State Estimation using Extended Kalman Filter for Fractional Model Predictive Control of Fractional Chaotic Rössler Oscillator

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Abstract:
In this research, we propose a robust state estimation framework for a fractional-order Rössler system by implementing Fractional Model Predictive Control (FMPC) with Kalman filtering. The Grünwald–Letnikov (GL) characterization is employed to model the fractional dynamics due to its computational efficiency and ease of implementation, distinguishing it from the more commonly used Caputo and Riemann–Liouville characterizations. Our FMPC framework directly incorporates fractional-order models into the predictive control structure, thereby enhancing control precision and stability. Two scenarios are investigated: one where the state z is measured to infer x and y, and another where x and y are measured to estimate z. In this scenario, realistic conditions with Gaussian noise are considered for the control design. By leveraging the memory properties of fractional derivatives and employing a tailored Extended Kalman Filter (EKF) to ensure state retrieval, our simulations demonstrate that the integrated FMPC–EKF strategy reliably tracks and estimates all state variables under various noise profiles. These results underscore significant improvements in the robustness and accuracy of state estimation for nonlinear, chaotic systems, offering promising implications for practical applications in complex dynamic environments.

Keywords: Chaotic fractional-order Rössler Oscillator, Grünwald-Letnikov Characterization, Fractional Model Predictive Control (FMPC), Steady-state convergence, Extended Kalman Filter (EKF), State Estimation.

1. INTRODUCTION

Fractional-order dynamical systems like the Rössler, Chua, Lü, Chen, Duffing, Hopf, and Lorenz models effectively capture complex behaviors arising from memory and non-local interactions (Lovoie et al. (1976) and Das et al. (2025b)). This study focuses on the fractional Rössler oscillator—comprising two linear and one nonlinear differential equation—well-suited for analyzing fractional dynamics. Using Caputo or Grünwald–Letnikov derivatives, the fractional order α governs the influence of historical states, enabling exploration of chaotic and hyperchaotic behavior (Oliveira et al. (2014)). Its phase space demonstrates deterministic yet chaotic trajectories, with embedded periodic orbits and sensitivity to initial conditions (Hunt and Ott (1996)).

Existing studies show the utility of fractional-order control methods like MPC and EKF for enhancing system stability and adaptability (Romero et al. (2012)). EKF approaches in fractional settings improve state estimation in nonlinear systems and perform robustly under uncertainty (Sierociuk and Dzieliński (2006)). This work introduces a novel integration of Grünwald–Letnikov-based differentiation with an EKF–MPC framework applied to the fractional Rössler system. The proposed method enhances control robustness and estimation accuracy.

Fractional Model Predictive Control (FMPC) is used to stabilize chaotic dynamics, with nonlinear optimization via MATLAB's *fmincon* ensuring constraint satisfaction. The FMPC is coupled with a consistent fractional-model-based EKF for state estimation. Two noise-affected measurement scenarios are analyzed: (i) measuring z, estimating x, y; (ii) measuring x, y, estimating z. Results demonstrate that the FMPC–EKF framework effectively achieves robust estimation and control in chaotic fractional systems.

In this work, the Extended Kalman Filter (EKF) is preferred over the Unscented Kalman Filter (UKF) and Particle Filter (PF) due to its computational efficiency, low memory requirements, and model simplicity (Konatowski et al. (2016)). The EKF uses Jacobian-based linearization to reduce the computational cost associated with nonlinear filtering, making it suitable for real-time and embedded applications. In contrast, the UKF and PF demand higher computational resources—UKF due to nonlinear propagation techniques and PF due to the need for numerous particles to estimate posterior distributions. When process and measurement noises are Gaussian and Jacobians are accessible, the EKF provides a reliable and efficient state estimation framework, offering a practical balance between accuracy and computational demands.

A notable application of EKF within an MPC framework is demonstrated in humanoid robot gait control by Zhang et al. (2022), where EKF estimates key state variables like the Center of Mass (CoM) in real-time. This enables the MPC to generate adaptive control actions, enhancing robustness on uneven terrain. Their approach shows that integrating EKF with MPC improves locomotion stability under disturbances. Building on this, the present research can extend EKF-based estimation within a FMPC framework to more complex fractional-order systems.

The article is structured as follows: Section 2 introduces the Grünwald–Letnikov fractional derivative; Section 3 reviews the fractional Rössler oscillator; Section 4 presents the EKF–FMPC integration methodology; Section 5 analyzes results on chaos suppression and stabilization under noise; and Section 6 concludes with key findings and future research directions.

2. FRACTIONAL DERIVATIVE CHARACTERIZATION

Fractional derivatives extend classical differentiation to non-integer orders, offering a comprehensive framework for capturing system dynamics that include memory effects. Unlike conventional derivatives, which quantify the instantaneous rate of change—fractional derivatives allow for differentiation to non-integer orders, thus providing a more refined analysis of temporal behaviors. In this study, we employ the Grünwald–Letnikov (GL) characterization due to its computational efficiency and straightforward implementation compared to the Caputo or Riemann–Liouville characterizations. The analytical expression for the Grünwald–Letnikov fractional derivative of a function $f(t)$ at time t_i and order α is given by

$$D^{\alpha}_{GL} f(t_i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t^{\alpha}} \sum_{n=0}^{i-1} (-1)^n \binom{\alpha}{n} f(t_i - n\Delta t) \qquad (1)$$

where $\binom{\alpha}{n}$ is the generalized binomial coefficient (Owolabi and Atangana (2019)). For practical computations, this limit is approximated using a finite time step dt:

$$D^{\alpha}_{GL} f(t_i) \approx \frac{1}{dt^{\alpha}} \sum_{n=0}^{i-1} (-1)^n \binom{\alpha}{n} f(t_i - n dt) \qquad (2)$$

This characterization, viewed as an extension of the Euler method—employs recursively defined, nonnegative fractional binomial coefficients that act as damping factors, thereby enhancing numerical stability and computational efficiency (Scherer et al. (2011)). The ease of implementation makes the GL approach particularly suitable for both analytical investigations and computational simulations of fractional-order systems.

3. FRACTIONAL CHAOTIC RÖSSLER OSCILLATOR

The fractional chaotic Rössler system is modeled by incorporating fractional derivatives defined via the GL characterization. Specifically, the system is represented as (Das et al. (2024)):

$$\begin{aligned} D^{\alpha}_{GL} x &= -y - z \\ D^{\alpha}_{GL} y &= x + ay \\ D^{\alpha}_{GL} z &= b + z(x - c) \end{aligned} \qquad (3)$$

where $0 < \alpha < 1$ is a real fractional order, x, y and z are state variables and D^{α}_{GL} denotes the commensurate fractional derivative as defined by the GL method (see Equation (1)). The fractional-order Rössler oscillator, defined by parameters a, b, c, and fractional order α, provides an efficient framework for investigating complex chaotic phenomena—bifurcations, synchronization, and control strategies Čermák and Nechvátal (2018). By employing the Grünwald–Letnikov characterization in our work, memory and non-locality into the state-update equations for x, y, and z are introduced, and thereby enriches the system's nonlinear and chaotic behavior compared to the classical integer-order model. Chaotic behavior is characterized directly and indirectly via phase portraits, time responses, control input evolution, Lyapunov exponent trajectories, and state error plots. Equilibria are found by setting the RHS of Equation (3) to zero, yielding fixed points independent of derivative order, while the fractional order chiefly influences stability and transient dynamics through its memory effect. Therefore, the fixed points can be computed in the same way for both systems. These two fixed points, denoted as $f_p^{\pm}$, are derived as:

$$\left(x_{f_p^{\pm}}, y_{f_p^{\pm}}, z_{f_p^{\pm}}\right) = \left(\frac{c \pm \sqrt{c^2 - 4ab}}{2}, -\frac{-c \pm \sqrt{c^2 - 4ab}}{2a}, \frac{c \pm \sqrt{c^2 - 4ab}}{2a}\right). \tag{4}$$

The Jacobian matrix associated with the fractional system, evaluated at these equilibria, is given by:

$$J = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ z & 0 & x-c \end{bmatrix} \tag{5}$$

(Das et al. (2024)). This matrix is employed to assess the local stability by examining its eigenvalues prior to any control implementation. For instance, using the parameter values $a = 0.38$, $b = 4$, $c = 9$, and $\alpha = 0.9$ for which the system is chaotic, the corresponding eigenvalues for f_p^+ are computed as $-0.0748 \pm 4.9207i$ and 0.3574, while those for f_p^- are $0.1645 \pm 0.9794i$ and -8.7768.

4. STATE ESTMATION WITH EXTENDED KALMAN FILTER WHILE USING FRACTIONAL MODEL PREDICTIVE CONTROL

MPC optimizes control inputs over a receding horizon using a system model, effectively handling constraints in multivariable systems. Its fractional-order extension captures memory effects, as in the Rössler oscillator. For linear systems with quadratic costs, it reduces to a finite-horizon LQ problem; with linear constraints, to a QP; and with absolute or peak error penalties, to an LP (Boudjehem and Boudjehem (2012), Maciejowski et al. (2007)).

The plant is commonly modeled by a state-space representation given by:

$$X_{k+1} = A_k X_k + Bu_k + EW_k \quad , Y_k = CX_k + V_k \tag{6}$$

Here, X_k is the 3D state vector, u_k the control input, and Y_k the measured output, with W_k and V_k denoting process and measurement noise. As the output matrix C in Equation (6) is typically non-square and non-invertible, the EKF is used to estimate the full state by recursively combining measurements with model predictions. This enables accurate reconstruction under partial observability. Uniform noise is added for robustness analysis. Control inputs are computed by minimizing a cost function tracking $r(k+i)$. In fractional-order MPC, the model and cost function incorporate fractional dynamics and outputs $y_{\text{out}_{\text{frac}}}(k+i \mid k)$. Specifically, the cost function to be minimized is expressed as:

$$J_{Fractional} = \sum_{i=0}^{N_p-1} \left[\left(y_{out_{\text{frac}}}(k+i|k) - r(k+i) \right)^T Q \left(y_{out_{\text{frac}}}(k+i|k) - r(k+i) \right) \right] + \left[\left(\Delta u(k+i-1) \right)^T R \left(\Delta u(k+i-1) \right) \right] \tag{7}$$

where N_p is the prediction horizon, Q is the weighting matrix associated with the output error, $r(k+i)$ is the reference trajectory at the future time step $k+i$, $\Delta u(k+i-1)$ represents the variation in the control input, and R is the weighting matrix for the control effort.

In this context, $y_{out_{\text{frac}}}$ represents the output of the fractional-order system, governed by the state-transition model in (6), with A_k as the Jacobian of the fractional Rössler oscillator (5). Unlike our previous work (Das et al. (2024)) where all states were assumed measurable, we now employ an EKF to estimate the full state vector $[x; y; z]$ from partial measurements. Two scenarios are considered: measuring only x and y, or only z. Due to intrinsic nonlinear coupling among the states, reliable estimation is achievable even with limited outputs, validating the effectiveness of the proposed filtering approach. The theory behind the EKF involves the following steps:

Nonlinear State Prediction: The system evolves through the nonlinear update $f(X, U)$ of the fractional Rössler oscillator. At each step, the EKF predicts $\hat{X}_{k+1|k}$ and its covariance using $f(\cdot)$, then refines this estimate via measurements. Linearizing the output model yields the Kalman gain, which balances process and measurement noise to improve estimation. This filtering enhances accuracy, enabling FMPC to ensure robust control in dynamic, uncertain settings like robotics and aerospace.

Linearization via the Jacobian: Due to system nonlinearity, the EKF linearizes the dynamics at each step by computing the Jacobian $A_k = \frac{\partial f}{\partial x}$ at $\hat{X}_k$, capturing local sensitivity and enabling accurate covariance propagation.

Covariance Prediction: With the linearized model, the error covariance P is propagated forward:

$$P_{k+1|k} = A_k P_k A_k^T + Q_{p_k}, \tag{8}$$

where Q_{p_k} is the process noise covariance matrix, which accounts for modeling uncertainties and unmodeled dynamics. P_k denotes the error covariance matrix at time step k, representing the uncertainty in the current state estimate $\hat{X}_k$.

Measurement Update: The measurement function is given by

$$h(x, y, z) = \begin{bmatrix} x \\ y \end{bmatrix}, \tag{9}$$

meaning that only the states x and y are observed. When z is known, the measurement function is given by

$$h(x, y, z) = z, \tag{10}$$

In the Extended Kalman Filter, $h(\cdot)$ maps the true state to the observed output, while its Jacobian H linearizes this mapping at the predicted state. H quantifies the sensitivity of measurements to state changes, aiding in innovation covariance and Kalman gain computation. The innovation, or measurement residual, is the difference between the actual measurement Y_{meas} and the predicted measurement $h(\hat{x}_{k+1|k})$. The innovation covariance S is computed as:

$$S = HP_{k+1|k}H^T + R_{m_k}, \tag{11}$$

where H is the measurement matrix, here,

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and R_{m_k} is the measurement noise covariance. When z is known, and the corresponding measurement matrix is

$$H = [0\ 0\ 1]$$

The Kalman gain K is then determined by (Zenere and Zorzi (2017))

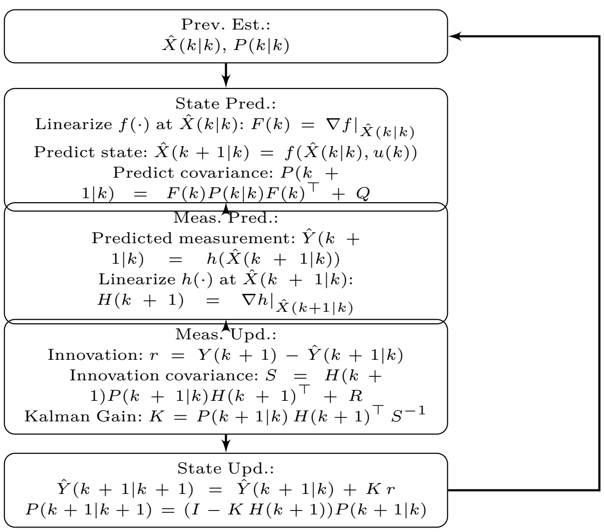
$$K = P_{k+1|k} H^T S^{-1} \tag{12}$$

State Update: Finally, the state estimate is updated using the Kalman gain:

$$\hat{X}_{k+1} = \hat{X}_{k+1|k} + K \left(Y_{\text{meas}} - h(\hat{X}_{k+1|k}) \right). \tag{13}$$

Here, Y_{meas} is the noisy EKF measurement, and $y_{\text{out}_{\text{frac}}} = h(\hat{X}_{k+1|k})$ the noise-free model prediction. When z is unmeasured, its correction depends on prediction and state coupling; if only z is measured, the innovation becomes $Y_{\text{meas}} - y_{\text{out}_{\text{frac}}}$, and the Kalman gain is a 3×1 vector. The measurement noise covariance R_{m_k} is tuned to reflect sensor reliability (Zenere and Zorzi (2017)). Aligning EKF covariances Q_{p_k} and R_{m_k} with MPC noise terms ensures robust estimation under uncertainty.

Below is the block diagram representing the EKF algorithm implemented in the system:



The EKF linearizes the nonlinear state transition $f(\cdot)$ to compute the Jacobian $F(k)$ (Eq. 5), capturing system sensitivity. Similarly, the measurement function $h(\cdot)$, used to map states to observables (e.g., z), is linearized at $\hat{X}_{k+1|k}$ to obtain the Jacobian $H(k+1)$. The predicted output $\hat{Y} = h(\hat{X})$ is compared with the noisy measurement Y_{meas}, and the estimate is updated accordingly. MPC simultaneously computes an optimal control sequence, applying only the first input u_k to ensure consistency with EKF predictions.

Following (Ribeiro (2004)), EKF implementation involves:

State Prediction: The predicted state $\hat{X}_{k+1|k}$ and its error covariance are computed by the nonlinear update $X_{k+1} = f(X_k) + EW_k$ around $\hat{X}(k|k)$ using the Jacobian A_k, and propagating the estimate via $f(x, u_k)$ as per Eq. (8).

Measurement Update: The measurement model $Y_k = h(X_k) + V_k$ is linearized at $\hat{x}_{k+1|k}$ to obtain $H(k+1)$. The innovation is used to compute the Kalman gain K, update the state estimate $\hat{X}_{k+1|k+1}$, and refine the covariance $P_{k+1|k+1}$.

Kalman Gain: The Kalman gain is computed as:

$$K = P_{k+1|k} H_{k+1}^T \left(H_{k+1} P_{k+1|k} H_{k+1}^T + R_{m_k} \right)^{-1}$$

where R_{m_k} depends on the measurement configuration. If only z is measured, $R_{m_k} = 0.1$; if x and y are measured, R_{m_k} is a diagonal matrix 2×2 with small entries (for example, $\text{diag}(0.1, 0.1)$).

The initial error covariance $P_0 = \text{diag}(s_x^2, s_y^2, s_z^2)$ reflects system uncertainty, where s_x, s_y, and s_z are the initial standard deviations of the state estimates. The process noise covariance $Q_p = \text{diag}(q_1, q_2, q_3)$ is tuned to model system evolution. As a one-step-ahead estimator, the EKF updates states based on new measurements, with the estimated state initializing the FMPC prediction horizon N_p to ensure accurate estimation and improved control performance in chaotic regimes.

5. RESULTS AND ANALYSES

Here is the list of parameters used in the MPC algorithm with their respective values:

$a = 0.38$, $b = 4$, $c = 9$, $\alpha = 0.9$, $N_p = 30$, $N_c = 10$, $B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $Q = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 10 \end{bmatrix}$, $R = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$.

The fractional Rössler oscillator parameters $a = 0.38$, $b = 4$, and $c = 9$ induce chaotic behavior (Rössler (1976) and Das et al. (2025a)). The control formulation is defined by the fractional order α, horizons N_p, N_c, and weighting matrices Q, R, and is evaluated over 3000 iterations. The objective is to steer the system from the origin $(0, 0, 0)$ to the unstable equilibrium $f_p^- = (0.1722, -0.4531, 0.4531)$, with equal weights on x and y, and a lower weight on z for improved performance. The EKF uses $Q_k = \text{diag}(0.1, 0.1, 0.1)$ and $R_{m_k} = 0.1$ (for z-only) or $\text{diag}(0.1, 0.1)$ (for x, y), with initialization $\hat{X}(0) = [0, 0, 0]^\top$ and $P(0) = I$, ensuring a balance between responsiveness and robustness under noise and model uncertainties.

Figure (1) shows the time response of the system when FMPC is applied and it is observed that the reference coordinates are achieved.

Figures (2) and (3) indicate that using the EKF it is possible to estimate the states x and y when z is known and estimate the state z when x and y are known.

When only x and y are measured, Figure 4 shows qualitatively similar system behavior. The initial cost peak re-

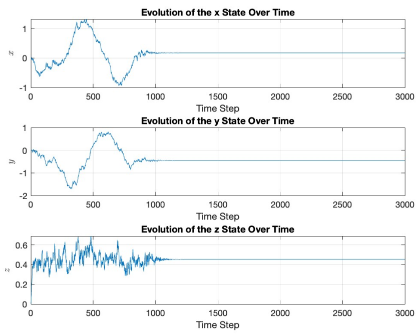


Fig. 1. Time Response of Fractional Rössler Oscillator after applying MPC under the influence of Gaussian Noise (when target $f_p^-: (0.1722, -0.4531, 0.4531)$ and initial condition is $(0, 0, 0)$)

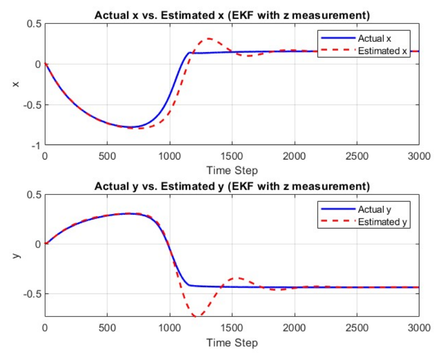


Fig. 2. Time Evolution of Actual vs. Estimated x and y when z is known for Fractional Rössler Oscillator after applying FMPC

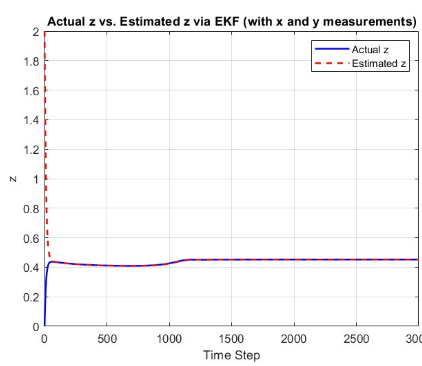


Fig. 3. Time Evolution of Actual vs. Estimated z when x and y are known for Fractional Rössler Oscillator after applying FMPC

flects transient errors due to noise and model uncertainty, resulting in large corrective inputs. As the EKF converges and FMPC stabilizes the system, cost decreases, indicating improved tracking.

Control robustness was also assessed under Gaussian white noise $x \sim \mathcal{N}(0, \sigma^2)$, representing zero-mean uncorrelated

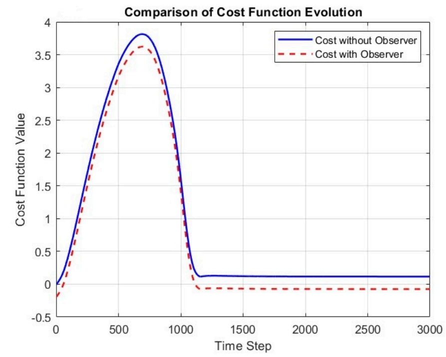


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Fig. 4. Time Evolution of cost function without using the observer and with using the observer for Fractional Rössler Oscillator after applying FMPC (when z is measured and x and y are estimated)

disturbances. MATLAB's *randn()* generates such noise by scaling standard normal samples with σ, consistent with the central limit theorem and common practice for modeling stochastic fluctuations.

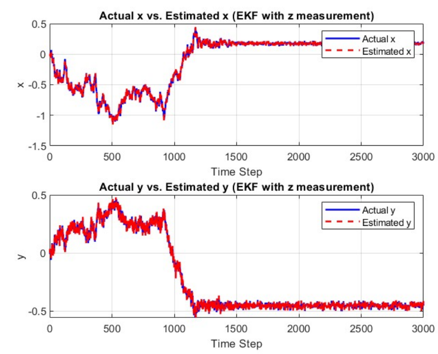

Fig. 5. Time Evolution of Actual vs. Estimated x and y when z is known for Fractional Rössler Oscillator after applying FMPC while guassian noise is present

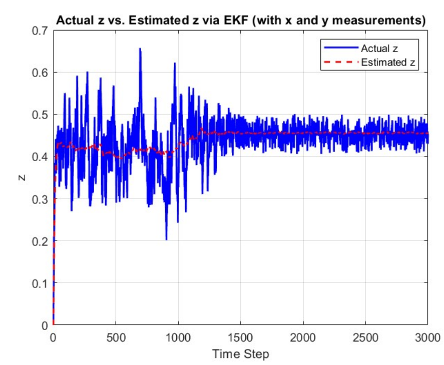

Fig. 6. Time Evolution of Actual vs. Estimated z when x and y are known for Fractional Rössler Oscillator after applying FMPC while guassian noise is present

Estimation quality is sensitive to noise, as illustrated in Figures (5) and (6). Increasing process noise covariance Q_p and decreasing measurement noise covariance

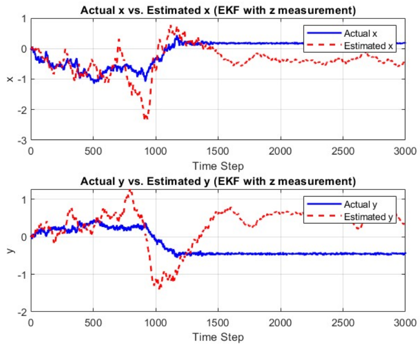

Fig. 7. Time Evolution of Actual vs. Estimated x and y when z is known for Fractional Rössler Oscillator after applying FMPC while guassian noise is present

R_m raise the Kalman gain K, enhancing responsiveness but risking instability if poorly tuned. Adjusting Q_p from diag([0.05, 0.05, 0.05]) to diag([0.1, 0.1, 0.1]) and R_m from diag([0.1, 0.1]) to diag([0.05, 0.05]) improves accuracy (Figures (7), (3)). FMPC stabilizes estimation when z is inferred from x and y, but degrades if only z is measured. MATLAB's *fmincon* ensures constraint satisfaction. Comparable cost trends for true-state and EKF-based control (Figure 4) confirm estimation reliability. Tracking performance depends on noise and horizon length; $\hat{z}$ is more noise-sensitive, whereas tuning R_m for $\hat{x}$ and $\hat{y}$ enhances accuracy. Longer horizons improve disturbance rejection but require careful tuning of Q_p, R_m, and prediction parameters for robust FMPC–EKF operation.

6. CONCLUSION AND FUTURE SCOPE

The integrated FMPC–EKF framework effectively stabilizes the fractional-order chaotic Rössler oscillator and ensures robust state estimation under noise. Leveraging the Grünwald–Letnikov formulation, it captures memory and non-local effects, outperforming integer-order models in accuracy and efficiency. Simulations confirm reliable state tracking, whether z is measured or estimated, validating the method's applicability to complex nonlinear systems. Future work will explore broader fractional systems and adaptive strategies for enhanced robustness.

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