Optimal Design of a Multi-Hub Battery Charging System for Rural Areas Electrification in the Global South

Federico Signorile, Paolo Scarabaggio, Raffaele Carli, Mariagrazia Dotoli

***Abstract*— Access to electricity is essential for socio-economic development, especially in rural areas of the Global South. Portable battery-based systems allow users to rent or recharge batteries at centralized stations powered by renewable sources, offering a viable alternative for off-grid electrification. This paper presents a methodology for designing a multi-hub battery charging system aimed at ensuring cost-effectiveness, efficiency, and scalability. The study addresses two main objectives: 1) determining the optimal number, location, and allocation strategy of households to charging hubs considering geographical and logistic constraints, and 2) estimating the daily power demand profile for each hub. To address the first goal, an integer linear programming problem is defined, while for the latter, an analysis-based Monte Carlo simulation of a discrete-time queue is proposed. The methodology is validated through simulations using data extracted from a real scenario, demonstrating its effectiveness and adaptability.**

I. INTRODUCTION

In 2022, 91.4% of the global population had access to electricity. This figure drops significantly in rural areas, where only 84% had access [1]. Such a disparity becomes even more pronounced in the global south, a term referring to developing countries, highlighting a stark urban-rural divide [1].

Access to modern energy services is essential for socio-economic development and is responsible for improving health by enabling the use of medical equipment, vaccine refrigeration, and reducing air pollution [2], [3]. It also improves educational opportunities through better lighting and access to digital tools, and supports communication and information access [4], [5]. Furthermore, electrification opens up new employment and income opportunities, contributing to gender empowerment [6] and stimulates economic growth by supporting small businesses, improving agricultural productivity, and reducing dependence on expensive diesel generators [7], [8]. These wide-ranging benefits make energy access a key enabler of numerous sustainable development goals (SDGs) [9], [10].

Two main solutions emerge to address electricity access in rural areas: grid extension and off-grid systems. In developing countries, power grid extension and development are essential long-term goals for rural electrification. Nevertheless, isolation, low population density, and limited electricity demand make rural areas unlikely to benefit from grid extension in the short term [11].

Among off-grid solutions for rural electrification, mini-grids are small-scale energy distribution systems that can operate independently or be connected to the main grid. Individual systems such as solar home systems (SHS) offer a viable solution for dispersed communities. These systems provide decentralized electricity for households, meeting basic needs such as lighting and small appliances. However, the SHS solution involves high upfront costs and requires significant investment in larger batteries to guarantee some days of autonomy during rainy or cloudy periods [12]. Consequently, battery charging systems (BCSs) emerge as the ideal solution for low-density, low-income communities, offering a more cost-effective and scalable alternative.

A BCS is an off-grid solution consisting of multiple charging hubs powered by solar energy. These hubs charge portable batteries, which are then used by households to power essential devices, such as lighting and mobile phones. In the related literature, several studies have explored rural electrification through the implementation of BCS projects. The study in [13] highlights how on-site surveys can guide system design and discuss the most appropriate business models for such systems. In [14], the concept of a multi-purpose BCS is presented, detailing its applicability, site selection, installation, and maintenance criteria. Additionally, [15] proposes a data-driven methodology to estimate end-use consumption based on a BCS case study. In [16] the BCS key challenges are identified, including battery malfunctions and the long distances between hubs and households, which hinder system usage. Authors in [17] propose the use of battery charging stations to improve electrification in rural areas of Rwanda, optimizing the use of existing solar panels and reducing costs for low-income households, also including heuristics for hub capacity and the number of batteries to be charged. Moreover, game-theoretic models from electricity markets [18] could be adapted to capture user behavior and system dynamics in BCS networks.

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Despite the recalled studies, a notable gap persists in the literature regarding integrated approaches to optimize BCSs. Specifically, there is a lack of methodologies that simultaneously address key aspects such as hub location, system sizing, user allocation, and accurate power demand estimation at the hub level. To design a BCS that is both cost-effective and accessible, a high-level strategy for hub location is essential. Subsequently, understanding the power demand profile at the hub level is crucial for appropriately sizing the hubs to meet the demand.

To address the challenges discussed above, this paper presents a novel sequential approach for the optimal design of BCSs. In the first step, an integer linear programming approach is proposed to optimize the hub location, taking into account geographical and logistical constraints. The results from this step serve as input for the second phase, which employs Monte Carlo simulations of a discrete-time queue to estimate the charging power profile at the hub level. The methodology is validated through simulations using data extracted from a real scenario, demonstrating its effectiveness and adaptability.

The remainder of the paper is organized as follows: Section II presents the system model and detailed methodology, formulating the problem statement and outlining the approach for optimizing hub placement and estimating daily power demand. In Section III, the proposed methodology is numerically validated through simulations, with results based on a real case study in Kenya. Finally, Section IV discusses the conclusions drawn from the study and outlines potential directions for future research.

II. The Proposed Methodology

The methodology proposed in this work addresses the optimal design of a BCS, focusing on three main objectives: (i) hub location, (ii) household-to-hub allocation, and (iii) power demand estimation for each hub. These objectives are tackled through a sequential procedure comprising two phases: the hub location problem (HLP), which addresses (i) and (ii), and the power profile estimation (PPE), which addresses (iii).

The HLP phase aims to identify the hub configuration that minimizes costs while ensuring user accessibility. The location of the hubs is crucial to ensure the system is both economically sustainable and logistically efficient. A key factor for the success of a BCS is its geographic accessibility, as the distance between hubs and households plays a vital role in determining the project's success, as demonstrated in [19] and [16].

The PPE phase aims to estimate the power demand at the hub level accurately and ensure the system is sized correctly. On the one hand, an incorrect estimation of the demand profile could lead to unnecessary operating costs, particularly critical in low-income rural areas. On the other hand, inadequate sizing could compromise the effectiveness and accessibility of the system, reducing the capacity of the hubs to correctly meet the end-user needs. Since the power demand at the hub level depends on random variables, such as consumption, hub visiting patterns, and charging behaviors, it is essential to adopt an approach that realistically models these uncertainties. To this aim, an analysis based on Monte Carlo simulations is proposed, using a discrete-time queue model, which accounts for the random nature of battery arrivals and charging times, to estimate the charging demand.

These two phases are interdependent, as the allocation of households to the hubs, determined in the first phase, directly influences the power demand estimation in the second phase.

A. System Modeling

The BCS design problem is defined within a specific rural area with known boundaries. Within this area, a finite set of households $\mathcal{N}$ is defined, where each household $i \in \mathcal{N}$ is equipped with a modular household battery of capacity C_{hb} and has an average end-use daily consumption β_i, with all β_i collected in a column vector $\boldsymbol{\beta}$.

A set of potential sites for hub installation $\mathcal{H}$ are identified, for instance, using site surveys that assess factors such as solar exposure, easy accessibility for households, and potential land availability. Each candidate site $j \in \mathcal{H}$ is associated with fixed costs K_j^{fix}, which include expenses that must be incurred regardless of the site's activity level, such as land acquisition, which may vary depending on the location (e.g., public or private land). Each hub j is equipped with s_j parallel charging sockets, which operate with constant current I and voltage V.

B. Hub Location Problem

The HLP, as discussed in Section II, addresses the optimal selection of hub location and the allocation of households to these hubs. Several studies addressed similar problems related to hub location and coverage, with well-established models such as the set covering problem (SCP) [20] and the maximal covering location problem (MCLP) [21]. Both aim to optimize hub locations while ensuring coverage of demand points. SCP seeks to minimize location costs while guaranteeing coverage of all demand points by at least one facility. Similarly, MCLP focuses on maximizing coverage with a fixed number of hubs. However, unlike SCP and MCLP, the proposed formulation introduces the additional complexity of logistical accessibility and ensures a 1-to-1 allocation between households and hubs, a factor not explicitly addressed in either of the recalled models. Furthermore, the proposed formulation emphasizes covering all households while minimizing installation costs, a primary focus that distinguishes it from SCP and MCLP.

In this paper, the HLP is formalized with the goal of minimizing the total installation costs while ensuring that all households are within an acceptable distance from a charging hub. The decision variables of the model are γ_j, a binary variable that is set to 1 if a hub is located at site j and 0 otherwise, and ϕ_{ij}, a binary variable that is set to 1 if household i is assigned to hub j and 0 otherwise:

$$\gamma_j, \phi_{ij} \in \{0, 1\} \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{H}. \tag{1}$$

All decision variables $\gamma_j, \forall j \in \mathcal{H}$ and $\phi_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{H}$ are collected in column vectors γ and ϕ, respectively. In particular, the vector ϕ is constructed by listing, for each household $i \in \mathcal{N}$, all corresponding variables ϕ_{ij} with $j \in \mathcal{H}$, one after the other; i.e., $\phi = (\phi_{11}, \phi_{12}, \ldots, \phi_{1|\mathcal{H}|}, \ldots, \phi_{|\mathcal{N}|1}, \phi_{|\mathcal{N}|2}, \ldots, \phi_{|\mathcal{N}||\mathcal{H}|})^\top$.

To guarantee that at least one hub is set, the following constraint is introduced:

$$\sum_{j \in \mathcal{H}} \gamma_j \geq 1, \tag{2}$$

while, as each household should be assigned to one and only one hub, the following equality is imposed:

$$\sum_{j \in \mathcal{H}} \phi_{ij} = 1, \quad \forall i \in \mathcal{N}. \tag{3}$$

Next, to ensure that the distance d_{ij} between each household i and its reference hub j does not exceed a maximum threshold ϵ, the following condition must hold:

$$d_{ij}\phi_{ij} \leq \epsilon, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{H}. \tag{4}$$

Each household i must be assigned to the closest hub:

$$\sum_{j \in \mathcal{H}} d_{ij}\phi_{ij} \leq \sum_{j' \in \mathcal{H}\setminus\{j\}} d_{ij'}\phi_{ij'} \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{H}. \tag{5}$$

Finally, to ensure that a household is only assigned to an operational hub, the following condition must be met:

$$\phi_{ij} \leq \gamma_j \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{H}. \tag{6}$$

Summing up, the HLP consists in determining the vector of binary decision variables ϕ and γ that simultaneously minimize the total installation costs while satisfying the integrality constraints (1), the inequality constraints (2), (4)-(6), and the equality constraint (3):

$$\begin{aligned} \min_{\phi,\gamma} \quad & \sum_{j \in \mathcal{H}} K_j^{\text{fix}} \gamma_j \\ \text{s.t.} \quad & (1) - (6). \end{aligned} \tag{7}$$

Problem (7) is an integer linear programming problem with $|\mathcal{N}||\mathcal{H}| + |\mathcal{H}|$ binary variables, $3|\mathcal{N}||\mathcal{H}| + 1$ inequality constraints, $|\mathcal{N}|$ equality constraints, and $|\mathcal{N}||\mathcal{H}|+|\mathcal{H}|$ integrality constraints.

C. Power Profile Estimation

Given the optimal hub location from the previous step, the PPE phase focuses on accurately forecasting the charging power profile at the hub level, considering uncertainties related to household consumption patterns and charging behaviors. In the related literature, a similar problem arises in estimating the demand load for electric vehicle charging stations. A widely used methodology for this is queueing network analysis, as demonstrated in several studies such as [22] and [23]. These works employ queueing models to optimize the management of charging stations and model demand fluctuations effectively. However, in the case of the problem addressed in this work, the assumptions of stationarity and infinite population do not hold, as the application involves a finite population. To capture the inherent uncertainties in charging demand, a Monte Carlo simulation-based approach is employed to simulate the dynamics of the queuing system over time.

Algorithm 1 Monte Carlo PPE for Hub j

```
1:  Input: β, s_j, C_hb, f(k), N_j, I, V
2:  Initialize: n_j(0) ← 0,  c_j(0) ← 0,  n_qj(0) ← 0,
      μ,  λ_Dj,  λ_j(k) ∀k ∈ [0, T]
3:  for each simulation t from 1 to M do
4:      for each time step k from 0 to T do
5:          Update E_j(k) through (10)
6:          Sample A_j(k) in accordance with (13)
7:          Update n_j(k), c_j(k), n_qj(k) through (9)
8:          Sample τ_c through (14)
9:          Update S_j(k + τ_c)
10:         Calculate P_j(k) through (15)
11:     end for
12:     Output: Power profile P̄_j^t(k) for simulation t
13: end for
14: Final Output: Daily power profile P̄_j(k), averaged over
      all simulations
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Leveraging the solution of the HLP ϕ_{ij}^*, the set of households assigned to hub j is defined as:

$$\mathcal{N}_j = \{i \mid \phi_{ij}^* = 1\}. \tag{8}$$

This represents the finite population of the queuing network referring to the hub j. The households, acting as customers of the charging service, are modeled using an $M_{1j}/M_2/s_j/N_j^0$ queue model at the hub j [24], where M_{1j} represents the arrival rate, varying throughout the day and modeled as a non-homogeneous Poisson process (NHPP), M_2 denotes the service time, modeled using a truncated exponential distribution, s_j is the number of parallel servers available at the hub j, representing the number of batteries that can be charged simultaneously, and N_j^0 is the finite population of costumers of the hub j, i.e. $|\mathcal{N}_j|$.

Algorithm 1 shows the Monte Carlo simulation process of the discrete-time queue. Specifically, M independent simulation runs are executed, each spanning T time steps (Algorithm 1, line 3-4, 11, 13). The system dynamics at each hub can be described by the following set of difference equations:

$$\begin{cases} n_j(k+1) = n_j(k) + A_j(k) - S_j(k) \\ c_j(k+1) = c_j(k) + E_j(k) - S_j(k) \\ n_{qj}(k+1) = n_{qj}(k) + A_j(k) - E_j(k) \end{cases} \tag{9}$$

where $n_j(k)$ represents the total number of batteries in the system (queued + charging batteries) at hub j in the discrete-time k, $c_j(k)$ represents the number of batteries currently being charged at hub j in the discrete-time k, $n_{qj}(k)$ represents the number of batteries in the queue at hub j in the discrete-time k, $A_j(k)$ is the number of new batteries arriving in the system (asking for a charging service) at hub j in the discrete-time k, $S_j(k)$ is the number of batteries that have finished charging and exits from hub j in the discrete-time k and finally $E_j(k)$ is the number of new batteries starting the charging process at hub j in the discrete-time k.

At each time step k (Algorithm 1, line 5), $E_j(k)$ can be determined as follows:

$$E_j(k) = \min\{\max\{0,\ s_j - C_j(k)\},\ n_{qj}(k)\}, \quad (10)$$

ensuring that new batteries start charging at each time step as long as there are available charging slots and batteries in the queue.

To model daily variations in charging requests, $A_j(k)$ is sampled using an NHPP (Algorithm 1, line 6), where the arrival rate $\lambda_j(k)$ changes throughout the day. On average, household i visits its assigned hub every $\frac{C_{\text{hb}}}{\beta_i}$ days, based on its energy consumption. Considering all the households assigned to hub j, the total daily arrival rate $\lambda_{\text{D}j}$ is the sum of individual household rates (initialized in Algorithm 1, line 2):

$$\lambda_{\text{D}j} = \sum_{i \in \mathcal{N}_j} \frac{\beta_i}{C_{\text{hb}}}. \quad (11)$$

To distribute this demand over a 24-hour period, a time-dependent distribution function $f(k)$ models the fluctuations in the arrival rate –with higher values during peak hours and lower values during off-peak hours– leading to a time-varying arrival rate $\lambda_j(k)$ (initialized in Algorithm 1, line 2):

$$\lambda_j(k) = \lambda_{\text{D}j}\, f(k). \quad (12)$$

The number of new arrivals $A_j(k)$ is determined by $\lambda_j(k)$ and the finite population (Algorithm 1, line 6):

$$A_j(k) \sim \text{Poisson}(\lambda_j(k), N_j^0, n_j(k)) \quad (13)$$

This limits the number of new arrivals when the queue is full (i.e., when n_j customers are in the system, at least $N_j^0 - n_j$ new arrivals can occur) [24].

Once $A_j(k)$, $E_j(k)$, and $S_j(k)$ (the latter being defined in previous iterations) are determined, the system state is updated according to (9) (Algorithm 1, line 7).

The service process at hub j is governed by $S_j(k)$, representing the number of batteries being served (finishing the charging process) at time step k. When a battery enters the system, for instance, at time step k', the service time (i.e., the charging duration) τ_c is sampled from an exponential distribution with rate μ, truncated to avoid assuming infinite charging times (Algorithm 1, line 8):

$$\tau_c \sim \text{Exp}(\frac{1}{\mu}) \quad (14)$$

The expected charging duration τ_c is then used to determine the discrete-time at which the battery completes the charging process, updating $S(k' + \tau_c)$ accordingly (Algorithm 1, line 9). The parameter $\frac{1}{\mu}$ represents the average charging time and is influenced by several factors, such as the household battery capacity C_{hb}, charging dynamics, and the state of charge (SOC). For simplicity, it is assumed that each of the s_j charging servers operates with constant current I and voltage V, neglecting non-linear variations in charging behavior. The instantaneous power $P_j(k)$ required at hub j at each time step k can be computed as (Algorithm 1, line 10):

$$P_j(k) = c_j(k)\, V\, I \quad (15)$$

This allows tracking the evolution of power demand throughout the day.

Finally, by applying Monte Carlo simulations to the queuing network model (9) the performance of the BCS can be simulated over a given observation period T. The average daily power profile $\overline{P}_j(k)$ is then obtained by averaging the instantaneous power values $P_j(k)$ over the days of each individual simulation (Algorithm 1, line 12), and subsequently averaging across all simulations (Algorithm 1, line 14).

The proper operation of the PPE model requires that the system's service capacity satisfies the following condition:

$$s_j \mu \geq \lambda_{Dj}, \quad (16)$$

i.e., the requests must be processed faster than they arrive. Conversely, if $s_j\mu < \lambda_{Dj}$, the system becomes saturated, leading to unserved demand and service interruptions.

III. Case Study

To validate the proposed methodology and assess its practical performance, a set of numerical simulations is carried out based on real-world data collected from rural electrification projects.

A. Simulation Setup

To calibrate simulations and define benchmark parameters, data collected from the solar nano grid (*SONG*) project in Kenya were used [19], [25]. This project involved the construction of two BCSs in villages in Nakuru County. From July 1st to November 22nd, 2016, data were collected from 51 households, monitoring their batteries' charging and discharging profiles at 30-minute intervals.

From the data analysis conducted on the dataset collected by the project, it was found that the average daily end-use consumption per household was $\overline{\beta} = 18.23[\frac{Wh}{day}]$, reflecting limited service usage primarily for lighting. Additionally, an empirical probability distribution $f(k)$ was derived (Fig. 1), describing user visiting patterns to the charging hubs. The data reveal a precise concentration of visits during daylight hours, with two significant peaks around 7:30 AM and 5:30 PM, likely corresponding to commuting times. These observed behavioral patterns are expected to be similar in other rural areas with comparable socio-economic and environmental conditions. Factors such as weather, daylight hours, and work routines significantly influence consumption and visiting behavior, making this analysis relevant for modeling and predicting charging hub usage in similar contexts.

B. Results Analysis and Discussion

The case study concerns a new unelectrified rural community located in the Kuresoi South constituency, in Nakuru County (Kenya). This area was selected due to its socio-economic conditions and geographical characteristics. Covering 2.67 km^2, the study area includes 374 households,

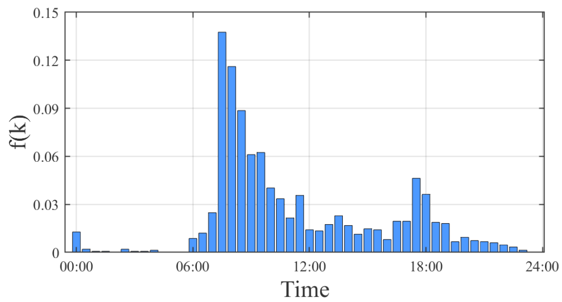

Fig. 1. Probability distribution of the arrival rate $f(k)$ extracted from the benchmark scenario.

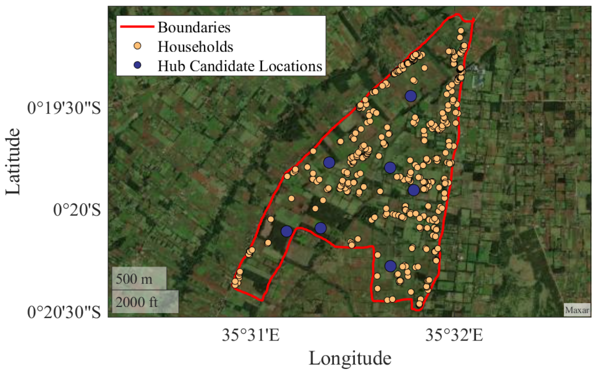

Fig. 2. Satellite view of the case study rural community.

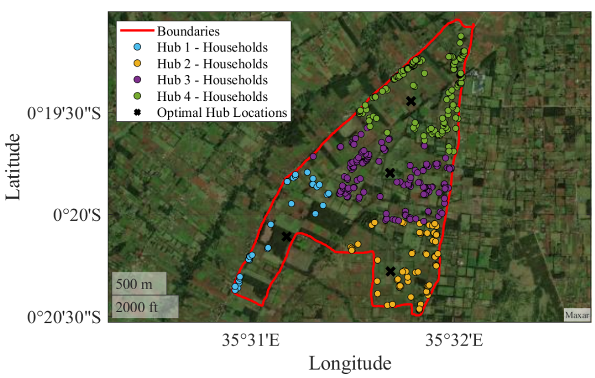

Fig. 3. Optimal hub configuration obtained in the case study rural community.

identified via satellite imagery, along with potential hub locations (Fig. 2).

The HLP, introduced in Section II-B, was solved using the GUROBI API for MATLAB [26], with a simulation environment replicating the real layout observed in Google Earth Pro. For the HLP, several parameters were assumed. Specifically, the maximum allowed distance (ϵ) is set to 0.8 km, ensuring that households are within a 10-minute round-trip walk to a charging hub. The fixed costs for the candidate hub locations are assigned randomly within a reasonable range, based on typical values observed in similar rural electrification projects. The resulting hub configuration, shown in Fig. 3, minimizes operational costs while ensuring that all households are within the set distance from a charging hub.

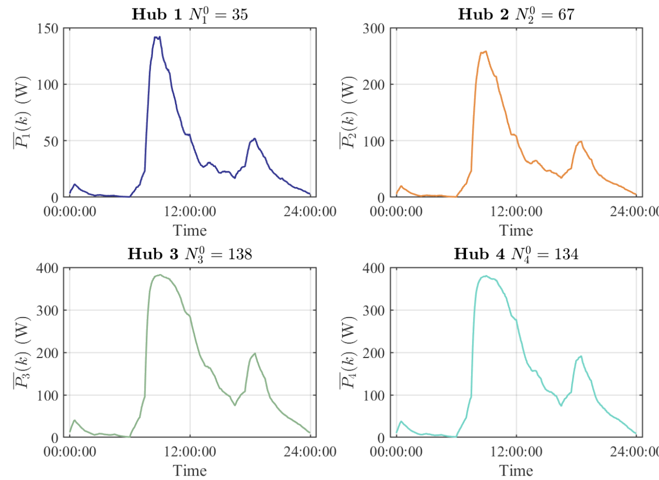

Fig. 4. Estimated daily power demand profiles for charging hubs across different population size.

After solving the HLP, Monte Carlo simulations over a 10-year period were performed to assess the expected charging demand profiles at the hubs. The key parameters assumed for all hubs were: $s_j = 5$, $I = 6.8\,\text{A}$, and $V = 12\,\text{V}$. For simplicity, all households were assumed to have a uniform average consumption $\overline{\beta}$, and their visit patterns were modeled using the empirical function $f(k)$, as indicated in the simulation setup. The household batteries were assumed to have capacity $C_{\text{hb}} = 82\,\text{Wh}$. The values of N_j^0 were derived from the optimal solution of the HLP.

Figure 4 illustrates the estimated daily charging demand at each hub, highlighting variations due to user arrival patterns and charging durations. The results provide insights into the approach's ability to handle demand fluctuations. Since all hubs are sized identically, it is observed that those with a smaller population efficiently handle demand, showing clear peaks with rapid depletion of queued requests. Conversely, in hubs serving larger populations, queue buildup occurs during peak hours, leading to prolonged power demands in the following hours and potential system saturation.

Furthermore, to assess the impact of key design parameters -such as charging intensity, current, and the parallel server capacity- on performance metrics, a sensitivity analysis was conducted on Hub 3, selected due to its larger population compared to other hubs in the study area. Five different scenarios were simulated, as reported in Table I, varying the number of parallel chargers s_3, as well as the current I and voltage V parameters, which directly influence the service time μ. Results are presented in Fig. 5, while Table I also includes the achieved performance metrics such as waiting times $W_q(k)$ and server utilization ρ. As expected, the results confirm that faster charging systems, with higher values for s_3, I, and V, efficiently meet demand, reducing waiting times (as in scenario 3). In contrast, slower systems lead to battery queue accumulation, increasing waiting times but maximizing server utilization (as in scenario 4).

Summing up, the proposed approach provides a foundation

TABLE I

SCENARIO ANALYSIS CONDUCTED ON HUB 3

Scenario	I (A)	V (V)	s	μ (min)	$\overline{W}_q(k)$ (min)	max($W_q(k)$) (min)	ρ
I	10	10	5	~ 39.5	~ 12	~ 25	~ 0.25
II	12	6.8	5	~ 48.5	~ 22	~ 50	~ 0.31
III	12	6.8	10	~ 48.5	~ 0.5	~ 9	~ 0.16
IV	12	3.4	5	~ 97.5	~ 107	~ 221	~ 0.62
V	12	3.4	10	~ 97.5	~ 10	~ 26	~ 0.31

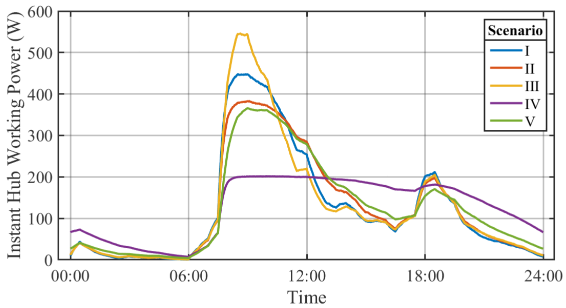

Fig. 5. Expected daily power profile estimation - hub 3 scenario analysis.

for future optimization by fine-tuning parameters (such as s_j, I, and V) to enhance charging efficiency. By optimizing these factors, the system can be designed to better balance energy supply and demand and reduce waiting times, ultimately improving the overall performance and sustainability of the charging infrastructure.

IV. CONCLUSION

This work has addressed two main challenges in battery charging system (BCS) design: (1) determining the optimal number, location, and allocation strategy for charging hubs, and (2) estimating the daily power demand for each hub. To tackle the first challenge, a model for the hub location problem has been developed, showing its effectiveness in optimizing hub locations in rural areas while minimizing costs and ensuring accessibility. For the second challenge, results from Monte Carlo simulations combined with a discrete-time queuing model have shown accurate estimates for PV systems sizing, revealing opportunities for performance improvements through scenario analysis.

Future research could include a comparison with existing literature and validation with real-world data, alongside the integration of demographic, social, and economic factors such as hub distance, energy consumption forecasts, and migration patterns.

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