)$$

All the trajectories in the closed-loop adaptive fractional order system are globally stable and converge to the desired dynamics., i.e.,  $\lim_{t \rightarrow \infty} e_i(t) = 0$ .

##### C. Proof

Let consider the following lyapunov candidate function:

$$V_1(t, e_1(t)) = \frac{1}{2} e_1(t)^T e_1(t) \quad (19)$$

According to Lemma 2, which provides an upper bound on the fractional derivative of a quadratic form, we have:

$$D^\alpha V_1 \leq e_1(t)^T D^\alpha e_1(t) \quad (20)$$

To ensure that  $D^\alpha V_1 < 0$ , we design the virtual control such that:

$$e_1(t)^T D^\alpha e_1(t) < 0 \quad (21)$$

This leads to:

$$e_1(t)^T D^\alpha e_1(t) = e_1(t)^T \cdot (D^\alpha x_1 - D^\alpha x_{1r}) \quad (22)$$

Then we propose to choose the following virtual control signal to stabilize the position error ,

$$u_v = -\lambda e_1(t) + D^\alpha x_{1r} \quad (23)$$

where  $\lambda > 0$  is a positive control gain.

To further analyze stability, we define the following composite Lyapunov function {red following backstepping approach:

$$V_2 = V_1 + \frac{1}{2} e_2(t)^T e_2(t) + \frac{1}{2} \tilde{\Delta}(t)^T \Gamma \tilde{\Delta}(t) \quad (24)$$

where  $\Gamma$  is a symmetric positive definite matrix.

According to Lemma 2, the fractional derivative of  $V_2$  satisfies:

$$D^\alpha V_2 \leq e_1^T D^\alpha e_1 + e_2^T D^\alpha e_2 + \tilde{\Delta}(t)^T \Gamma D^\alpha \tilde{\Delta}(t) \quad (25)$$

According to (14) :

$$D^\alpha \tilde{\Delta}(t) = -D^\alpha \hat{\Delta}(t) \quad (26)$$

We aim to design the control input  $\mathbf{u}$  to make  $D^\alpha V_2 < 0$ . From the previous results:

$$e_1^T D^\alpha e_1 = e_1^T (-\lambda e_1 + e_2) \quad (27)$$

Substituting into the expression for  $D^\alpha V_2$ , we obtain:

$$D^\alpha V_2 \leq -\lambda \|e_1\|^2 + e_2^T (e_1 + D^\alpha e_2) - \tilde{\Delta}(t)^T D^\alpha \hat{\Delta}(t) \quad (28)$$

The derivative  $D^\alpha e_2$  can be computed as:

$$D^\alpha e_2 = M(x_1)^{-1} (\mathbf{u} - Cx_2 - G + \Delta(t)) - D^\alpha u_v \quad (29)$$

The proposed control law is then:

$$\mathbf{u} = M(D^\alpha u_v - e_1 - \lambda e_2) + Cx_2 + G - \hat{\Delta}(t) \quad (30)$$

Substituting this into the expression for  $D^\alpha V_2$ , we get:

$$D^\alpha V_2 \leq -\lambda \|e_1\|^2 - \lambda \|e_2\|^2 + e_2^T M^{-1} \tilde{\Delta}(t) - \tilde{\Delta}(t)^T \Gamma D^\alpha \hat{\Delta}(t) \quad (31)$$

To eliminate the disturbance term, according to [9] we define the adaptation law as:

$$D^\alpha \hat{\Delta}(t) = \Gamma^{-1} M^{-T} e_2 \quad (32)$$

Thus, the final expression becomes:

$$D^\alpha V_2 \leq -\lambda \|e_1\|^2 - \lambda \|e_2\|^2 < 0 \quad (33)$$

This ensures the exponential stability of the closed-loop system, in accordance with Lemma 1 and Lemma 2 using the extended Lyapunov stability theorem.

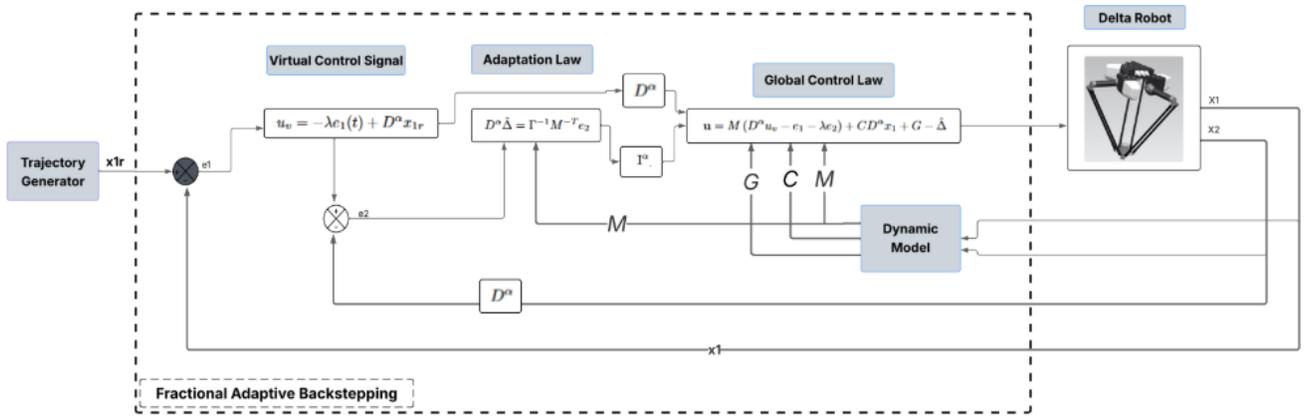


Fig. 2. Closed loop control scheme

## V. SIMULATIONS AND DISCUSSIONS

To validate the effectiveness of the proposed fractional-order backstepping control strategy, simulations {red ( For  $\alpha = 0.8$ , the model is designed to closely approximate the dynamics of an equivalent integer-order system, while effectively capturing the intrinsic fractional characteristics of the real system without introducing significant disruption to its behavior. ) are performed in MATLAB/Simulink. The DELTA robot's end-effector is tasked with a sinusoidal trajectories on its joints angular position. This trajectory is carefully designed to ensure smooth motion, along with continuous velocity and acceleration profiles, which are crucial for precise tracking and stability analysis.

The Table bellow summarizes the parameters of the robot , According to [15].

TABLE I  
DELTA ROBOT PARAMETERS

Parameter	Notation	Value	Unit
The Arm	$L_A$	240	mm
The Forearm	$L_b$	480	mm
Diameter of the Nacelle	$D_n$	30	mm
Motor's Inertia	$I_m$	$1.8 \cdot 10^{-4}$	$\text{Kg.m}^2$
Diameter of the Base	$D_b$	180	mm
Mass of the Nacelle	$m_n$	0.3815	Kg
Mass of the Forearm	$m_b$	0.2209	Kg

By observing the joint position curve in Fig. 6, we notice a perfect tracking of the reference. The FO Backstepping controller delivered excellent performance with a significantly low rise time (about 0.005 s), highlighting the stability of the closed-loop system. Furthermore, the controller demonstrates a high level of performance generally required for robotic applications.

The tracking performance of the system can also be evaluated by analyzing the curves along the 3d space trajectory, in Fig.6. As observed, all the end-effector response exhibits excellent tracking of the reference trajectory, demonstrating the effectiveness of the control strategies implemented.

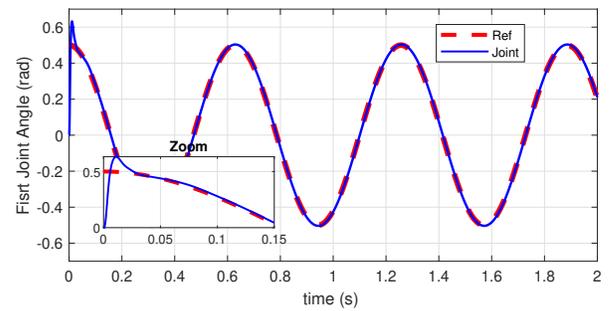


Fig. 3. Joint angle trajectory

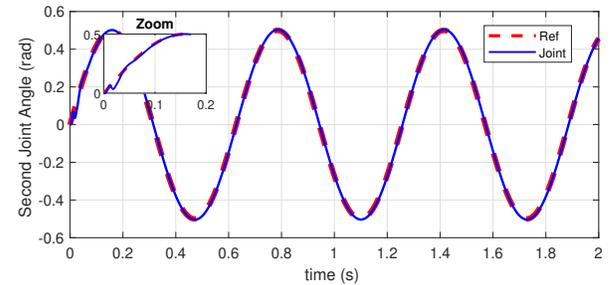


Fig. 4. Joint angle trajectory

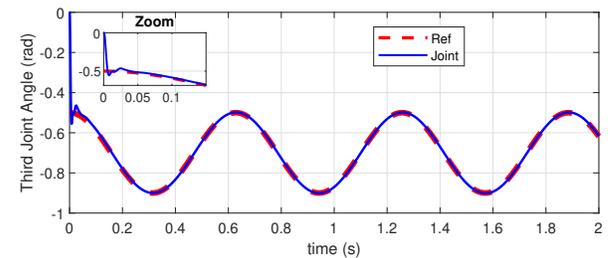


Fig. 5. Joint angle trajectory

To analyze the performances of the control law, the root mean square error (RMSE) of the trajectories for both con-

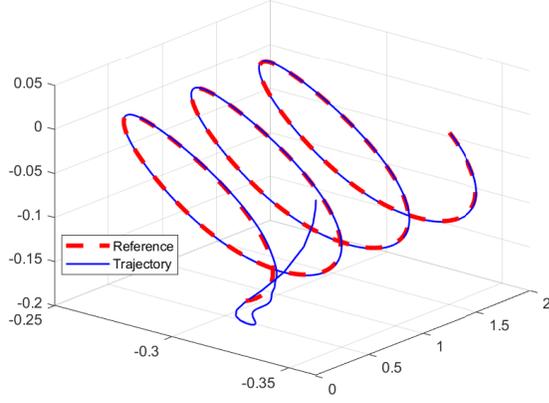


Fig. 6. 3D space trajectory

trollers is summarized in Table II.

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{ir} - x_i)^2} \quad (34)$$

TABLE II  
RMSE VALUES

Controller	RMSE (x-axis)	RMSE (y-axis)	RMSE (z-axis)
FO Backstepping	$1.8 \times 10^{-4}$	$3.2 \times 10^{-10}$	$6.4 \times 10^{-5}$

The RMSE values highlight the tracking accuracy of the control law along the x, y, and z axes, highlighting that the control objective is achieved.

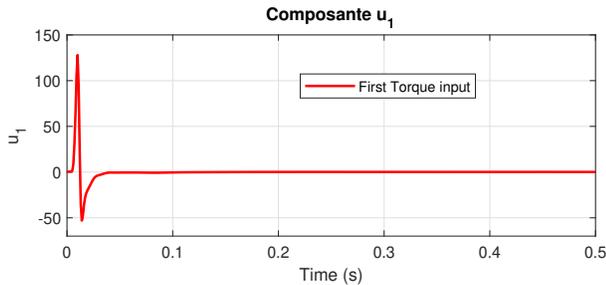


Fig. 7. Induced Control Signal

The figures 7, 8, and 9 illustrate the torque evolution over time for the FO Backstepping controller. Initially, a significant torque overshoot is observed during the transient phase due to the influence of the fractional dynamics on rise time and performance. However, it is rapidly stabilized around 0.05 ( worst case )in the steady state.

Implementing fractional derivatives in real-time is challenging due to their non-local nature, requiring memory of past states and complex approximations like Oustaloup filters. This increases computational load, making high-speed control on embedded systems resource-intensive.

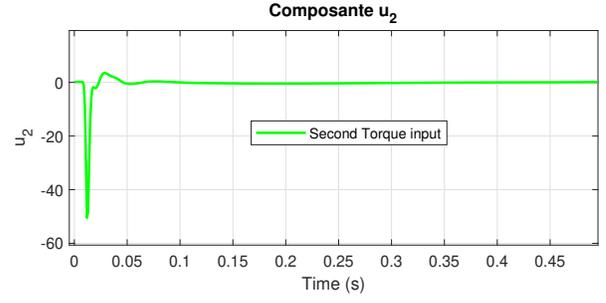


Fig. 8. Induced Control Signal

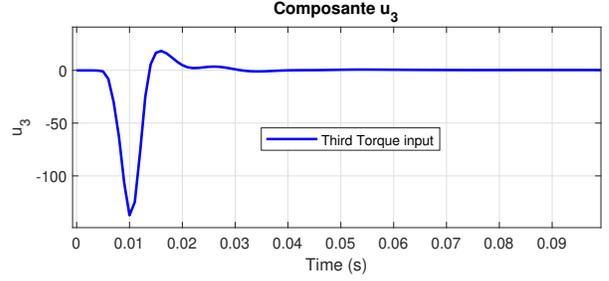


Fig. 9. Induced Control Signal

## VI. CONCLUSION

In this paper, we proposed a robust control strategy for a {redDELTA robot using a Fractional-Order (FO) Adaptive Backstepping approach. The study began with a detailed modeling of the {redDELTA robot, followed by an introduction to fractional-order calculus and its benefits in modeling and control applications. An adaptive backstepping control law was developed to enhance the system's performance, ensuring accurate trajectory tracking and improved stability. Extensive simulations demonstrated that the FO Backstepping controller achieves perfect tracking accuracy, with the position curves along the x and z axes showing excellent reference tracking. The RMSE values confirm the enhanced precision of the proposed method. Furthermore, torque evolution analysis revealed that the FO Backstepping approach provides smoother control actions, effectively reducing undesired oscillations and contributing to overall system robustness.

## REFERENCES

- [1] H. Bouyedda and S. Ladaci, *Optimal tuning of fractional order PI $\lambda$ D $\mu$  controllers using genetic algorithms*, 2016 8th International Conference on Modelling, Identification and Control (ICMIC), IEEE, pp. 207-212, 2016.
- [2] Y. Hatem, S. Ihadadene, and S. Ladaci, *Robust Optimal Control Design for a Parallel DELTA Robot*, Proc. the 2024 IEEE International Conference of the African Federation of Operational Research Societies (AFROS), Tlemcen, Algeria, pp. 1-6, 3-5 November, 2024.
- [3] K. Wen, Y.-C. Cheng, H.-H. Chiang, I.-H. Li, L.-W. Lee, *FPGA-based Control Design and Implementation of DELTA robots Using Horizontal-Axial Pneumatic Actuators*, IFAC-PapersOnLine, vol. 56, No. 2, pp. 11633-11638, 2023.
- [4] A. Gasparetto and L. Scalera, *From the Unimate to the Delta robot: the early decades of Industrial Robotics*. In: Zhang, B., Ceccarelli, M. (Eds) Explorations in the History and Heritage of Machines and Mechanisms.

- History of Mechanism and Machine Science, vol 37. Springer, Cham, 2019.
- [5] K. Rabah and S. Ladaci, *A fractional adaptive sliding mode control configuration for synchronizing disturbed fractional-order chaotic systems*. Circuits, Systems, and Signal Processing, vol. 39, no. 3, pp. 1244–1264, 2020.
  - [6] A. O. Aguilar-Mejia, J. M. Escorcia-Hernandez, R. Tapia-Olvera, H. Minor-Popocatl, and A. Valderrabano-Gonzalez, *Adaptive Control of 3-DOF Delta Parallel Robot*, 2019 IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC), Ixtapa, Mexico, 2019.
  - [7] W.S. Huang, P.L. Hsu, *Design the Adaptive Controller for Delta Robots with the Decoupling-Current Servo Motor*. In: Liu, H., Kubota, N., Zhu, X., Dillmann, R., Zhou, D. (eds) Intelligent Robotics and Applications. ICIRA 2015. Lecture Notes in Computer Science(), vol 9245. Springer, Cham, 2015.
  - [8] J. d. D. Flores-Mendez, H. Schiøler, S. Bai and O. Madsen, *Force estimation and control of Delta Robot for assembly*, 2021 IEEE Conference on Control Technology and Applications (CCTA), San Diego, CA, USA, pp. 640-647, 2021.
  - [9] Y. Wei, Y. Chen, S. Liang, and Y. Wang, *A novel algorithm on adaptive backstepping control of fractional order systems*, Neurocomputing, Accepted Manuscript, Elsevier, 2015. DOI: <http://dx.doi.org/10.1016/j.neucom.2015.03.029>.
  - [10] T.-L. Bui, T.-H. Nguyen, X.-T. Nguyen, *A Controller for Delta Parallel Robot Based on Hedge Algebras Method*, Journal of Robotics, 2271030, pp. 1-20, 2023.
  - [11] R. d. J. Portillo-Vélez, Burgos-Castro, I. A., Vásquez-Santacruz, J. A., and Marín-Urías, L. F. *Integrated Conceptual Mechatronic Design of a Delta Robot*, Machines, vol. 10, no. 3, 186, 2022.
  - [12] M. A. Duarte-Mermoud, N. Aguila-Camacho, J. A. Gallegos, and R. Castro-Linares, *Using general quadratic Lyapunov functions to prove Lyapunov uniform stability for fractional order systems*, Communications in Nonlinear Science and Numerical Simulation, vol. 22, no. 1–3, pp. 650-659, 2015.
  - [13] L. Rey, and R. Clavel, *The Delta Parallel Robot*. In: Boër, C.R., Molinari-Tosatti, L., Smith, K.S. (eds) Parallel Kinematic Machines. Advanced Manufacturing. Springer, London, 1999.
  - [14] H. K. Khalil, *Nonlinear Control*. Prentice Hall, NJ, 2015.
  - [15] A. Codourey, *Dynamic modeling of parallel robots for computed-torque control implementation*, The International Journal of Robotics Research, vol. 17, no. 12, pp. 1325-1336, 1998.
  - [16] S. Ladaci and A. Charef, *On fractional adaptive control*, Nonlinear Dynamics, vol. 43, pp. 365-378.
  - [17] H. Benchaita and S. Ladaci, *Fractional adaptive SMC fault tolerant control against actuator failures for wing rock supervision*, Aerospace Science and Technology, vol. 114, 106745, 2021.
  - [18] Y. Ali-Larnene, S. Ladaci and A. Belmeguenai, *Fractional-order model-based robust identification for a vehicle semi-active electro-rheological suspension system*. International Journal of Advanced Mechatronic Systems, vol. 11, no. 2, pp 111-119, 2024.
  - [19] S. Ladaci, S.E. Khelas, A.R. Ynineb, D. Copot and C.M. Ionescu, *Fractional order MRAC control design for a lightning system based on a fractional order second degree model*, IFAC-PapersOnLine, vol. 58, no. 12, 95-100, 2024.
  - [20] Z.-X. Yang, J.-Q. Peng, and M.-Y. Chen, *Adaptive Control of a Linear Delta Robot Based on Backstepping Design*, 2018 Joint 10th International Conference on Soft Computing and Intelligent Systems (SCIS) and 19th International Symposium on Advanced Intelligent Systems (ISIS), Toyama, Japan, 2018.
  - [21] S. Ladaci, J. Loiseau and A. Charef *Using fractional order filter in adaptive control of noisy plants*, Proceedings of the International Conference on Advances in Mechanical Engineering And Mechanics, ICAMEM 2006 Hammamet, Tunisia. 17-19 December, 2006.
  - [22] N. Aguila-Camacho, M. A. Duarte-Mermoud, and J. A. Gallegos, *Lyapunov functions for fractional order systems* Communications in Nonlinear Science and Numerical Simulation vol. 19, no. 9, pp. 2951–2957, 2014.
  - [23] Y. Li, Y. Chen, and I. Podlubny, *Mittag-Leffler stability of fractional order nonlinear dynamic systems*, *Automatica*, vol. 45, no. 8, pp. 1965–1969, 2009.
  - [24] H. Liu, Y. Pan, S. Li, and Y. Chen, *Adaptive fuzzy backstepping control of fractional-order nonlinear systems*, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, [25] S. Amrane, C. E. Boudjedir, and D. Boukhetala, *Adaptive control of a parallel robot via backstepping technique*, *International Journal of Systems, Control and Communications*, vol. 1, no. 1, pp. 45–55, 2009.
  - [26] M. Litim, A. Omari, L. Vermeiren, and A. Benyamine, *Control of 2-DOF planar parallel manipulator using backstepping approach*, in *Proc. Int. Conf. on Advanced Robotics (ICAR)*, 2015, pp. 1–6.
  - [27] Y. Zhang *et al.*, *Backstepping sliding mode robust control for a wire-driven parallel robot based on a nonlinear disturbance observer*, *Mathematical Problems in Engineering*, vol. 2020, Article ID 3146762, pp. 1–11, 2020.
  - [28] W. Zhao, *Design of an adaptive backstepping controller for 2 DOF parallel robot*, M.S. thesis, Dept. Mech. Eng., Univ. of Florida, 2013.
  - [29] A. Ahmadi *et al.*, *Adaptive backstepping controller based on a novel framework for dynamic solution of an ankle rehabilitation spherical parallel robot*, *Robotica*, vol. 41, no. 6, pp. 1234–1250, 2023
  - [30] A. Mujumdar, B. Tamhane, and S. Kurode, *Fractional order modeling and control of a flexible manipulator using sliding modes*, *Proc. of the American Control Conference (ACC)*, Portland, OR, USA, pp. 1–6, 2014.
  - [31] K. Bingi, B. R. Prusty, and A. P. Singh, *A Review on Fractional-Order Modelling and Control of Robotic Manipulators*, *Robotics*, vol. 12, no. 1, p. 20, 2023. Available: <https://www.mdpi.com/2218-6581/12/1/20>