Generalized Predictive Proportional Integral Controller Robust Stability Design*

Alejandro Rojas[1], Pedro Muñoz[2] and Hugo Garcés[2]

Abstract— **This work tackles the challenge of ensuring closed-loop stability for discrete-time first-order plant models with significant time delays, regardless of their stability, amidst uncertainties in plant gain, pole, or time delays. We propose a generalized predictive proportional integral (GPPI) controller designed specifically for these discrete-time plant models. Our primary objective is to ensure robust stability in the face of uncertainties related to plant gain, pole, and time delay. The design of the GPPI controller is founded on a complementary sensitivity approach, and when necessary, it is further developed into a two-degree-of-freedom controller to separate reference tracking from the design aiming for robust closed-loop stability. We validate the effectiveness of our robust stability design through simulations that include both stable and unstable plant models with considerable time delays.**

Index Terms— **discrete-time plant mode, time delay, generalized predictive proportional integral controller, pant model uncertainty, robust stability.**

I. Introduction

Control theory has established results since in the mid-20th century [1]. Applications of such theoretical results have focused mostly on the use of the proportional integral derivative (PID) controller, which in turn relates to processes represented by the first-order, or at most second-order models. Nevertheless, there is still room for new theoretical and practical results in such an established research area. For example, the authors of [2] developed a filtered version of the PID controller to improve the overall closed-loop performance and the size of the controller parameter region, increasing the applicability of the proposed filtered PID structure. On the other hand, applications of such an established controller structure can be found quite recently and can also address very challenging processes such as energy-efficient dividing-wall distillation columns [3], where the authors were able to achieve disturbance rejection by maintaining a degree of physical insight, or in [4], where three different designs (two PI-based and the last one a model-based predictive approach) are compared for the best performance for a speed control of drive systems with elastically coupled loads. In this last case, the challenge derives from the unwanted mechanical dynamics which justified the need by the authors to include a predictive approach option to achieve the required robustness against these unmodelled dynamics. The challenge for the use of PID controllers might also come from an extension of classical control named networked control systems (NCS) where explicit communication channel models are considered in the feedback loop. In [5], a PI controller is used over a wireless network to control a continuous-time direct-drive-wheel system. The designed PI controller cannot only control the given process but also reject the effects of network-induced delays and stochastic packet dropouts through a Lyapunov–Krasovskii functional-based design.

When the process to control is dominated by a long time delay, it is well known that the controller must include some form of predictive component to better manage the delay. For example, in [6], a prediction function for future states is examined for a class of nonlinear pure-feedback discrete-time systems using adaptive neural networks. The nonlinear closed-loop system achieves stability while keeping the output tracking error bounded within a neighborhood around zero. On the application side, as seen in [7], we again find a case of nonlinear control incorporating a generalized predictive component based on exponential functions to adjust the predictive period for the attitude position control of a spacecraft. The predictive component is initially calibrated by a known invariant period, and then, based on Lyapunov theory, a dynamic adaptive control law is refined for period prediction, subject to unknown nonlinear disturbances. A linear generalized predictive control is proposed in [8] for the current control of grid-connected voltage source inverters. Here, the control objective focuses on robust performance and a fast response to current reference step changes, despite variations in converter parameters. More recently, [9] investigates the frequency performance of a control method combining active disturbance rejection with generalized predictive control, offering insights into the integration of PI controllers with predictive strategies; [10] reports a back-stepping control method that integrates a generalized proportional integral observer with a Kalman filter, aiming to enhance disturbance estimation and noise filtering in DC-DC buck converters; and [11] proposes a novel adaptive algorithm combining generalized predictive control (GPC) and backpropagation (BP) neural network for improved temperature control in a lithography system.

When the focus is on linear models, commonly used to approximate nonlinear processes near their operating points, the issue of long time delays can be addressed using a modified PI structure that incorporates the required predictive component. In [12] a generalized predictive proportional integral (GPPI) controller is proposed. The controller ap-

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proach is based on an NCS optimization approach, which then surpasses the standard PID controller in performance for an improved plant unstable parameter region. In [13] the robust stability question for the GPPI controller is studied in the NCS context when the network is characterized by its communication delay. The robust design is achieved using a Nyquist criterion approach. In [14] the GPPI robust stability question is posed for continuous time systems through a β tuning for which the open control loop can achieve greater gain and phase margins. An LMI approach is used in [15] to achieve robust performance for a class of discrete time uncertain systems, by means of a prediction-based controller. Applications of these ideas can be found for example in [16] where a generalized proportional integral observer based predictive current control is proposed for industry applications, and in [17] where now a discrete-time reduced order generalized proportional-integral observer is used to propose output speed predictions that are then used to solve a constrained receding horizon optimization problem for an uncertain generic dc–dc buck converter-driven permanent magnet dc motor model subject to lumped disturbances.

The contribution of this work is the robust stability design of the GPPI controller for first-order plant models (either stable or unstable) with long time delay. The plant model uncertainty is studied separately, either as plant gain uncertainty, plant pole uncertainty, or plant time delay uncertainty. We propose a complementary sensitivity design approach for the GPPI controller robust stability design. We offer simulations to verify the obtained design.

The paper follows the following structure: Section II introduces the required background needed for our results. In Section III we develop our contribution for a stable plant model. In Section IV we repeat the process for the discrete time unstable plant model with long time delay. In Section V we showcase the proposed robust stability design through simulations of each case. We conclude the exposition with final remarks and future research opportunities in Section VI.

II. Preliminaries

In this section we present the preliminaries required for our results in the following two sections. The nominal plant model under study is given by the structure in eq.(1)

$$G_o(z) = \frac{K_o}{z-\rho_o} \cdot \frac{1}{z^{N_o}} \tag{1}$$

with $K_o \in \mathbb{R}^+$, $\rho_o \in \mathbb{R}$, and $N_o \in \mathbb{N}$. The proposed nominal plant model is not as ideal as can be initially thought, since any modeling from first principles, in particular from a starting balance equation, will result in a system dominated by a first order derivative, in continuous time, or a first order difference equation in discrete time. Also, even if the plant model is dynamically more complex, it can always be reduced to its dominant dynamics since the approximation plant model error can be compensated by the control feedback loop; see for example, Fig. 1. The plant model includes $G_{os}(z)$ which represents stable dynamics. Therefore, the controller can cancel them by an added inverted term $G_{os}^{-1}(z)$. Thus, we observe that the model in (1) is quite generalizable. The type of plant model uncertainty is assumed to be of the multiplicative type, that is we propose that the *real* plant model, instead of the nominal plant model, is obtained as

$$G(z) = G_o(z)\,(1+\Delta G) \tag{2}$$

When focused on the plant gain K_o uncertainty, we can then observe that

$$\Delta G(z) = \frac{\Delta K}{K_o} \tag{3}$$

If the uncertainty is on the value of the plant model pole ρ_o, the plant multiplicative uncertainty is

$$\Delta G(z) = \frac{\Delta\rho}{z-\rho_o-\Delta\rho} \tag{4}$$

and finally, if the uncertainty is on the plant time delay value, we get that

$$\Delta G(z) = z^{-\Delta N} - 1 \tag{5}$$

When $N_o = 0$, or at most 1, the standard controller structure for stabilizing the nominal plant model (1) in a control feedback loop is usually the PI controller (a PID controller with the D component set to zero), see Fig. 1. In discrete time, this type of controller is given by eq.(6)

$$C(z) = K_p + K_I\frac{z}{z-1} = K_c\frac{z-z_c}{z-1} \tag{6}$$

with $K_c = K_p + K_I$ and $z_c = \frac{K_p}{K_p+K_I}$, $K_p, K_I \in \mathbb{R}^+$.

On the other hand, the presence of a long nominal time delay $N_o \geq 2$ requires introducing a predictive structure in the controller, see Fig. 2. A known predictive structure is the Smith Predictor, but it has limitations when dealing with an unstable plant model. Another type of predictive structure given by

$$z^{-N_o}\sum_{j=0}^{N_o-1}\beta_j z^j \tag{7}$$

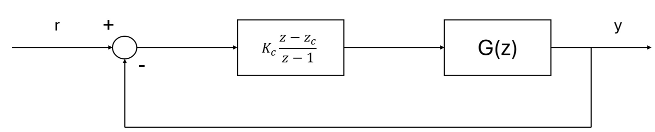

Fig. 1. PI control feedback loop.

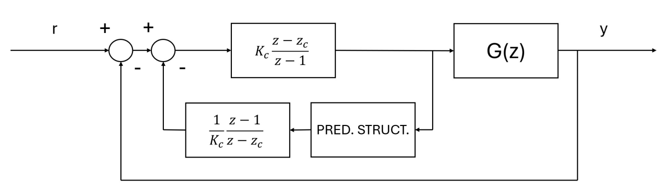

Fig. 2. GPPI control feedback loop.

with $\beta_j = (1-z_c)(1-\sum_{i=N_o-1}^{j}\alpha_{N_o-1-i})$ results in the overall GPPI controller given by

$$C(z) = K_c \frac{(z-z_c)z^{N_c}}{z^{N_c}(z-z_c)-(1-z_c)\sum_{j=0}^{N_c}\alpha_j z^j} \quad (8)$$

with $N_c \in \mathbb{N}$, $K_c, z_c \in \mathbb{R}^+$, $|z_c| < 1$, $\alpha_j \in \mathbb{R}$ and $\sum_{j=0}^{N_c}\alpha_j = 1$. The imposition on the magnitude of the GPPI controller zero z_c is to avoid a non-minimum phase zero, and it is in line with the zero location in the case of the PI controller in (6). Fig. 2 shows the existing connection between the GPPI controller and the PI controller, where if $N_c = 0$ the GPPI controller becomes the PI controller.

Lemma 1: **[18, §5.9.2] (Control Feedback Loop Robust Stability)** For a given plant model multiplicative uncertainty ΔG, the controller $C(z)$ will achieve robust stability for the control feedback loop if

$$|T_o \Delta G|_\infty \leq 1, \ \forall \omega \quad (9)$$

where T_o is the nominal control feedback loop complementary sensitivity given by

$$T_o(z) = \frac{C(z)G_o(z)}{1+C(z)G_o(z)}$$

Corollary 1: If $T_o(z)$ is low pass, and the plant model uncertainty ΔG is all-pass or low pass, then the controller $C(z)$ will achieve robust stability for the control feedback loop if

$$\|T_o\|_\infty \leq \frac{1}{\|\Delta G\|_\infty} \quad (10)$$

Proof: From Lemma 1 and the observation that $\|H\|_\infty = \max_\omega \left\{H(e^{j\omega T_s})\right\}$, , with $T_s \in \mathbb{R}^+$ the sampling time, we have then

$$\|T_o\|_\infty \|\Delta G\|_\infty \leq 1$$

and (10) follows, which concludes this proof. ■

Remark 1: The above condition is easier to verify, since it is independent of the frequency, and as long as the low pass assumptions are satisfied.

We now, based on Corollary 1, develop in the next section a complementary sensitivity design approach for the robust stability of the control feedback loop, subject to the uncertainty models in (3), (4), and (5), when the plant is stable.

III. Stable plant model with long time delay

We consider in this section the case of $|\rho_o| < 1$,meaning the nominal plant model is stable. We start by declaring the target complementary sensitivity which in turn results in a GPPI controller structure.

Theorem 2: **(Robust Stability Design for Stable Plants)** The GPPI controller, with $N_c = N_o$, is given by

$$C(z) = \frac{1+a_0}{K_o} \frac{(z-\rho)z^{N_o}}{z^{N_o}(z-\rho)-(1-\rho)\sum_{j=0}^{N_o}\alpha_j z^j} \quad (11)$$

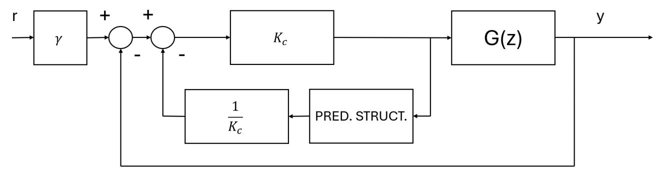

Fig. 3. 2 DOF control feedback loop.

where a_0 is a design parameter used to shape the desired complementary sensitivity function and

$$\alpha_{N_o} = -\frac{\rho+a_0}{1-\rho}, \ \alpha_{N_o-1} = \cdots = \alpha_1 = 0 \ \alpha_0 = \frac{1+a_0}{1-\rho} \quad (12)$$

The GPPI controller above achieves the following target complementary sensitivity for the control feedback loop

$$T_o(z) = \frac{1+a_0}{z^{N_o}(z+a_0)} \quad (13)$$

with $|a_o| < 1$.

Proof: We start by observing that for the given target complementary sensitivity, the resulting controller can be obtained as

$$\begin{aligned} C(z) &= T_o(z)\left(1-T_o(z)\right)^{-1} G_o^{-1}(z) \\ &= \frac{1+a_0}{z^{N_o}(z+a_0)}\left(1-\frac{1+a_0}{z^{N_o}(z+a_0)}\right)^{-1}\frac{(z-\rho)z^{N_o}}{K_o} \\ &= \frac{1+a_0}{K_o}\frac{(z-\rho)z^{N_o}}{z^{N_o}(z+a_0)-(1+a_0)\pm\rho z^{N_o}} \end{aligned}$$

from which adjusting terms in the denominator of the last development we recover the GPPI controller presented in (11), with the predictive structure coefficients defined as in (12), which concludes this proof. ■

Remark 2: We observe that the target complementary sensitivity in (13) achieves asymptotic setpoint tracking, since $T_o(1) = 1$.

We now consider the uncertainty types proposed in Section II, starting with uncertainty in the plant model gain K_o.

A. Plant Gain Uncertainty

When the real plant model $G(z)$ is subject to a plant gain uncertainty, we refer to the following case

$$G(z) = \frac{K_o+\Delta K}{z-\rho_o}\cdot\frac{1}{z_o^N} \quad (14)$$

If the above model is the real plant model versus the nominal plant model in (1), then the expression for the resulting multiplicative uncertainty is as in (3). We observe from Theorem 2 that $\|T_o\|_\infty = 1$. On the other hand, given ΔG as in (3) we now have

$$\|\Delta G\|_\infty = \left|\frac{\Delta K}{K_o}\right| \quad (15)$$

We then observe from Corollary 1 that, as long as $|\Delta K| < |K_o|$, then $C(z)$ as in Theorem 2 achieves the required robust stability objective for the control feedback loop in Fig. 2.

On the other hand, if $|\Delta K| > |K_o|$, then a target complementary sensitivity with $\|T_o\|_\infty = 1$ is not suitable anymore. To solve this issue, we then are faced with the need of decoupling the robust stability objective from the asymptotic tracking objective by means of a two degree of freedom (2DOF) controller, see Fig. 3. In this alternative control feedback loop, the inner controller is designed for robust stability, and the parameter γ is tuned for the setpoint asymptotic tracking. As a result of this discussion, we propose next a corollary to Theorem 2 to address the 2 DOF robust stability design.

Corollary 3: **(2 DOF Robust Stability Design for Stable Plants)** The GPP controller, with $N_c = N_o$, is given by

$$C(z) = \frac{K_T}{K_o} \frac{(z-\rho) z^{N_o}}{z^{N_o}(z-\rho) - (1-\rho)\sum_{j=0}^{N_o} \alpha_j z^j} \tag{16}$$

with

$$\alpha_{N_o} = -\frac{\rho + a_0}{1-\rho},\ \alpha_{N_o-1} = \cdots = \alpha_1 = 0,\ \alpha_0 = \frac{K_T}{1-\rho} \tag{17}$$

This GPP controller achieves the following target complementary sensitivity for the control feedback loop

$$T_o(z) = \frac{K_T}{z^{N_o}(z+a_0)} \tag{18}$$

with $|a_o| < 1$. The gain γ for asymptotic tracking is then obtains as

$$\gamma = T_o^{-1}(1) \tag{19}$$

Proof: The proof follows the same steps as for the proof of Theorem 2. The value of γ is such to equalize the steady state gain between the setpoint and control feedback loop at steady state. ■

Remark 3: From the GPP controller proposed in (16) we have that the sum of the α_j coefficients is not one anymore, and thus the controller does not achieve steady state setpoint tracking anymore, thus the label of GPP, instead of GPPI, controller.

We now use Corollary 3 to solve the robust stability design requirement which then reports the need to satisfy

$$K_T < \frac{(1+a_0)|\Delta K|}{|K_o|} \tag{20}$$

As long as K_T is designed as above the control feedback loop will satisfies the robust stability objective.

B. Plant Pole Uncertainty

We now consider the case of a plant model multiplicative uncertainty as the one defined in (4). Since we are still considering the stable plant case, we are left to decide the use of either Theorem 2 or Corollary 3. To decide this, making use of Corollary 1, we observe that

$$\|\Delta G\|_\infty = max\left\{ \frac{|\underline{\Delta\rho}|}{|1-\rho_o-\underline{\Delta\rho}|}; \frac{|\overline{\Delta\rho}|}{|1-\rho_o-\overline{\Delta\rho}|} \right\} \tag{21}$$

Again, if $\|\Delta G\|_\infty$ is less than one we can use Theorem 2, otherwise we propose the ue of Corollary 3.

C. Plant Time Delay Uncertainty

For the last type of plant model multiplicative uncertainty under study, we have that its frequency response is given by

$$\Delta G(j\omega) = e^{-j\omega T\Delta N} - 1 \tag{22}$$

Thus, the worst case considers the fact that $e^{-j\omega^(T\Delta N} = 1$ for some frequency ω^* and then $\|\Delta G\|_{infty} = 2$, independently of the value of ΔN. This las fact shows that for this type of multiplicative uncertainty the only viable option, in order to achieve control feedback loop robust stability, is to make use of Corollary 3.

IV. Unstable plant model with long time delay

We now consider in this section the case of an unstable nominal plant model with long time delay, that is a model as in (1) with $|\rho_o| > 1$. The presence of an unstable plant pole requires for the complementary sensitivity to satisfy the interpolation condition

$$T_o(\rho_o) = 1 \tag{23}$$

as to avoid an internal unstable cancellation. More so, the presence of such an unstable plant pole triggers the presence of a fundamental limitation, which reflects on a minimum H_∞ norm greater than zero. We address this in the next theorem.

Theorem 4: Given the nominal plant model in (1), the GPP controller that achieves the minimal H_∞ norm for $T_o(z)$, is given by

$$C(z) = \frac{(\rho_o^2 - 1)\rho_o^{N_o-1}}{K_o} \frac{z^{N_o}}{z^{N_o} - \sum_{j=0}^{N_o-1} \alpha_j z^j} \tag{24}$$

with

$$N_c = N_o - 1,\ z_c = 0,\ \alpha_j = \left(1-\rho_o^2\right)\rho_o^{N_o-1-j} \tag{25}$$

The resulting optimal complementary sensitivity is

$$T_o(z) = \frac{(1-\rho_o^2)\rho_o^{N_o}}{-\rho_o z + 1} \cdot \frac{1}{z^{N_o}} \tag{26}$$

In turn, the H_∞ norm for this complementary sensitivity is

$$\|T_o\|_\infty = \left|(\rho_o + 1)\rho_o^{N_o}\right| \tag{27}$$

Proof: The proof is based on the observation that the norm minimization result from [19], which is based on a Youla parametrization argument, achieves the minimum complementary sensitivity norm, independent if the norm is H_2 or H_∞, [20]. From [19, Cor. 4] we derive the minimal norm complementary sensitivity in (26). The controller structure and parameters are then obtained after some algebra, from the expression $C(z) = T_o(z)\left((1-T_o(z))G_o(z)\right)^{-1}$ and the interpretation of the resulting controller is a GPP with $z_c = 0$ and $N_c = N_o - 1$. Finally, the norm expression in (27) is obtained from (26) as follows

$$\begin{aligned}\|T_o\|_\infty &= \left\| \frac{(1-\rho_o^2)\rho_o^{N_o}}{-\rho_o e^{j\omega T_s} + 1} \cdot \frac{1}{e^{j\omega T_s N_o}} \right\|_\infty \\ &= \left|(1-\rho_o^2)\rho_o^{N_o}\right| \left\| \frac{1}{-\rho_o e^{j\omega T_s} + 1} \cdot \frac{1}{e^{j\omega T_s N_o}} \right\|_\infty\end{aligned}$$

where we replaceed $z = e^{j\omega T_s}$, with $T_s \in \mathbb{R}^+$ the sampling time. Since $-1 \leq |e^{jx}| \leq 1$, for any $x \in \mathbb{R}$, we then have that

$$\begin{aligned}\|T_o\|_\infty &= \left|(1-\rho_o^2)\rho_o^{N_o}\right| \left\|\frac{1}{-\rho_o e^{j\omega T_s}+1}\right\|_\infty \\ &= \left|(1-\rho_o^2)\rho_o^{N_o}\right| \frac{1}{|-\rho_o+1|}\end{aligned}$$

which results in the expression presented in (27). This concludes the present proof. ■

Remark 4: We observe that minimal norm complementary sensitivity in (26) satisfies the interpolation condition for the unstable pole ρ_o, that is $T_o(\rho_o) = 1$.

The fact that the nominal plant is unstable makes achieving any control objective more challenging. This includes of course the robust stability objective we are studying here. In particular the norm expression in (27), together with Corollary 1, will allow us to estimate the maximum achievable uncertainty region for each of the plant parameters uncertainties.

A. Plant Gain Uncertainty

As in the previous section, we start our analysis with the case of plant gain uncertainty, which for robust stability can be seen to have to satisfy the following bounds

$$\frac{-K_o}{|(1+\rho_o)\rho_o^{N_o}|} \leq \Delta K \leq \frac{K_o}{|(1+\rho_o)\rho_o^{N_o}|} \tag{28}$$

From the above inequalities we observe that in this situation we do not have free reign to design the controller as to achieve any given plant gain uncertainty interval. More so, the tolerable uncertainty range is reduced by the interplay between the unstable pole $|\rho_o| > 1$ and the plant time delay N_o. The greater the magnitude of the unstable pole and/or the greater the plant time delay, the smaller the tolerable uncertainty range for the plant gain parameter to achieve robust stability. Finally, we observe that there is no promise of an alternative controller design, since as observed in Theorem 4 the present design is the one achieving the minimal H_∞ norm for $T_o(z)$.

B. Plant Pole Uncertainty

We consider now the case of plant pole uncertainty, as defined in (4). For these type of uncertainties, using Theorem 4 results in

$$\begin{aligned}|(\rho_o+1)\rho_o^{N_o}| \left\|\frac{\Delta\rho}{z-\rho_o-\Delta\rho}\right\|_\infty &\leq 1 \\ |(\rho_o+1)\rho_o^{N_o}| \left|\frac{\Delta\rho}{1-\rho_o-\Delta\rho}\right| &\leq 1\end{aligned} \tag{29}$$

Since we offer here a preliminary analysis we assume for this subsection that $\rho_o > 1$ and $\Delta rho > 0$. We then study the resulting lower bound, and conclude that

$$\Delta\rho < \frac{\rho_o - 1}{(\rho_o+1)\rho_o^{N_o} - 1} \tag{30}$$

We again observe that as ρ_o increases and/or N_o increases the lower bound on $\Delta\rho$ becomes tighter.

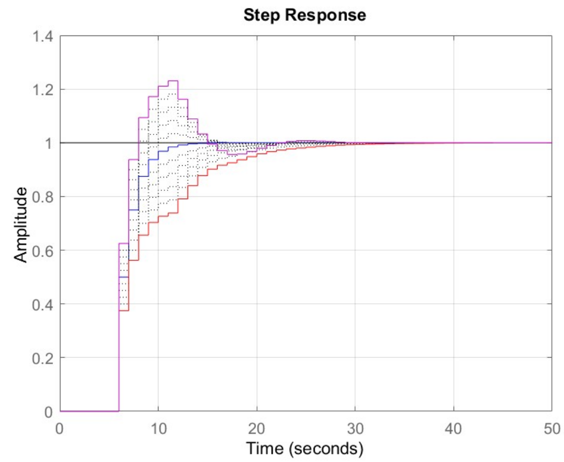

Fig. 4. Step responses for stable plant gain uncertainty . $\Delta K = -0.5$, red line; $\Delta K = 0$, blue line; $\Delta K = 0.5$, magenta line. Intermediate ΔK values, black dotted lines.

C. Plant Time Delay Uncertainty

The last uncertainty type under study is the one reported in (5). For this type of uncertainty we have that $\|\Delta G\|_\infty = 1.9953$. Again, from Theorem 4 and Corollary 1, we observe that

$$|(\rho_o+1)\rho_o^{N_o}| - 0.5012 \leq 0 \tag{31}$$

The above equation has to be studied numerically for a valid set of $\Delta\rho$ values, depending on the sign of ρ_o, and further depending if N_o is odd or even.

V. Examples

In this section we test the ideas developed in the previous sections. For the stable nominal plant example we consider

$$G_o(z) = \frac{2}{z-0.5} \cdot \frac{1}{z^5} \tag{32}$$

We propose the complementary sensitivity for the uncertainties in (3) and (4) to be

$$T_o(z) = \frac{0.5}{z+0.5} \cdot \frac{1}{z^5} \tag{33}$$

For these choices, and using Theorem 2, we achieve $\Delta K \in [-0.5; 0.5]$, see Fig. 4, or $\Delta\rho \in [-0.1; 0.1]$, see Fig. 5. For the uncertainty in (5), and the stable plant model example, we make use of Corollary 3 with a modified complementary sensitivity defined as

$$T_o(z) = \frac{0.05}{z+0.5} \cdot \frac{1}{z^5} \tag{34}$$

With this last choice of $T_o(z)$ we were able to achieve $\Delta N \in [-2; 2]$, see Fig. 6. We now study the case of an unstable nominal plant model such as

$$G_o(z) = \frac{3}{z-2} \cdot \frac{1}{z^5} \tag{35}$$

For each type of uncertainty, when dealing with an unstable nominal plant model, we make use of Theorem 4. The

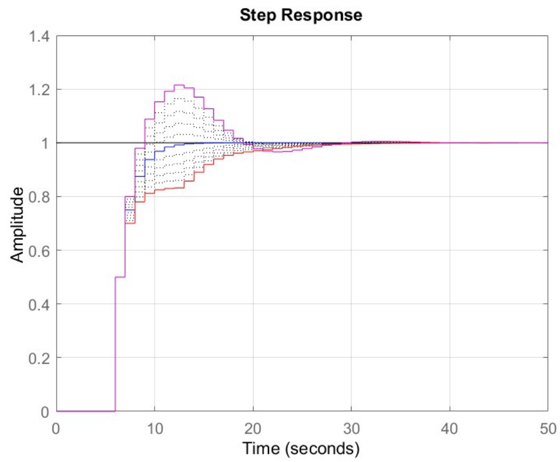

Fig. 5. Step responses for stable plant pole uncertainty . $\Delta\rho = -0.1$, red line; $\Delta\rho = 0$, blue line; $\Delta\rho = 0.1$, magenta line. Intermediate $\Delta\rho$ values, black dotted lines.

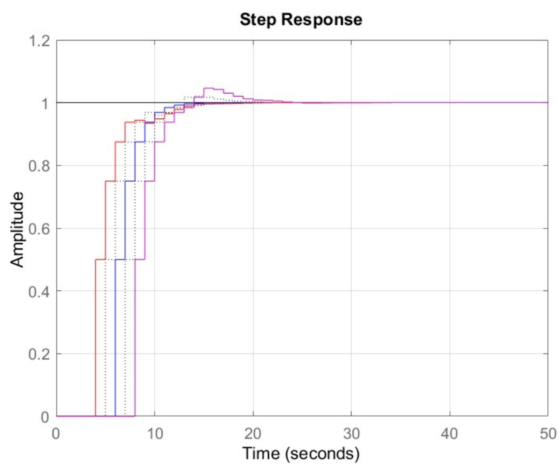

Fig. 6. Step responses for stable plant time delay uncertainty . $\Delta N = -2$, red line; $\Delta N = 0$, blue line; $\Delta N = 2$, magenta line. Intermediate ΔN values, black dotted lines.

analysis report that for these choices we can achieve $\Delta K \in [-0.03; 0.03]$, see Fig. 7. For $\Delta\rho$ on the other hand we obtain a subset defined by $\Delta\rho \in [-0.01; 0.01]$, see Fig. 8. Finally, for ΔN we have that it is quite difficult to achieve control feedback loop robust stability. Nevertheless studying the polynomial in (29) we observe that a feasible value of $\rho_o \in [-1.2008; -1]$. We thus choose the option of $\rho_o = -1.05$ as below

$$G_o(z) = \frac{3}{z + 1.05} \cdot \frac{1}{z^5} \tag{36}$$

We then apply Theorem 4, but observe that Corollary 1 low pass assumption is not satisfied, see Fig. 9. More so, Lemma 1 is not satisfied for ΔN odd, but it is satisfied for ΔN even. We then work with values of $\Delta N \in \{-4; -2; 2; 4\}$ for which indeed we achieve stability as reported in Fig. 10. Through these examples we have applied our results from

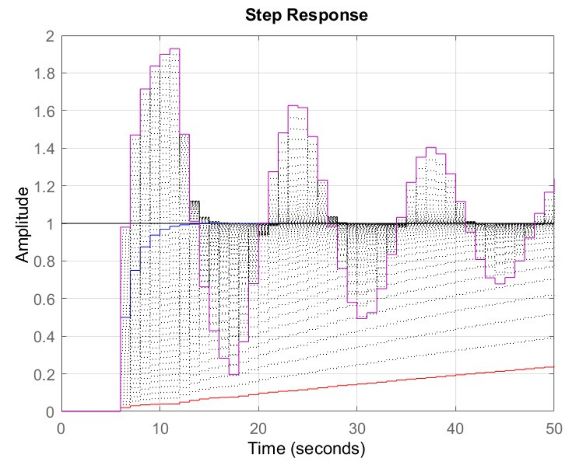

Fig. 7. Step responses for unstable plant gain uncertainty . $\Delta K = -0.03$, red line; $\Delta K = 0$, blue line; $\Delta K = 0.03$, magenta line. Intermediate ΔK values, black dotted lines.

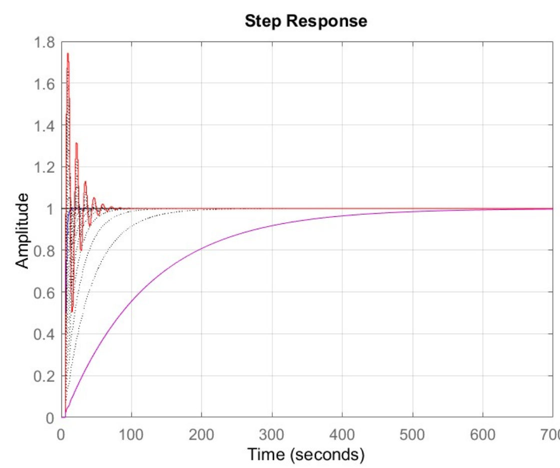

Fig. 8. Step responses for unstable plant pole uncertainty . $\Delta\rho = -0.01$, red line; $\Delta\rho = 0$, blue line; $\Delta\rho = 0.01$, magenta line. Intermediate $\Delta\rho$ values, black dotted lines.

Theorem 2, Corollary 3 and Theorem 4. As expected from the theoretical development, the stable nominal plant case is less challenging than the unstable nominal plant case when we wish to achieve control feedback loop robust stability.

VI. Conclusions

In this work, we propose a robust stability GPPI control feedback loop design for discrete-time first-order plant models with long time delays, whether stable or unstable, and subject to uncertainties in their plant gain, plant pole, or plant time delays. The design objective of robust stability is considered alongside the performance objective of steady-state setpoint tracking, which is achieved either by placing a pole at $z = 1$ in the controller or by utilizing a 2 DOF controller setup, depending on the robust stability requirements. The GPPI controller's robust stability design

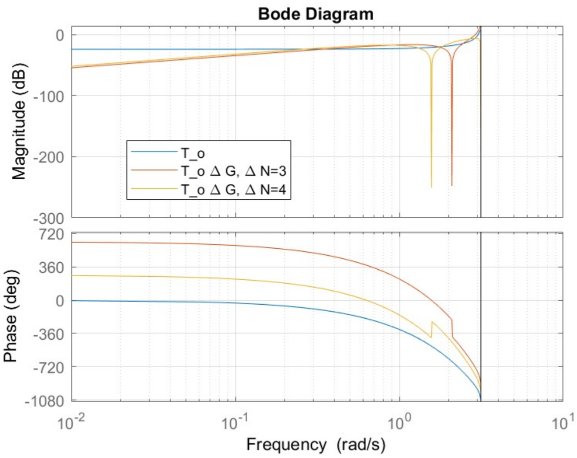

Fig. 9. Frequency responses for unstable plant time delay uncertainty . T_o, blue line; $T_o\Delta G$ for $\Delta N = 3$, red line; $T_o\Delta G$ for $\Delta N = 4$, yellow line.

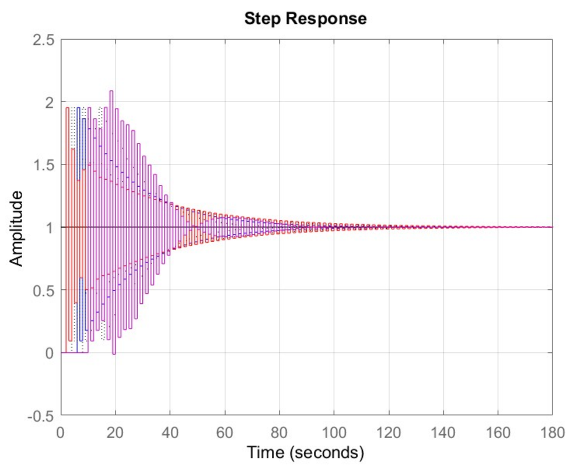

Fig. 10. Step responses for unstable plant time delay uncertainty . $\Delta N = -4$, red line; $\Delta N = 0$, blue line; $\Delta N = 4$, magenta line. Intermediate even $N = \pm 2$ values, black dotted lines.

was based on a complementary sensitivity design approach and was extended to the 2 DOF controller when necessary. The effectiveness of the proposed robust stability design was verified through simulations for both stable and unstable plant models with long time delays. Future research could explore a small gain design and/or the simultaneous presence of uncertainties in more than one plant model parameter.

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