Strategic Investment for Healthcare System Resilience: A Scenario-Based Optimization Approach

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Abstract— **Healthcare systems are essential to society but face numerous challenges in maintaining their resilience and long-term viability. Disruptions, much like those experienced in supply chains—can significantly hinder operational continuity. To mitigate such risks, strategic investments can be made to reduce the likelihood of service interruptions. However, given the constraints of a limited protection budget, it is crucial to allocate resources efficiently to enhance the resilience of selected critical facilities. The objective of this research is to develop an optimal protection strategy that minimizes both patient travel distances for accessing care and the number of patients left without timely treatment due to disruptions. Since accurately predicting facility failures remains difficult, robust optimization provides a valuable framework. In particular, a scenario-based robust decision-making approach allows for the evaluation of multiple plausible disruption scenarios, ensuring that the selected strategy performs well under various conditions. Our methodology leverages scenario analysis to identify the most effective investment strategy for sustaining healthcare system performance over time. Each scenario corresponds to a different allocation of protective investments, and we assess their respective impacts on system resilience. By analyzing the relationship between the fortification of specific facilities and overall system performance, we generate insights to inform budget allocation decisions aimed at maximizing resilience. This process helps quantify the relative importance of each facility, enabling an optimized distribution of resources within fixed budgetary limits.**

I. Introduction

Nowadays, healthcare systems face numerous challenges, including demographic changes, global warming and pollution, emerging diseases, and worldwide pandemics. Additionally, a declining number of physicians impacts both the quality and accessibility of care. Disruptions can also occur at the facility level, for instance, due to equipment failure or staff shortages, rendering a facility unavailable and forcing patient redirection to other facilities. All these factors affect the performance of healthcare systems, which must remain resilient and adaptable. At the facility level, various mitigation strategies can be implemented to address everyday challenges, each with its own costs and outcomes.

In decision-making, the environment in which choices are made plays a crucial role in determining the best approach. Frank Knight [1] introduced back in 1929 the distinction between decision-making environments, he divides them into three categories: (i) certainty; (ii) risk or disturbance; and (iii) uncertainty. In deterministic environment all parameters are known with certainty and outcomes are fully predictable. In risk situations, there are uncertain parameters whose values are governed by probability distributions known by the decision-maker, meaning the risk can be assess. In uncertainty situations, parameters are uncertain, and furthermore, no information about probabilities is known. Problems under uncertainty are known as robust optimization problems and often attempt to optimize the worst-case performance of the system.

Disruptions in healthcare systems, as in supply systems in general, can have severe effects on operational continuity. Supply systems have implemented various mitigation strategies to address disruptions, such as managing inventories, diversifying sourcing, collaborating with external partners, or strategically locating facilities to enhance resilience [2]. Plants, distribution centers, and other facilities typically operate for years or even decades, during which time the environment in which they function may change significantly. Costs, demands, travel times, and other inputs in classical facility location models can be highly uncertain. Snyder et al. [3] and Govindan et al. [4] provide literature reviews on facility location within stochastic and robust optimization communities, highlighting the growing interest in this subject.

While the impact of facility disruptions can be mitigated in the initial design, redesigning an entire system may be infeasible or prohibitively expensive an alternative approach involves fortifying of key assets against disruptions and adaption to future conditions. This enhancement of system reliability can be achieved through efficient investment measures. In the supply system fortification problem, no new location decisions are considered. A limited amount of resources, referred to as the protection budget, is available to enhance the resilience of certain facilities. The goal is to identify the optimal protection strategy that minimizes the system's vulnerabilities. This study focuses on the long-term resilience of healthcare systems; therefore, no distinction is made between investment measures that can be implemented rapidly and those requiring a longer time horizon.

As it can sometimes be very challenging to determine the probability disruption of facilities inside a system, robust optimization helps to address all possible scenario. In this research we will extend fortification problem in

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uncertain environment. To the best of our knowledge, all research on the fortification problem in robust decision environments focuses on minimizing the maximum damage. At the opposite, the scenario based approach in robust decision theory involves testing multiple plausible scenarios to make informed decisions. This approach allows for a more nuanced analysis, finding a solution that performs well on average. Then, our approach will be based on scenario analysis to identify the best investment strategy that maintains the healthcare system's performance on average. We will define each scenario as a potential fortification strategy and will evaluate the performance of each strategy. Hence, we will determine a correlation between fortifying certain facilities and the overall performance of the system. This correlation will then guide the allocation of funds to optimize system performance under the constraint of a fixed budget. These coefficients represent the importance of increasing the reliability of the corresponding facilities. Our model will help policymakers understand, pinpoint the strengths and weaknesses within their systems, determine the strategic facilities and build long-term resilience.

The remainder of this paper is structured as follows. Section II presents a literature review on the fortification problem. Section III outlines the different steps of the proposed framework. A numerical example is provided in Section IV. Finally, the paper concludes with a discussion of key insights and directions for future research.

II. Literature review

We conduct a comprehensive literature review on the fortification problem. To do so, we first define relevant keywords, categorizing them into two groups. The first group includes keywords related to the logistics system and its underlying infrastructure: *Supply system, supply network, facilities,* and *critical infrastructure*. The second group pertains to system fortification, including *fortification, protection,* and *protective resource allocation*. The research articles found can be categorized into two main groups: those focusing on protecting supply systems from probabilistic disruptions and those addressing uncertain disruptions. Only the most relevant articles will be discussed in detail in the following.

Researchers have studied protection models for systems facing probabilistic disruptions, meaning they have information on disruptions such as their frequency or impact. For example, Wu et al. [5] examine disruptions in an energy network where each link transporting energy is subject to failure and then become inoperable. They use a defender-attacker game, modeling different failure scenarios, each with a certain probability of occurring. Zhang et al. [6] introduce a simultaneous game on a graph where vertices can be attacked. Both the attacker and the defender have information about the probabilities of their opponent's attack or defense choices, respectively, and each seeks to make the optimal decision from their own perspective. Harnnarong et al. [7] study a warehouse network where disruptions are probabilistic and partially affect a warehouse's operational capacity. However, a fortified warehouse remains immune to disruptions. Baret et al. [8] analyze patient behavior within a healthcare system, considering their preferences for facilities and the impact of healthcare facility failures. The propose model tends to minimize the distance traveled by patient to get care and the number of patients on hold for care. Indeed, if a facility is disrupted, patients are redirected to other facilities and several facility can be unavailable at the same time. Different levels of fortification can be implemented to reduce failure probabilities.

However, determining the probability of disruptions can be highly challenging. As a result, many studies focus on fortification problems in uncertain scenarios, aiming to minimize the worst possible outcome. Aliakbarian et al. [9] propose a fortification problem for a multi-level service system, where the objective is to minimize clients' travel distance. When a facility is disturbed it becomes unavailable, whereas a fortified facility remains immune to disruption. They model the problem as a min-max problem. Roboredo et al. [10] introduce a covering location-fortification problem in which client loss due to non-coverage is penalized. The system's efficiency is measured by the total covered demand. Facilities are subject to interdictions, meaning that when a facility is interdicted, it can no longer serve customers. To mitigate the negative impact of interdictions on system efficiency, the system planner fortify a subset of facilities, rendering them immune to interdictions in order to minimize the worst case loses. Hesam et al. [11] tackle a trilevel optimization problem for protecting critical depots in a routing network against an intelligent adversary. They formulate the problem as a defender-attacker-defender game. The defender (upper level) selects u depots to protect among m existing ones. The attacker then destroys r of the remaining $(m - u)$ unprotected depots to maximize disruption. Finally, in the lower level, the defender optimizes vehicle routes, deciding which customers to serve after the attack. Parvasi et al. [12] establish on the basis of leader–follower game a game in the form of defender–attacker–defender for defense facilities. At the first level, the defender tries to decide on the best location of defense facilities among potential sites such that they receive minimum damage from the attacker in the entire time horizon, considering a budget constraint. At the second level, the objectives of the model are in contrast to the first level objectives. Considering his budget conditions, the attacker decides which service facilities are attacked at each period such that maximum damage would be inflicted upon the defender's service facilities. Finally, at the third level, the defender decides how to allocate customers to service facilities considering the remaining capacity at each period to achieve maximum benefits.

In robust decision theory, the choice between a min-max approach and a scenario-based approach depends on the nature of uncertainty and the decision-maker's objectives. To the best of our knowledge, all research addressing fortification under uncertainty has relied on the min-max approach. The goal of the approach min-max is to minimize the maximum possible loss in the worst-case scenario, the concern is a rare but critical catastrophic event such as natural disaster [13]. It can also be used to face an informed attacker who coordinates his attacks to maximize damage, in the case of terrorist attack for example ([9], [12]) or military defense [11]. On the other hand, the scenario-based approach explores multiple possible futures, aiming for decisions that perform well across a broad range of disruption scenarios. This approach aims to find a solution that performs well on average. When modeling the long-term performance of a healthcare system facing recurring facility disruptions, a scenario-based approach is more suitable for identifying an investment strategy that maintains overall system robustness. According to our literature analysis, the fortification problem in stochastic environments appears more flexible, as it allows for level-based protection. This means that facilities can be fortified to different degree, each with different impact on probability of disruption. Level-based protection offers more detailed and accurate solutions, making it well-suited for real-life applications. Stochastic environment also allows imperfect fortification, which is to suppose that even a protected facility can fail but with a reduced probability. Our framework is inspired by stochastic analysis to extend robust analysis by using scenario analysis to incorporate level-based fortification and imperfect protection. We divide the system behavior analysis into several stages. First, we define scenario as fortification scenario, where certain facilities are assumed to be fortified, making them immune to attacks corresponding to disruption. Each scenario is evaluated based on the worst-case coordinated attacks on a set of facilities, taking into account the corresponding fortification strategy. Hence, we will determine a correlation between fortifying certain facilities and the overall performance of the system. This correlation will guide the allocation of funds to optimize system performance within the constraints of a fixed budget.

III. Problem formulation

A. Overview of the proposed approach

We assume the existence of a healthcare system with operating facilities. Let $\mathcal{K}$, indexed by k, be the set of existing facilities. A facility can be disrupted and subsequently become unavailable; however, no information regarding the probability of such disruptions are available. We also consider a set of patients $\mathcal{I}$, indexed by i. The travel distance between a facility k and patient i is d_{ik}. Moreover patients have preference over facilities and tend to go to their preferred facility if available, and some facilities are not admissible for a patient, for being too far for example. K_i is the ordered set by preferences of admissible facilities for patient i. If all facilities admissible for a patient are unavailable, the patient cannot be treated and it occurs a penalty f_i. The goal is to minimize the average distance travel by patients to get care and the penalty incurs by patients on hold for care due to disruption.

The overall flow of our approach is as follows: first a set of scenarios corresponding to fortification scenario are generated, in which certain facilities are fortified, making them immune to attacks, corresponding to disruption.

Each scenario is then evaluated based on the worst possible coordinated attacks knowing that certain facilities are immune to attacks. To do so, we use an attacker model where an informed attacker coordinates his attacks on a number of sites, with the aim of maximizing the distance patients must travel to receive care and the penalty of having patients on hold.

After analyzing a sufficient number of scenarios, we identify a correlation between strengthening specific facilities and the overall system performance. The resulting coefficients serve as indicators of the relative importance of improving the reliability of each facility. This insight informs the strategic allocation of funds to maximize efficiency within a fixed budget. Figure 1 depicts the main steps of the method.

B. Scenario evaluation

The goal is to assess whether it is crucial to reinforce a specific facility or not. This involves evaluating the impact each facility has on the overall system's resilience and identifying which ones require more significant investment to minimize the risk of disruptions. In order to do so, scenarios corresponding to fortification strategies will be evaluated. During this process, no cost are considered; only the fact that certain facilities are immune to disruption is considered. It is essential to set a balance between having a sufficient diversity of scenarios to capture all uncertainty and limiting the number of scenarios to maintain a reasonable computation time. In general, the higher the number of scenarios tested, the more precise the performance estimation becomes. However, beyond a certain threshold, the results tend to converge. To determine this threshold, stability tests can be conducted. Scenarios are randomly generated, but it is important to ensure that all facilities are fortified roughly the same number of times. This helps maintain a balanced representation of different fortification strategies across the system.

A scenario is a fortification strategy in which each facility is either fortified or not. Let $\mathcal{S}$ be the set of scenarios, for all $s \in \mathcal{S}$, we have a vector $z^s = \{0,1\}^K$, with $z^s_k = 1$ if facility k is fortified, 0 otherwise. The effectiveness of the strategy is then assessed by analyzing the worst-case attack scenario under the given fortifications. An attacker will aim to maximize the damage by targeting facilities that will become unavailable and then unable to treat patients. Fortified facilities are immune to attacks. Attacks will correspond to disruption.

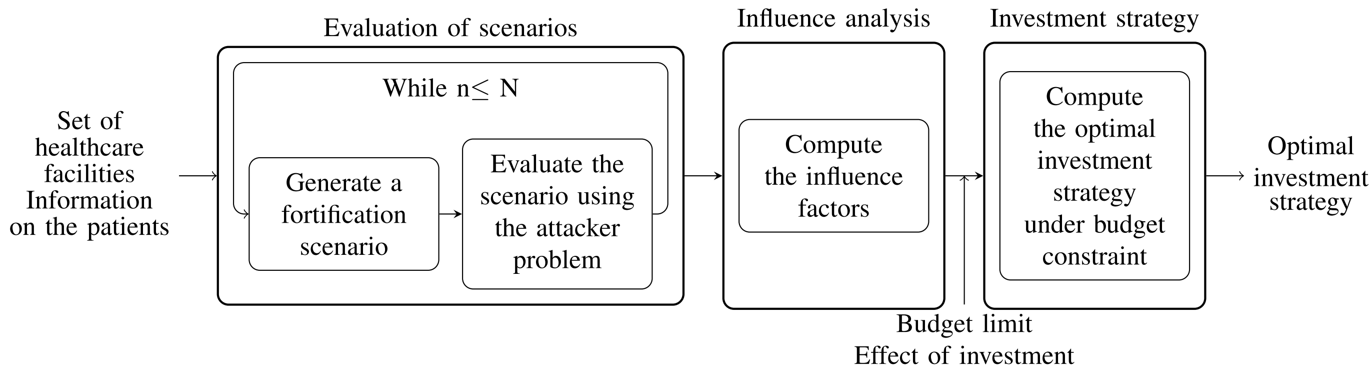

Fig. 1. Conceptual framework of the proposed method

The healthcare system aims to provide the best possible conditions for patients, minimizing their travel distance and ensuring service. The decision maker, who is the manager of the healthcare system, aims to minimize these conditions. In contrast, the attacker's goal is then to maximize the distance patients travel and increase the number of patients on hold for care (objective 1). The constraint 2 sets the auxiliary variable p_i to 1 if a patient is not assigned to any facility, indicating that the patient is on hold. The constraint 3 limits the number of attacked facilities; otherwise, the attacker may target all unprotected facilities. This reflects real-life situations, as it is rare for all facilities to be unavailable simultaneously. The constraint 4 ensures that a protected facility cannot be attacked. The constraint 5 ensures that each patient is assigned to at most one facility. The constraint 6 is an adaptation of the closest assignment constraint (CA). The CA constraint, first introduced by [14] and widely used in the literature, enforces the post-attack assignment of customers to the closest non-interdicted facilities. In our case, the constraint enforces the allocation of patient to their preferred non-interdicted facility, with $K^{'}_{ik}$ being the position of facility k in K_i and $K_i(\tau)$ the τ-th facility in K_i. The attacker problem will be solved for each scenario $s \in \mathcal{S}$.

Parameters

- $\mathcal{I}$: set of cluster of patients
- $\mathcal{K}$: set of facilities
- d_{ik} : distance i and facility k
- K_i : ordered set of admissible facilities for i
- $K^{'}_{ik}$ is the position of k in K_i
- f_i : penalty if patient i is not assigned to any facilities
- A : maximal number of attacks
- $z_k = \begin{cases} 1 \text{ if facility } k \text{ is fortified} \\ 0 \text{ otherwise} \end{cases}$

Decision variables

- $x_{ik} = \begin{cases} 1 \text{ if patient } i \text{ is assigned to facility } k \\ 0 \text{ otherwise} \end{cases}$
- $y_k = \begin{cases} 1 \text{ if facility } k \text{ is attacked} \\ 0 \text{ otherwise} \end{cases}$

Auxiliary variables

- $p_i = \begin{cases} 1 \text{ if patient } i \text{ is not assigned} \\ 0 \text{ otherwise} \end{cases}$

$$max \sum_{i=1}^{I} \sum_{k=1}^{K} d_{ik} x_{ik} + \sum_{i=1}^{I} f_i p_i \tag{1}$$

$$(1 - \sum_{k=1}^{K} x_{ik}) = p_i \qquad \forall i \in \mathcal{I} \tag{2}$$

$$\sum_{k=1}^{K} y_k \leq A \tag{3}$$

$$y_k \leq (1 - z_k) \qquad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{k=1}^{K} x_{ik} \leq 1 \qquad \forall i \in \mathcal{I} \tag{5}$$

$$\sum_{\tau=1}^{K^{'}_{ik}} x_{iK_i(\tau)} \geq (1 - y_k) \qquad \forall i \in \mathcal{I}, \forall k \in \mathcal{K} \tag{6}$$

C. *Influence analysis*

To compute the influence factor between the fortification of a facility and the overall performance of the system, we generate a significant number of scenarios S. Each scenario $s \in \mathcal{S}$ represents a different fortification strategy and is associated with a weight w_s, corresponding to the worst-case attacks that is the objective of the attacker problem from section III-B. The higher the value of w_s, the less impact fortified facilities have. The objective is to identify a relationship between the selection of fortified facilities that lead to the best performance. By understanding these relationships, the decision maker can optimize resource allocation to fortify the most critical facilities in the most effective manner.

We compute the influence factor following the steps below:

- For each facility $k \in \mathcal{K}$ compute its mean weight corresponding to scenarios in which it is fortified:

$$W_k = \frac{1}{\sum_{s=1}^{S} z_k^s} \sum_{n=1}^{S} w_s * z_k^s$$

- Then, in order to have a value between 0 and 1 compute for all facilities:

$$\alpha_k = 1 - \frac{W_k}{w_0}$$

with w_0 the score of the scenario such as $z^0 = \{0\}^K$

The influence factor α_k represents the importance of fortifying facility k to prevent major disruptions. A higher value of α_k indicates greater importance and will be used to guide a robust investment strategy aimed at enhancing the resilience of the healthcare system.

D. Investment strategy

Investments can be made in facilities to reduce their probability of failure. Unlike the previous steps, where fortifying a facility provided complete protection against disruptions, investments now serve to reduce the probability of failure rather than eliminate it entirely. By analyzing the influence between reinforcing specific facilities and overall system performance, we can guide budget allocation to maximize system resilience. The importance of strengthening each facility has been quantified, allowing for an optimized distribution of funds within the fixed budget constraints.

Each facility comes with L possible investment levels, each of which corresponds to different monetary values, with level 0 corresponding to no investment. The more we invest on a facility the more its reliability is improved. For example, hiring one doctor and hiring two doctors will have different costs, but recruiting two doctors may provide twice the benefit of hiring just one. Level-based protection offers more detailed and accurate solutions, making it well-suited for real-life applications. When an investment at level l is made in facility k, it incurs a cost c_{kl} and an outcome coefficient q_{kl}. If no investment are made the outcome coefficient is set to 0 ($q_{k0} = 0$) . As previously defined, the influence coefficient α_k set the importance of fortifying the facility k to prevent major disruptions.

The goal is to maximize the overall outcome. The contribution of each facility to resilience enhancement can be quantified by computing its weight, which is given by the influence coefficient α_k multiplied by the outcome coefficient q_{kl} corresponding to the chosen investment level l (objective 7). The constraint 8 insures that the overall budget is not exceeded and constraint 9 insures that only one level of investment is chosen per facility. The formulation of the problem is given below.

Parameters

- $\mathcal{K}$: Set of facilities
- $\mathcal{L}$: Set of levels of investment
- c_{kl}: cost of investing at level l on facility k
- q_{kl}: improved reliability at facility k at level l
- α_k: influence coefficient
- B : the overall budget

Decision variables

- $u_{kl} : \begin{cases} 1 \text{ if facility } k \text{ is fortified at level } l \\ 0 \text{ otherwise} \end{cases}$

$$max \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_k q_{kl} u_{kl} \tag{7}$$

$$\sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl} u_{kl} \leq B \tag{8}$$

$$\sum_{l=1}^{L} u_{kl} = 1 \qquad \forall k \in \mathcal{K} \tag{9}$$

IV. Numerical case study

A random instance generator was developed to create problem instances. It allows for specifying the number of clusters of patients and the number of facilities randomly generates their locations within a specified range. For each cluster, we compute their distance to each facility and rank the facilities in increasing order of distance to establish the patient's preference set K_i. All patient in cluster i have the same preference set. For every patient, the three closest facilities from this ranking are considered feasible options.

We consider a case with 200 patients and 10 facilities, each with unlimited capacity. Both linear models presented are solved using IBM ILOG CPLEX Optimization Studio. Scenarios are generated randomly, with a maximum of five facilities being fortified in each scenario. To determine the optimal number of scenarios, we conducted a convergence test on facilities's weights and found that 140 scenarios is sufficient. After evaluating the scenarios using the Attacker problem described in Section III-B, we observe that, for instance, in scenarios with 5 fortified facilities, the improvement in score compared to the worst case (without any fortification) ranges from 12% to 69%. Similarly, for scenarios with 4 fortified facilities, the improvement ranges from 0% to 63%. Furthermore, two different scenarios may have similar improvement rates, for example the scenario where facilities 1, 6, 7, 9 and 10 are fortified has an improve rate of 68.6% and the scenario where facilities 1, 2, 3, 6, and 10 are fortified has an improve rate of 66.0%.

We compute the weights using the method describe in section III-C, resulting in the following parameters: 0.54, 0.58, 0.57, 0.61, 0.62, 0.65, 0.61, 0.51, 0.51, 0.54. This indicates that fortifying facility 5 or 6 has the more impact.

Those weights are used to solve the model for optimal distribution of fund to the facilities. We suppose that we have three level of investment on facilities, the level 0 being that we don't invest in this facility, level 1 a small investment with a small impact and level 2 a bigger investment with bigger impact. The budget available is computed as a percentage of the overall cost of investing at maximum level in all facilities. Investment helps improve the facility resilience.

First, assuming that investment has the same effect on all facilities, we set $q_{k0} = 0, q_{k1} = 0.1$ et $q_{k2} = 0.2$ for all facilities. As expected, when the budget is 25% of the total cost, facilities 5 and 6 are improved to the maximum level, while facility 7 is improved at level 1. Similarly, when the budget is increased to 50% of the overall cost, facilities 2, 4, 5, 6 and 7 are improved to level 3.

Now, assuming that investments have different impacts, we randomly generate q_{k1} for all facilities within the range 0.05 to 0.2. The step between q_{k1} and q_{k2} is also determined randomly. This results in the following parameters: q_1 =[0 0.15 0.27], q_2=[0 0.07 0.2], q_3=[0 0.19 0.29], q_4=[0 0.05 0.15], q_5=[0 0.04 0.26], q_6=[0 0.17 0.23], q_7=[0 0.12 0.35], q_8=[0 0.17 0.24], q_9=[0 0.1 0.2], q_{10}=[0 0.10 0.23]. With a budget of 25% of the total cost, facility 7 is fortified to the maximum level, while facilities 3, 6 and 8 are fortified at level 1. When the budget increases to 50% of the total budget, facilities 1, 3, 6 and 8 are fortified at level 1, while facilities 5, 7, and 10 are fortified to the maximum level.

V. Conclusion

Healthcare systems face numerous challenges. Disruptions can occur at the facility level due to factors such as equipment failures or staff shortages, making certain facilities temporarily unavailable and requiring patients to seek care elsewhere. To address daily operational issues, various mitigation strategies can be implemented at the facility level, each with associated costs and outcomes. However, budget constraints often require decision-makers to prioritize which facilities to strengthen. Our model provides policymakers with insights into the strengths and vulnerabilities of their healthcare systems, helping them identify key facilities for investment to enhance long-term resilience.

Research on the fortification problem can be categorized into two main groups: studies focusing on protecting supply systems from probabilistic disruptions and those addressing worst-case disruptions. In the first category, researchers develop protection models based on known probabilities of disruptions, such as their frequency or impact. However, estimating these probabilities can be highly complex. Consequently, many studies focus on fortification in uncertain scenarios, aiming to minimize the worst possible outcome, indeed, to the best of our knowledge, all research in this area adopts a min-max approach. In contrast, the scenario-based approach in robust decision theory evaluates multiple plausible scenarios. This method allows for a more balanced analysis, identifying solutions that perform well on average while maintaining robustness.

In this paper, we propose a framework that integrates level-based fortification and imperfect protection with scenario analysis. Our approach divides system behavior analysis into multiple stages. First, we define a fortification scenario, in which specific facilities are fortified, making them immune to disruptions. Each scenario is then evaluated based on the worst possible coordinated attacks targeting multiple sites, given the chosen fortification strategy. Attacks will represent disruptions. By analyzing these scenarios, we aim to identify the influence between fortifying certain facilities and the overall system performance. This insight will guide the optimal allocation of resources, ensuring the best possible system resilience within a fixed budget.

As research perspective, first, we need to test the proposed model on a case study and develop a heuristic or metaheuristic approach to handle large instances efficiently. Second, the model can be further enhanced by incorporating additional factors, such as facility capacity, recovery time from disruption, implementation time of investment measures to better reflect real-world constraints and improve decision-making. Thirdly, the model can be studied as a bi-objective, as our two aggregate performance indicators are contradictory and the impact of the penalty value of being on hold plays a major role in the value of the objective function.

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