

LPV supervised reinforcement learning based control for autonomous vehicles

András Mihály and Van Tan Vu and Olivier Sename and Péter Gáspár

Abstract—In the recent research of autonomous vehicle (AV) control methods, machine learning techniques are gaining more and more popularity due to their possible performance advantages over classical model based methods. As lane keeping or trajectory tracking through steering intervention is one of the key feature of the autonomous vehicle, several methods had been proposed and utilized both with model and machine learning based controllers. Present paper focuses on the integration of a robust Linear Parameter Varying (LPV) and a Reinforcement Learning (RL) controller in a supervisor structure, with the aim to guarantee stability against disturbances not considered during the training process of the RL agent. Hence, with the supervision of the proposed LPV controller, noisy or faulty GPS signals can be handled safely, without compromising the trajectory tracking performance of the RL agent during normal operation of the autonomous vehicle. For the demonstration of the proposed LPV supervised RL control method, highway simulations have been carried out and compared in CarSim simulation environment.

I. INTRODUCTION

Autonomous vehicles are recently of great interest to both scientists, researchers and automobile manufacturers worldwide, as AVs have great potential to reduce the risk of traffic jams and road traffic accidents, see [1]. Moreover, autonomous vehicles often use novel energy sources such as electricity, adapting to the trend of reducing the use of fossil energy, which is greatly encouraged by governments and vehicle users as well. Through this development trend, autonomous vehicles are receiving increasing investment, becoming a potential market for investors. Effective and safe applications of AVs can be considered in traffic environments where confidence in the correct functioning of autonomous technology is guaranteed, which poses several interesting challenges for researchers.

For autonomous cars to operate without a driver, important functions such as sensors, actuators and algorithms must be considered consistently and integrated effectively.

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First, traffic environment and surrounding vehicles must be considered in the design, being an important condition for describing directional control commands as well as dynamical vehicle status. The most important sensors of an AV include GPS, LIDAR, IMU, and cameras around the vehicle, see [2], [3], [4]. Through data from these sensors, the designed control method must provide algorithms to determine velocity, direction of movement and appropriate speed and steering control operations. This designed control should be similar to when there is a driver present, in order to remain predictable for human driven vehicles and to preserve comfort for the passenger, see [5], [6], [7], [8], [9]. State-of-the-art approaches of model-based lateral vehicle control algorithms have been compared considering path tracking ability and passenger comfort, see [10].

Reinforcement Learning (RL) is an important branch of machine learning techniques. RL is considered one of the most effective solutions in applying trajectory control for autonomous vehicles. RL methods can be efficient in autonomous vehicles to understand their surroundings, accurately determine the optimal path, making smart driving decisions, and control the vehicle safely. RL plays an increasingly important role in realizing and continuously improving automated driving features, see [11]. RL algorithms when equipped for autonomous cars, capable of navigating when changing lanes with predetermined routes, include Deep Neural Network (DNN) and Deep Reinforcement Learning (DRL) [12]. Models based on Q-learning, Q-network and Deep Deterministic Policy gradient (DDPG) algorithms that can learn to efficiently handle unforeseen situations on the road, see [13], [14]. This ability is continuously learned and updated while interacting with the environment, helping autonomous vehicles ensure safety and efficiency when operating in different conditions and emergencies while on the move. This is a very important and necessary factor in handling situations, from complex interactions in traffic to adapting to unexpected weather conditions [15], obstacle avoidance [16], and longitudinal control when changing lanes.

Although there are outstanding performance advantages of the RL method applied to autonomous vehicles, it still has some difficulties such as slow computational speed due to having to process a lot of data and to adapt to different traffic scenarios immediately. Moreover, in emergency situations, difficulties might occur due to the measurement noises of the sensors, specifically in the GPS positioning system. Handling of such events is the main topic of this paper. Linear parameter varying (LPV) control method has

also been proven to be particularly effective in handling complex nonlinear systems such as autonomous vehicle control, see [17], [18]. In addition, due to its ability to extend some linear concepts to nonlinear systems such as \mathcal{H}_∞ , sensitivity shaping, D-stability, it makes it applicable to nonlinear systems on vehicles, see [19]. Thus, when LPV is integrated into the automatic vehicle controller, it is capable of autonomous driving control with time-varying speed and external disturbances [20].

Through analyzing the advantages and disadvantages of two methods of controlling the trajectory (RL, LPV), it can be seen that these two methods can be combined to control the trajectory of autonomous vehicles. That is, an LPV supervised RL-based trajectory tracking control method is proposed with the aim of ensuring vehicle stability in the presence of interference of sensors, such as interference of the GPS signal. The novelty of this method includes a supervised control structure, in which the RL is supervised by the LPV trajectory tracking controller with the aim of handling sensors noises, which may be present in the GPS system. Therefore, in case of GPS signal interference detection, the RL-based trajectory tracking of the AV is overtaken by the LPV controller, prescribing a more robust steering intervention. In this way, special RL controllers that cannot handle disturbances and noise robustly and in stable manner are replaced by LPV controllers with built-in stability guarantees to handle such cases.

The paper is organized as follows: Section II introduces the vehicle model applied for the proposed supervised trajectory tracking method and the LPV control design. Section III describes the training process for designing the RL agent used for the basic trajectory tracking task, while Section IV details the supervised control structure considering the effect of GPS sensor noise. Section V demonstrates the advantages of the supervised control method through CarSim simulations with a highway cornering example. Finally, concluding remarks are given in Section VI.

II. MODEL-BASED ROBUST LPV CONTROL DESIGN

A. Autonomous vehicle model

For modeling the planar plane vehicle dynamics the well-known bicycle model shown in Figure 1 is applied, with the corresponding motion equations given in (1) [21].

$$\begin{aligned} J\ddot{\psi} &= c_1 l_1 (\delta - \beta - \dot{\psi} l_1 / \dot{x}_v) - c_2 l_2 (-\beta + \dot{\psi} l_2 / \dot{x}_v) \\ m\dot{x}_v(\dot{\psi} + \dot{\beta}) &= c_1 (\delta - \beta - \dot{\psi} l_1 / \dot{x}_v) + c_2 (-\beta + \dot{\psi} l_2 / \dot{x}_v) \\ \ddot{y}_v &= \dot{x}_v (\dot{\psi} + \dot{\beta}) \end{aligned} \quad (1)$$

where ψ represents the yaw and β the side-slip angle, J stands for the yaw inertia, m for the mass of the AV, l_1 and l_2 are the distances from the front and rear axles to the center of gravity, c_1 and c_2 are the front and rear cornering stiffness of the tires, which can be identified [22]. Note, that $\dot{\xi}$ and \ddot{y}_v represents the longitudinal and lateral acceleration, which are measured by the accelerometer of the AV.

The aim of the lane tracking control design is to guarantee minimal lateral error for the AV from the predefined road

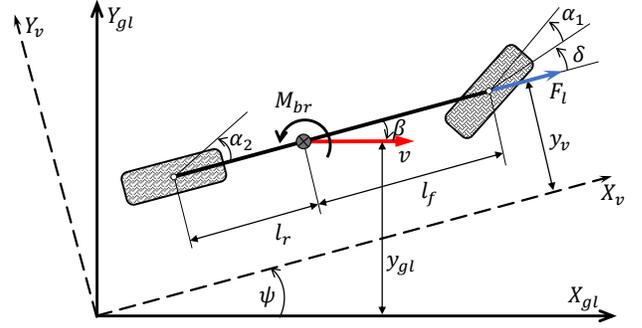


Fig. 1. Single track bicycle model

center line, which is given by the road reference. Since the latter is given in the world coordinate system X_{gl} and Y_{gl} , a coordinate transformation is necessary as shown in Figure 1, in order to transform the road data points in the AVs own rotating coordinate system X_v and Y_v , which rotates by the yaw angle ψ . Hence, as the predefined road center line coordinates $x_{gl,r}$ and $y_{gl,r}$ are given in the world coordinates, the lateral position of the vehicle in its own reference frame is calculated as $y_{v,r} = -\sin(\psi) x_{gl,r} + \cos(\psi) y_{gl,r}$.

B. LPV control design

In order to design the supervisor LPV controller, the dynamic equation (1) is reorganized in the LPV state-space representation as given in equation (2), where the varying parameter ρ characterizes the LPV system.

$$\begin{aligned} \dot{\mathbf{x}} &= A(\rho)\mathbf{x} + B_1\mathbf{w} + B_2(\rho)\mathbf{u} \\ \mathbf{z} &= C_1\mathbf{x} + D_{11}\mathbf{w} + D_{12}\mathbf{u} \\ \mathbf{y} &= C_2\mathbf{x} + D_{21}\mathbf{w} \end{aligned} \quad (2)$$

where the state vector $\mathbf{x} = [\dot{\psi} \ \beta \ \dot{y}_v \ y_v]^T$ contains the yaw-rate, side-slip angle, lateral velocity and position of the AV, while the control input $\mathbf{u} = \delta$ is the steer angle of the front wheels. The measured outputs $\mathbf{y} = [\dot{x}_v \ y_v]^T$ are the longitudinal velocity and lateral position of the vehicle, which are assumed to be measured by on-board sensors.

Next, the performances of the control system are defined in a vector $\mathbf{z} = [z_1 \ z_2]^T$. Here, $z_1 = |y_{v,r} - y_v|$ contains the lateral error of the vehicle from the lane center, where $y_{v,r}$ is the road reference given by R in Figure 2. The goal of the control design is to minimize z_1 , i.e. $z_1 \rightarrow 0$ (optimization criterion). Also, the second performance is chosen as $z_2 = \delta$, with the aim to minimize the steering angle during the trajectory tracking of the AV. This performance is important to avoid the saturation of the actuator and to ensure a prescribed high-level input for the AV which is feasible by the low-level control system.

Note, that the longitudinal velocity \dot{x}_v of the AV is one of the parameters that varies constantly but can be easily measured, see [23]. Moreover, the vehicle model described in equation (1) has nonlinear properties due to the velocity

\dot{x}_v , thus in the LPV state-space formulation in equation (2) considers the varying parameter $\rho = \dot{x}_v$.

The LPV control design is founded on a a weighting strategy, poised via a closed-loop interconnection structure as depicted in Figure 2. Here, the weighting function W_p stands for the previously detailed performance specifications of the controller, which is crucial in reaching a balance between good trajectory tracking without the saturation of the steering input. The aim of the weighting function W_n is to consider the sensor noises of the system, namely the disturbances or noises in the GPS signal of the AV. Note, that in order to guarantee stability when sensor noise is present, this function has been designed with bigger weights. Also, the weighting function W_δ representing the steering intervention has been shaped to avoiding actuator saturation. Note, that W_p , W_n and W_δ has been selected in a first-order proportional form in the frequency domain as follows:

$$\zeta \frac{1}{T_1 s + 1}, \quad (3)$$

where T_1 and ζ are design parameters carefully tuned to realized optimal performance of the closed-loop LPV controller.

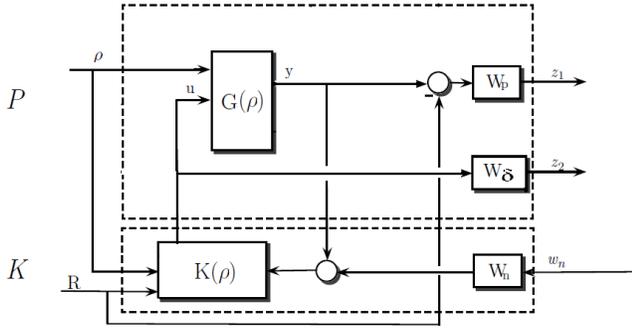


Fig. 2. Closed-loop interconnection structure

The minimization problem for the LPV design criteria detailed in [24]–[27] is given as follows:

$$\inf_K \sup_{\rho \in \mathcal{F}_P} \sup_{\|w\|_2 \neq 0, w \in L_2} \frac{\|z\|_2}{\|w\|_2}. \quad (4)$$

The aim of the design is to select a parameter-varying controller with quadratic stability by which the induced \mathcal{L}_2 norm from the disturbance ω to the performances z remains smaller than a predefined γ value. Hence, γ is the \mathcal{L}_2 norm of Linear Fractional Transformation (P,K), as described in [28]. Finally, the state space formulation of the LPV controller $K(\rho)$ is built. With the above defined control design structure, the LPV control algorithm iteration attempts to find a solution that minimizes the induced \mathcal{L}_2 norm of the LFT(P,K), resulting in a γ value of 0.0776. This small value of γ achieved guarantees exceptional disturbance rejection for the closed-loop LPV control system. Hence, the LPV controller achieves robust performance, since both robust stability and nominal performance requirements are satisfied.

For an example, in Figure 3 singular value plots are depicted for a selected $\rho = 27.7 \text{ m/s}$ value.

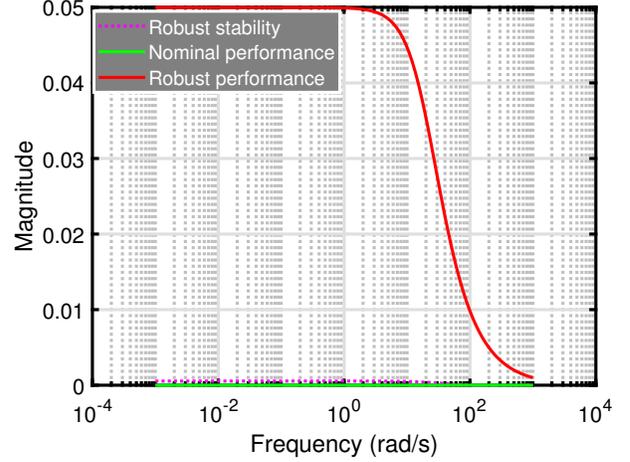


Fig. 3. Singular value plots

III. REINFORCEMENT LEARNING CONTROL DESIGN

For the design of the reinforcement learning controller, a deep Q-learning network has been applied to solve the lane tracking task. The scheme of the reinforcement learning training process is depicted in Figure 4. The ego vehicle dynamics has been modeled with the bicycle model shown in Figure 1, while for the trajectory tracking training process the road geometry of the simulation environment depicted in Figure 6 has been used with a road length of one kilometer. The training has been evaluated with a sampling time of $T_s = 0.02$ seconds, while the total simulation time has been set to $T = 1000/\dot{x}_v$, where \dot{x}_v has been set in the range of $[60 \text{ } 110]$ km/h to represent typical highway speeds. The output of reinforcement learning agent is the front steering angle of the vehicle constrained in the range of $[-5 \text{ } 5]$ degree, with discrete steps of 0.01 degree.

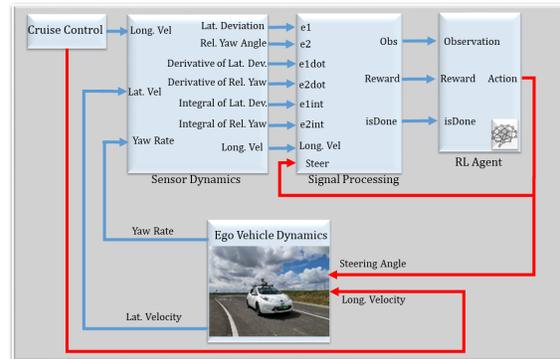


Fig. 4. Reinforcement learning training process

The observations from the vehicle environment given by the vehicle sensors are the following: model for the ego vehicles has been set as follows:

- Lateral deviation: e_y

- Relative yaw angle: e_ψ
- Derivative of lateral deviation: \dot{e}_y
- Derivative of relative yaw angle: \dot{e}_ψ
- Integral of lateral deviation: $\int e_y$
- Integral of relative yaw angle: $\int e_\psi$
- Vehicle longitudinal velocity: \dot{x}_v

The reward function r_t for the training process has been designed as $r_t = -(40e_y^2 + 20u^2)$, with the goal to minimize the lateral error of the lane tracking and to avoid chattering of the steering angle at the same time. Note, that the cumulative value of the reward provided at each time step t is the episode reward, while in the action space the agent selects from 1001 possible discrete steering angles between -5 to 5 degrees.

The maximal episode number during the training process has been set to 10000, and the training is stopped when the episode reward reaches -2.5 . Note, that in order to test and verify the operation of the RL steering controller, the trained RL agent has been tested in CarSim simulation environment. Hence, the design of the RL agent has been evaluated in an iterative manner, based on the results of the CarSim simulations. Thus, the training parameters and the reward function has been modified based on the results of the CarSim simulation, until the operation of the designed RL controller has been satisfactory.

IV. LPV SUPERVISED REINFORCEMENT LEARNING CONTROL

The supervision of the robust LPV controller is necessary to handle GPS sensor noises or faulty signals, which are not considered during the training process of the reinforcement learning agent, while the focus of the LPV design has been on handling sensor noises without instability. Thus, the supervised control structure depicted in Figure 5 works in the manner of enabling the operation of the more efficient reinforcement learning agent during normal conditions, while switching to the parallel operating LPV controller in case of a noise in the GPS signal is detected. For sensor noise detection of the GPS systems, several methods have been proposed not investigated in present paper, see [29]–[31]. Hence, a decision logic is considered in the input selection process as follows:

$$\delta = \begin{cases} \delta_{RL}, & \text{without noise} \\ \delta_{LPV}, & \text{with noise} \end{cases} \quad (5)$$

where δ_{LPV} stands for the steering angle prescribed by the LPV controller, while δ_{RL} represents the steering input designed by the reinforcement learning agent.

V. SIMULATION RESULTS

The operation of the proposed LPV supervised reinforcement learning control has been simulated in CarSim environment through different real-life scenarios. In order to demonstrate the supervision of the LPV controller, a GPS sensor noise has also been considered during one of the simulations. The physical and dynamical parameters of the

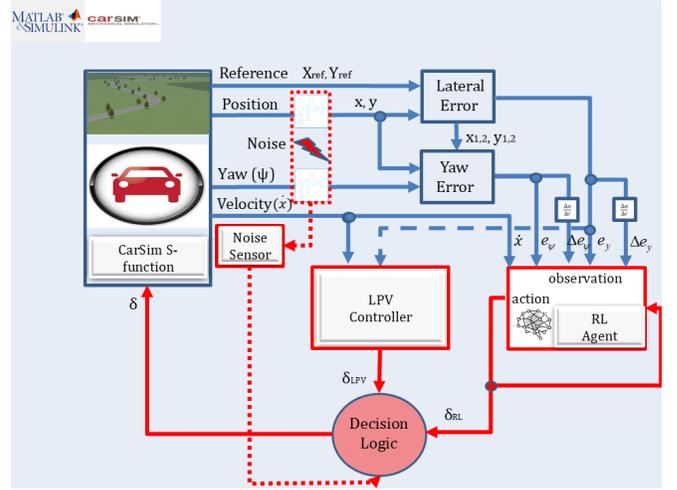


Fig. 5. RL agent supervised controller

simulation vehicle has been selected as shown in Table I, which are similar to that applied during the bicycle model based LPV control synthesis and the reinforcement learning training procedure.

TABLE I
PARAMETERS OF THE VEHICLE

Parameter	Value	Unit
Vehicle mass (m)	1742	kg
Yaw moment of inertia (J)	1523	kgm^2
Distance from C.G to front axle (l_1)	1.161	m
Distance from C.G to rear axle (l_2)	1.539	m
Tread front (b_f)	1.54	m
Tread rear (b_r)	1.535	m
Height of COG (h_{COG})	0.438	m
Cornering stiffness front (c_1)	70	kN/rad
Cornering stiffness rear (c_2)	55	kN/rad
Aerodynamic drag co-efficient (c_w)	0.3431	$-$
Front contact surface (A)	1.6	m^2

The simulations had been performed on a highway road section with straights and curves, as depicted in Figure 6 (a). The velocity of the simulated vehicle had been set to alter between 70 to 110 km/h , while to represent the GPS sensor noise a band-limited white noise signal had been used with a maximal amplitude of 0.2 meter (with noise power of 0.001), see Figure 6 (b).

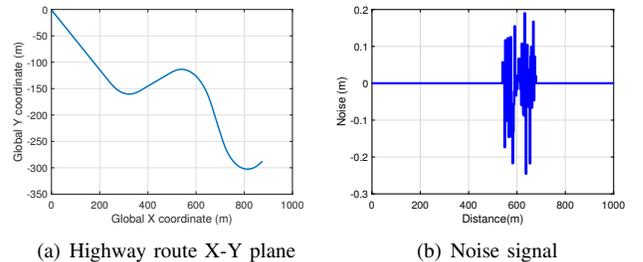


Fig. 6. Simulation environment

A. Baseline controls without sensor noise

For the baseline simulation, both the LPV controller and the RL agent has been operated independently without any disturbance in the GPS system. The goal of this first scenario is to compare the performance of each controller, as depicted in Figure 7. Note, that the cruise control of the simulated autonomous vehicle performed the reference velocity tracking as shown in Figure 7 (a), showing similar results for both cases. The steering angles prescribed by the LPV and RL controllers are shown in Figure 7 (b), showing a faster and more sensitive behavior for the RL controller. As expected with this kind of behavior, both lateral and yaw error depicted in Figure 7 (c)-(d) show better performance of the RL controller, with significantly smaller peak values. Thus, without sensor noise the trajectory tracking of the RL controller achieves better results than the robust LPV controller.

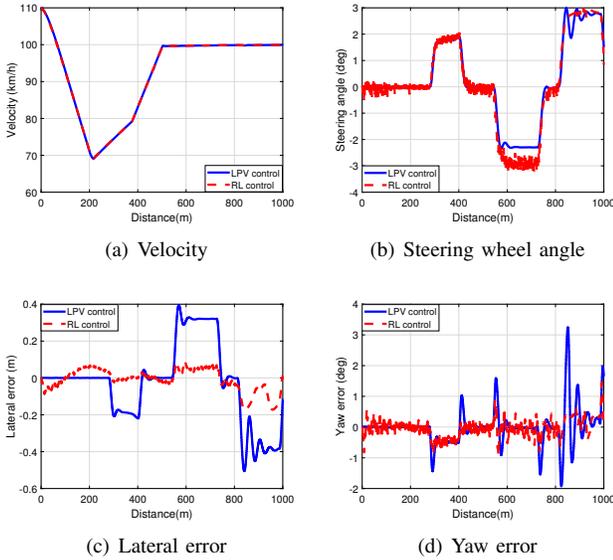


Fig. 7. LPV and RL control without noise

B. Reinforcement learning control with GPS noise

Next, simulation of the RL controller had been performed with the added GPS noise signal depicted in Figure 6 (b). Thus, the measured position of the vehicle had been affected by the noise between 23 – 28 seconds, during the entering of the second curve. Although the velocity of the vehicle governed by the cruise control system remained unaffected (see Figure 9 (a)), stability loss of the vehicle appeared without the supervision of the LPV controller, see Figure 8.

This resulted in the lateral deviation of the vehicle to reach 2 meters during the period of the sensor noise (see Figure 9 (c)), while the yaw error depicted in Figure 9 (d) shows the oscillations in the vehicle direction resulting from the disturbed steering input shown in Figure 9 (b).



Fig. 8. Stability loss of the AV

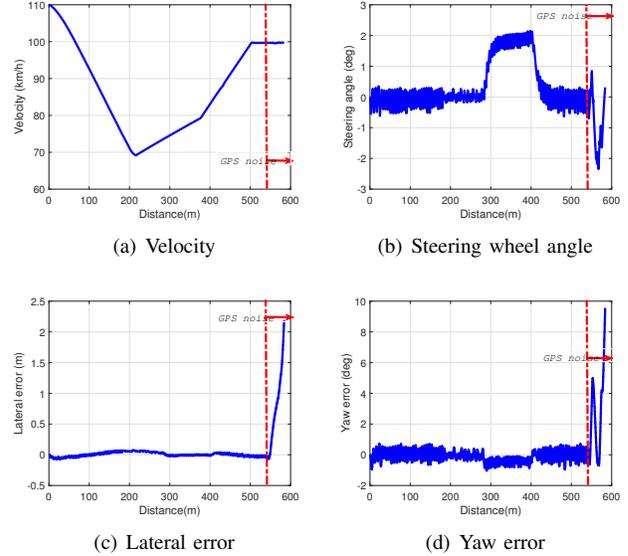


Fig. 9. Reinforcement learning with noise

C. Supervised reinforcement learning control with GPS noise

Finally, a simulation scenario had been performed with the previously detailed GPS sensor noise present, using the proposed LPV supervised control structure. Here, the steering input of the vehicle shown in Figure 10 (b) is prescribed by the reinforcement learning controller until the sensor noise is detected at 23 seconds, when the robust LPV controller overtakes the steering action until 28 seconds, stabilizing the autonomous vehicle. It is well demonstrated, that although lateral error depicted in Figure 10 (c) and yaw rate error shown in Figure 10 (d) had increased significantly, stability loss and vehicle oscillation had been prevented due to the robust performance of the LPV controller.

VI. CONCLUSION

The paper proposed an LPV supervised reinforcement learning control method for the safe lane keeping of an autonomous vehicle even during disturbance in the global positioning system of the vehicle. Thus, the goal of the presented control scheme is to guarantee the stability of the vehicle in case sensor noises are present, for which the LPV supervisor had been designed with great disturbance rejection properties. Hence, in case a GPS signal noise is detected, the operation of the autonomous vehicle is overtaken by the LPV

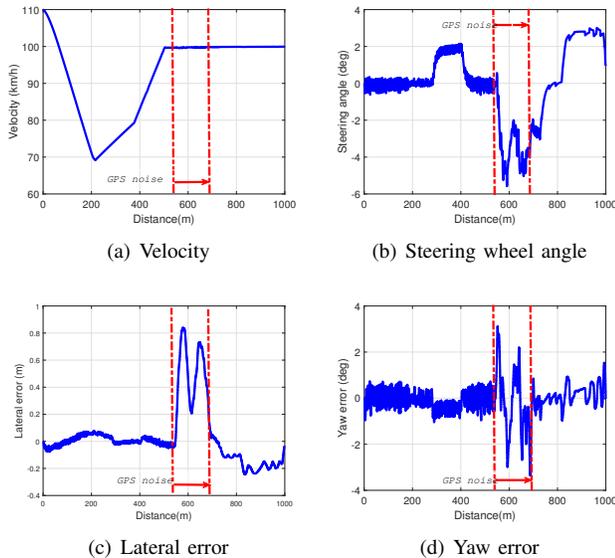


Fig. 10. LPV supervised reinforcement learning with noise

controller from the reinforcement learning agent, by which stability can be preserved. The proposed method has been validated through multiple simulations analysis performed in CarSim environment with a real-life highway scenario. In future work, a more detailed robust control switching algorithm and the validation of the proposed supervised control algorithm is considered in a real-life test environment.

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