A Quantum Optical Systems Simulator: Assumptions and Requirements in the Framework of Second Quantization and Fock Space*

Kamil Wereszczyński, Agnieszka Michalczuk, Krzysztof A. Cyran

Abstract— **We present a conception of a novel quantum optical circuits designer, simulator, and visualization tool based on second quantization and the Fock space formalism. Our approach integrates a graph-based representation of quantum optical circuits with an augmented reality (AR) visualization system, enabling intuitive modeling and analysis. The framework ensures quantum correctness, supports real-time circuit generation, and bridges theoretical quantum optics with computational simulation, facilitating interactive quantum system design.**

I. Introduction

In this work, we present a novel concept for a quantum optical circuit simulator based on second quantization and the Fock formalism, designed for integration with an augmented reality (AR) visualization system. Our approach provides a structured framework for modeling and simulating quantum optical devices while ensuring compatibility with interactive AR representations, enabling an intuitive and precise visualization of quantum phenomena.

A. State of the art

Quantum optical circuit simulation has significantly progressed, with multiple high-performance tools developed for research and industrial applications. Wang [1] introduces an efficient quantum circuit simulator that eliminates external dependencies to improve performance. The FPGA-based quantum circuit simulator by Hong at al. [2] optimizes gate operations, reducing computation time and improving scalability. Qulacs in the work of Suzuki at al. [3] is a widely used high-performance quantum circuit simulator for research applications, leveraging acceleration techniques for efficiency. Google's qsim [4] is an advanced wave-function simulator utilizing gate fusion and multi-threading, supporting circuits with up to 40 qubits. Virtual Lab by Quantum Flytrap [5] provides a real-time, no-code simulation of optical tables, enabling interactive quantum optical experiments. However it is not a 3D visualization. QuEST (Quantum Exact Simulation Toolkit) [6] is designed for large-scale simulations, supporting multi-threading, GPU acceleration, and distributed computing. Additionally, IonSim.jl [7] offers a specialized environment for simulating the interactions of trapped ions with laser light, crucial for quantum computing with ion-based qubits.

While these simulators are powerful, most do not fully integrate the second quantization formalism and Fock space representations, which are fundamental for modeling quantum optical systems, particularly in augmented reality (AR) visualization frameworks. The second quantization formalism, extensively discussed e.g., in [8], introduces creation and annihilation operators to dynamically describe quantum states. This approach is essential for handling many-body quantum systems, where the number of particles varies, and is widely used in quantum optics and condensed matter physics.

B. Motivation

The growing interest in quantum technologies highlights the need for accurate quantum optical simulators. However, existing tools lack support for AR visualization and do not implement the Fock formalism or second quantization. Most rely on matrix-based methods but fail to integrate Fock space and creation-annihilation operators, essential for modeling quantum optical circuits (QOCs). Additionally, AR integration requires handling both quantum properties and spatial constraints, which current simulators overlook. To address this, we propose a new framework for simulating and visualizing QOCs in an AR-compatible environment.

C. Contribution

In this work, we propose a structured framework for modeling quantum optical circuits by combining graph-based representations with the Fock space formalism and creation-annihilation operators. Our approach bridges theoretical quantum optics with practical simulation and supports integration with augmented reality (AR).

The main contributions are:

1. A formal graph-based definition of quantum optical circuits incorporating second quantization principles.
2. A correctness framework ensuring physical and logical validity of modeled circuits.

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All authors are in merQlab, Department of Computer Graphics, Vision and Digital Systems Silesian University of Technology, Gliwice, Poland

Corresponding author: kamil.wereszczynski@polsl.pl

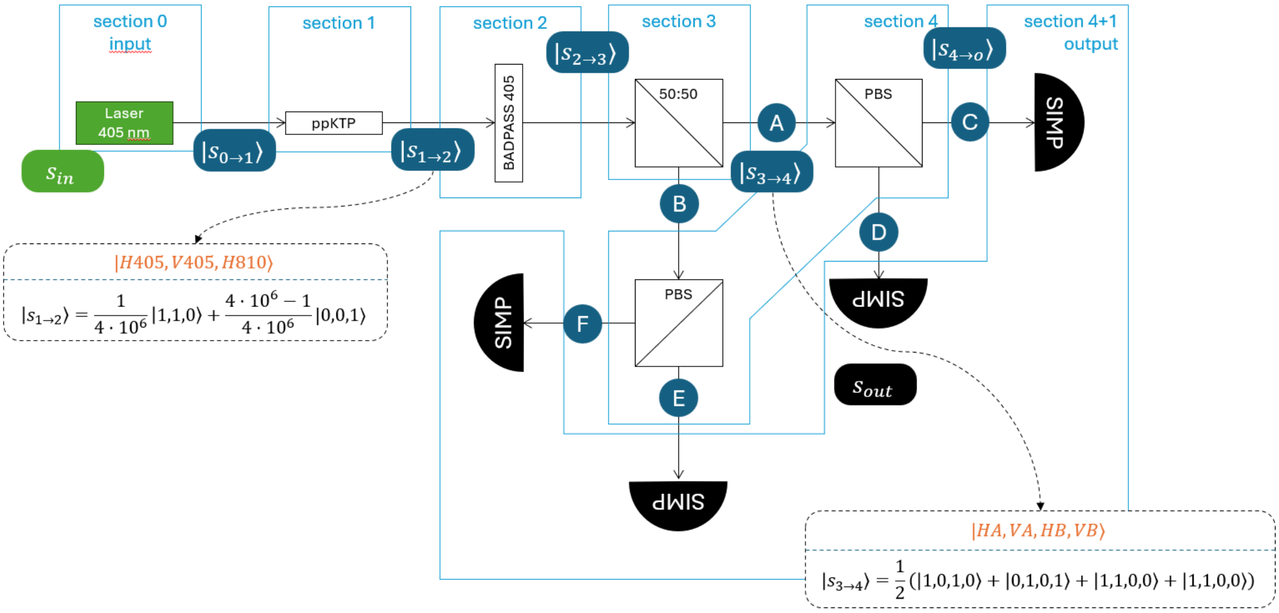

Figure 1. The illustration of Quantum Optical Circuit design and connection with Fock space formalism

3. A dual design methodology—geometry-driven and structure-driven—that enables both intuitive AR-based layout and systematic circuit construction.

These elements form the basis for a quantum optical simulator that unites theoretical rigor, spatial modeling, and interactive visualization.

II. Materials and Methods

In this chapter, we propose a definition of a quantum optical system in the context of a simulator based on second quantization, namely the Fock formalism and the language of creation and annihilation operators, along with their theoretical foundations.

A. Quantum Optical Circuit

Quantum Optical Circuit (QOC) in the context of software, is an directed graph (V, E), where:

1. There are featured nodes that have no input, called light sources and are denoted by s_j. There is at least one light source in the circuit.
2. There are featured nodes that have no output, called detectors and are denoted by d_j . There is at least one detector in the circuit.
3. We assume that each detector has exactly one output, that is not an element of the circuit. This output is electrical signal which is used for analysis of the quantum measurement arising at the latest in detector.
4. All other nodes are at least one input and at least one output. These nodes are called (optical) devices and are denoted v_j
5. Vertices represents possible optical trajectories of the photons generated in laser light source.
6. The graph fulfills the condition: for each photon generated in the light source there is at least one detector, which it could achieve. The case is allowed, when one photon can achieve more then one detector, which means that it may move many different trajectories. This case represents the phenomenon of superposition.

For QOC there are defined:

1. **Initial state (IS)** which describes how many photons are present at the beginning of the simulation at each light source. Formally, the states are denoted by vectors in the Fock space – it will be clarified further.
2. **Measurement state (MS)** which describes how many photons can be obtained on each detector, described in the Fock space formalism as well.
3. **Section of QOC which** is the set of devices that have the same length of the path connecting the light source and the given device. If there are more then one path this condition has to be fulfilled for at least one of them (path). We assume that photons achieve all devices in the same time with some accuracy, and we call this feature n*-th simultaneity or (time) slot of section*, where n is the length of mentioned paths. Therefore each section has:
   a. k inputs, which is the number of active inputs to devices that constitute a section. By *active* input we mean this input on which could appear at least one photon, potentially.
   b. l outputs, defined analogously to the inputs.
4. **Intermediate state (NS)** for the given section, which describes how many photons can be obtained after they pass through all of devices belonging to this path.
5. **The flow of the QOC** is the set family consisting of:
   a. set of initial states, which we call *section nr* 0,
   b. section nr 1, …, section nr j, …, section nr D,
   c. set of detectors, which we call *section nr* $S+1$ or *last section*.

The section in within the flow has to fulfill the condition: the number of inputs of j-th section is equal to the number of outputs of the $j-1$ section, for all $j \in [1, \dots, S+1]$. Denote that while IS and MS are proper circuit quantum states, which will be physically implemented and measured in a real setup, the NS only is virtual one, that cannot be gathered in reality[1]. This concept is illustrated in Figure 1. The 405 nm laser serves as the input device. There are four intermediate sections and one output section. Intermediate states are marked in blue, the input state in green, and the output state in black.

B. *Fock space formalism*

In this section, we present the formalism of Fock space, but only to the extent necessary to understand how the simulation should function, with reference to the previously defined representation of the quantum optical system as a directed graph. Therefore, we do not provide a formal definition of Fock space but rather introduce it in the context of its application for describing the consecutive states in the QOC flow. We focus on a finite-dimensional subspace of Fock space, considering only a limited number of modes relevant to the quantum optical system under study. For section j, we denote the state at its output in Fock space as $|S_j\rangle_F$, distinguishing it from states in Hilbert space, which, by convention, are denoted without the index **F**. Thus, $|S_0\rangle_F$ represents the initial state, while $|S_D\rangle_F$ denotes the state that should be obtained at the detectors (detectors have no outputs, hence the final section does not have them either). The number of beams (outputs, vertices) in the **j-th** state (section) is denoted by $|S_j\rangle_F$. Unlike the Hilbert space, where a state represents a single-particle wavefunction, the Fock space allows for the superposition of states with varying numbers of photons.

As in standard quantum mechanics, the Fock space formalism employs bra-ket notation:·⟩ - ket state or ⟨·| - bra state. For now, we will only use ket states. The number of modes (analogous to "slots" in computer science) corresponds to the number of possible particle states present in the system. For example, if there are two optical paths in the system where a photon may be present, there will be two modes. If we additionally consider the vertical and horizontal polarization of the photon, there will be four modes. In the case of the j-th section, the number of rays in the system corresponds to the number of outputs from this section. If, for a given section j, we have $|S_j\rangle = 6$ and we include the polarization, Fock state will have 12 modes. Since the number of optical paths (beams) may change dynamically due to optical elements such as beam splitters or diffraction gratings, the number of available modes in the Fock space representation also varies accordingly[2]. For convenience, we propose using the following notation for Fock space states: ·· ;··; … ;··⟩ where each pair of numbers, separated by a semicolon, represents a single optical path. The first number indicates the number of horizontally polarized photons, while the second represents the number of vertically polarized photons[3].

As illustrated in mentioned Figure 1, each section of the quantum optical circuit corresponds to a distinct Fock state configuration, with potentially varying numbers of particles and different mode structures. Two intermediate states are explicitly described: one between sections 1 and 2, and another between sections 3 and 4. The first case highlights the probabilistic nature of photon pair generation: a 405 nm photon passing through the ppKTP crystal may either remain unchanged or split into two 810 nm photons with orthogonal polarizations, leading to a superposition of Fock states with up to three particles. In the second case, after filtering out the 405 nm component, the state involves two optical paths (trajectories), each allowing for horizontal or vertical polarization of 810 nm photons, yielding four modes. This example illustrates that there is no universal or fixed Fock state for the entire system—rather, the structure of the Fock state evolves throughout the circuit. We acknowledge that our representation slightly departs from the traditional quantum field theory formalism, particularly in how we denote and index Fock states across sections. This is intentional and reflects the needs of the simulation architecture and the circuit's modular nature.

C. *Creation – Anihilation operators formalism*

This section introduces creation and annihilation operators from a practical perspective for system design, omitting full mathematical rigor. These operators modify states in Fock space, representing abstract transitions rather than the physical appearance or disappearance of particles. The vacuum state, |0⟩, denotes a state where no particles occupy the defined modes, but this does not imply an empty physical space. External factors, such as air molecules, environmental noise, or stray photons, may still be present. These disturbances, categorized as classical noise, quantum noise, and decoherence, are not explicitly included in the Fock space description and form what is known as the system's bath. While these external effects influence quantum systems, they are beyond the scope of this work and will be addressed in a later phase of simulator development.

The creation and annihilation (C-A) operators are mathematically defined by their action on Fock states, as follows:

[1] The connection between NS and quantum mechanics is quantum operator O, which is subject to the time-dependent Schrödinger equation. J-th NS is the initial state after action of this operator for a time t_j. This time can be determined by multiplication of the light speed and the length of the light trajectory for the j-th section. This side note is just for clarification and the described physical setup does not have influence for the simulation hence, formalism of creation – annihilation operators will be used in it.

[2] This situation, which is completely different then in classical physics where the phase space has to be defined once for the whole system, in quantum science (especially in quantum field theory) is allowed and prevalent – e.g., beam splitter can divide one beam into two, and scattering on a diffraction grating will create a number of new beams. The definition is consistent because of annihilation – creation operator formalism.

[3] It is good practice to emphasize in menus and descriptions (articles, documentation) which number (first or second) means which polarity, as there is no convention in this regard.

$$\hat{a}^{\dagger}|n\rangle_F = \sqrt{n+1}|n+1\rangle_F$$
$$\hat{a}|n\rangle_F = \sqrt{n}|n-1\rangle_F, n \geq 0 \quad (1)$$

Where $\hat{a}^{\dagger}$ is the creation operator, which increases the number of particles in a given mode, and $\hat{a}$ is the annihilation operator, which decreases it.

Note that applying the annihilation operator to a single-particle state leads to the vacuum state: $\hat{a}|1\rangle_F = |0\rangle_F$. Conversely, a single application of the creation operator to the vacuum generates a one-particle state: $\hat{a}^{\dagger}|0\rangle_F = |1\rangle_F$. However, applying the annihilation operator to the vacuum state yields the zero vector: $\hat{a}|0\rangle_F = 0$. Since the zero vector represents the null state, any subsequent application of the creation operator to it also results in zero:$\hat{a}^{\dagger}0 = 0$. If Fock space consists of multiple modes, the C-A operators are defined separately for each mode j:

$$\hat{a}_j^{\dagger}\left|\ldots, n_j, \ldots\right\rangle_F = \sqrt{n_j+1}\left|\ldots, n_j+1, \ldots\right\rangle_F$$
$$\hat{a}_j \left|\ldots, n_j, \ldots\right\rangle_F = \sqrt{n_j}\left|\ldots, n_j-1, \ldots\right\rangle_F \quad (2)$$

For clarity, it is sometimes useful to introduce more descriptive notation. For example, $\hat{a}_{j,V}^{\dagger}$ denotes the creation operator for a photon in the j-th mode with vertical polarization, while $\hat{a}_{j,H}^{\dagger}$ represents the creation operator for a horizontally polarized photon in the same mode. Since creation and annihilation operators modify the number of particles in a mode, they do not commute. Their commutator satisfies the fundamental relation: $[\hat{a}^{\dagger}, \hat{a}] = \hat{a}^{\dagger}\hat{a} - \hat{a}\hat{a}^{\dagger} = 1$

D. *Quantum Definition of Optical Device*

A Quantum Optical Device (QOD), which is an element of a Quantum Optical Circuit (QOC), can be understood as a "black box" with a certain number of input and output ports. Unlike in electrical systems, the distinction between "inputs" and "outputs" in a quantum optical device is largely conventional. This means that one can connect incoming photons to ports conventionally labeled as "outputs" and observe the device's effect on ports labeled as "inputs," or even create a mixture of both.

To avoid ambiguity, in this document, we will refer to these sides as the "A-side" and "B-side" of the device. The ports on the A-side will be denoted as $a_1, \ldots, a_N$ while those on the B-side will be denoted as $b_1, \ldots, b_M$. Importantly, this distinction between sides is not related to how the device is physically connected but rather to how its functionality is mathematically defined.

In the most general case, a quantum optical device can be described as either: A-side as a function of B-side ($A = f(B)$) or B-side as a function of A-side ($B = f(A)$). n this document, we will adopt the $B = f(A)$ convention, meaning that the device's output states are expressed in terms of its input states. However, this is merely a convention, and in the literature, the opposite approach may also appear.

The formal definition of a QOD relies on the Fock space representation and the creation-annihilation (C-A) operator formalism, described above. The QOD is thus defined by the following set of equations:

$$\hat{b}_1^{\dagger} = \varphi_1(\hat{a}_1^{\dagger}, \ldots, \hat{a}_M^{\dagger})$$
$$\hat{b}_2^{\dagger} = \varphi_2(\hat{a}_1^{\dagger}, \ldots, \hat{a}_M^{\dagger}) \quad (3)$$
$$\ldots$$
$$\hat{b}_N^{\dagger} = \varphi_N(\hat{a}_1^{\dagger}, \ldots, \hat{a}_M^{\dagger})$$

As seen in the definition, the system consists of N equations, which, in the most general case, could take any mathematical form. However, in practical implementations, these functions φ_j are typically polynomials in terms of the input mode operators with complex coefficients.

Since the creation and annihilation operators obey the bosonic commutation relations:

$$\left[\hat{a}_j, \hat{a}_k{}^{\dagger}\right] = \delta_{jk} \quad (4)$$

it is necessary that the transformation $B = f(A)$ preserves these fundamental commutation relations to ensure the device's physical validity. This means that the functions φ_j must be chosen in such a way that the new set of creation operators satisfies:

$$\left[\hat{b}_j, \hat{a}_k{}^{\dagger}\right] = \delta_{jk} \quad (5)$$

This requirement imposes constraints on the possible transformations a QOD can implement. In practice, this condition is often satisfied automatically when the transformation is unitary, such as in the case of beam splitters, phase shifters, and interferometers.

III. Results

In this chapter, we present the results of our theoretical work, which focused on developing a coherent framework for defining quantum optical devices and circuits using the formalism of second quantization and Fock space. These results directly extend the concepts and definitions introduced in the previous methodological sections (II.B–II.D), where we described the structure of Fock states, the role of creation and annihilation operators, and their application in defining quantum optical devices. In this section, we demonstrate how these theoretical elements are synthesized into a consistent modeling approach.

Specifically, we provide a set of operator equations describing the internal behavior of optical components, define correctness criteria that ensure physically valid circuit construction, and show how these elements form the basis for a simulation-ready representation of quantum optical circuits. Additionally, we include a requirements specification that translates this theoretical foundation into a structured guideline for implementation. This specification ensures that each device modeled in the simulator adheres to quantum mechanical principles and fits seamlessly within the larger graph-based architecture of the system.

The implementation phase of the simulator is currently in progress. Future work will report experimental validations and simulation results in a separate article.

A. *Quantum Optical Circuit Design*

This section outlines the process of creating a QOC from the perspective of its designer, a key system actor. Two approaches are considered: geometric and structural, which will be detailed later. First, we define the three fundamental elements of a QOC: the light source, optical devices, and detectors. The terminology introduced here was developed as part of the OptiQ project to support software for designing quantum optical systems, from device-level definitions to 3D hologram mappings in augmented reality. This work also contributes to the analysis of requirements for design, simulation, and visualization systems, offering a novel approach not previously addressed in the literature. Light sources (LS), optical devices (ODev), and detectors (DT) are treated as optical objects, each defined by its entry configuration, geometry, and quantum description. The geometry of an optical object is further divided into intrinsic and extrinsic. Intrinsic geometry describes how light propagates within the object, while extrinsic geometry defines its position and orientation in the real world, such as on an optical table.

The entry configuration defines the A-side and B-side of an optical object. Light sources (LS) typically have only one side and one entry, though this is not a strict requirement. Detectors (DT) have only an A-side and usually one entry, while optical devices (ODev) have both sides. Each entry must be classified as IOE (input-only entry), OOE (output-only entry), or GPE (general-purpose entry).

The center of an optical device is a point or segment through which light must pass between entries. A device may have one or multiple centers. A pathway is a triple (a_j, c_k, b_l). defining an allowed light trajectory inside the device. If both a_j and b_l are GPE, the pathway is bidirectional, meaning two trajectories are allowed: $a_j \to c_k \to b_l$ and $b_l \to c_k \to a_j$. If a_j is an IOE, then b_l must be an OOE or GPE. Entries classified as OOE cannot serve as inputs, and IOE entries cannot serve as outputs.

Each pathway has an associated operator in the creation-annihilation (C-A) formalism, given by: $\hat{a}_j^\dagger = f(\hat{b}_1^\dagger, \dots, \hat{b}_M^\dagger)$. The reverse operators can be derived if the transformation between $\hat{a}_j^\dagger$ and $\hat{b}_k^\dagger$ is unitary or otherwise invertible, as in the case of lossless optical elements like beam splitters and interferometers. However, if the device introduces irreversible processes such as absorption, decoherence, or non-unitary transformations, the reverse operators cannot be uniquely determined and must be explicitly defined.

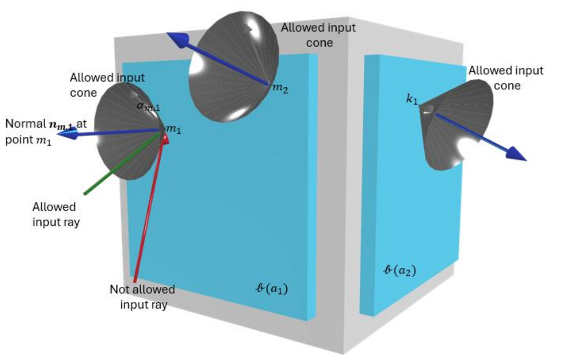

Figure 2. Bounding boxes for entries and allowed input cones.

B. Extrinsic and intrinsic geometry of an optical device

To model both internal behavior and spatial placement of quantum optical devices, we define two geometric layers: **intrinsic geometry**, which describes how light propagates and transforms within the device, and **extrinsic geometry**, which defines the device's position and orientation on the optical table (see Figure 2).

Extrinsic geometry includes:

1. **Geometric center** – the device's reference point in 3D space;
2. **Device border** – an abstract frame used to position ports and internal elements;
3. **Constructional rotation** – a reference vector defining the default orientation.

Each entry port is visually represented in the AR model as a **point** or **surface**, with triangle symbols indicating type: IOE (input), OOE (output), or GPE (bidirectional). We assign orange to A-side entries and green to B-side for visual clarity.

Photon **pathways** within the device are categorized as **transmissive**, **reflective**, or **deflective**, depending on how light is redirected internally. These pathways are linked to operator equations introduced earlier and define how quantum states evolve during propagation through the device.

further work progresses.

C. Parameters and features of an optical device

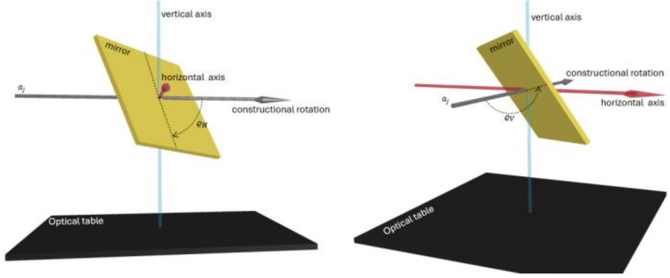

Figure 3. Reflection core parameters in reflective pathways

An optical device has two parameter groups. The first group, derived from its intrinsic geometry, consists of two reflection core angles—ϱ_H and ϱ_V—which define the virtual mirror responsible for internal reflection. Here, ϱ_H is the angle around a horizontal axis (parallel to the optical table) perpendicular to the construction rotation, while ϱ_V is the angle around the axis perpendicular to the optical table. In a case of deflection, there are two deflection angles δ_H, δ_V defined similar to reflection core angles.

Secondly, parameters of quantum description, which are derived from the C-A operator equations.

Optical device, in the program, should have at least the following features:

1. Name.
2. Graphical symbol that will appear in the designer.
3. Entries, which is the list of A-side and B-side entries.
4. Centers
5. Pathways. Each pathway has at least: Name, Configuration, which is a triple (a_j, c_k, b_l), Type (IOE, OOE, GPE) and Event kind.

6. Optical efficiency, which is the distribution of efficiencies (in [0,1] range, where 1 is best) over the wavelength spectrum.
7. Quantum description, which is the list of equations, where each entry has exactly one equation

D. *Correctness of the Quantum Optical Circuit*

In our framework, an optical device is considered correct according to three distinct aspects, as detailed in the requirements specification. First, intrinsic correctness ensures that the quantum description of the device is well-defined, meaning that the systems of equations describing the transformation of the input and output operators are solvable for all parameter values. Second, consistency correctness is achieved when the two sets of equations—the one mapping the A-side operators from the B-side operators and vice versa—are mutually consistent; substituting the operators from one side into the equations of the other yields identities, thereby confirming the coherence of the quantum representation. Finally, connectivity correctness requires that every entry and exit point within the device is properly linked by at least one pathway, with the additional condition that the presence of an equation for a given operator is directly tied to the existence of a corresponding physical pathway, such as one of the GPE type. This comprehensive view of correctness guarantees that the device functions as intended both in its quantum behavior and its physical connectivity.

E. *Generating Quantum Optical Circuit.*

Quantum optical circuits (QOC) in our framework can be generated using two complementary approaches: geometry-driven and structure-driven. In the geometry-driven approach, the layout is derived from the spatial distribution of 3D holograms of optical devices placed on a virtual optical table. Each device is linked to a quantum description and defined by a geometric center and orientation vector. Bounding boxes and spatial hierarchy (e.g., in Unity 3D) ensure that transformations, such as rotation, are reflected in the device's optical behavior. Light sources are initialized with defined rays, Fock states, and spectra, and the resulting QOC graph encodes trajectories as graph vertices. In contrast, the structure-driven approach starts from an abstract QOC graph and reconstructs the spatial layout by placing devices on a predefined grid of mounting points. Each device is positioned according to its grid coordinates and direction, ensuring appropriate physical spacing and interconnections. Both methods rely on the same formal optical device descriptions and 3D mappings. The geometry-driven approach emphasizes intrinsic optical properties, while the structure-driven method ensures coherent physical placement. Together, they provide a robust framework for accurate and AR-compatible QOC construction.

IV. Conclusions and Discussion

In this work, we introduced a novel approach to simulating quantum optical circuits based on second quantization and the Fock formalism, designed for integration with augmented reality visualization. Our framework systematically bridges the gap between theoretical quantum optics and practical computational modeling, ensuring both mathematical rigor and real-world applicability. By defining quantum optical circuits as directed graphs and incorporating intrinsic and extrinsic geometries, we provide a structured representation of quantum devices that aligns with both quantum mechanics and software engineering principles. Our contributions include a formal definition of quantum optical circuits, a correctness framework ensuring their validity, and two complementary methods for their generation: geometry-driven and structure-driven. These approaches allow for both accurate quantum behavior modeling and spatial coherence within optical setups.

Future work will focus on expanding the simulator's capabilities, including the implementation of real-time reconfiguration of optical circuits, support for dynamic quantum states, and enhanced AR-based visualization. Additionally, improving the efficiency of the computational model and integrating experimental validation will further strengthen the applicability of the proposed framework. This research lays the foundation for more interactive and precise quantum optical simulations, providing a valuable tool for both theoretical exploration and practical system design.

It is important to note that the goal of this paper is to present the conceptual and theoretical foundation for a quantum optical circuit simulator. The results described above focus on the formal modeling of quantum optical systems and the design of an operator-based simulation framework. As the implementation is currently in progress, numerical simulation results are not included in this manuscript. These outcomes will be reported in a dedicated scientific article upon completion of the development phase.

V. References

[1] C.-C. Wang, "QueenV2: Future of Quantum Circuit Simulation," 2024, *arXiv*. doi: 10.48550/ARXIV.2409.14697.

[2] Y. Hong, S. Jeon, S. Park, and B.-S. Kim, "Quantum Circuit Simulator based on FPGA," in *2022 13th International Conference on Information and Communication Technology Convergence (ICTC)*, Oct. 2022, pp. 1909–1911. doi: 10.1109/ICTC55196.2022.9952408.

[3] Y. Suzuki *et al.*, "Qulacs: a fast and versatile quantum circuit simulator for research purpose," *Quantum*, vol. 5, p. 559, Oct. 2021, doi: 10.22331/q-2021-10-06-559.

[4] Q. A. team and collaborators, *qsim*. (Sep. 2020). Zenodo. doi: 10.5281/zenodo.4023103.

[5] P. Migdał, K. Jankiewicz, P. Grabarz, C. Decaroli, and P. Cochin, "Visualizing quantum mechanics in an interactive simulation – Virtual Lab by Quantum Flytrap," *Opt. Eng.*, vol. 61, no. 08, Jun. 2022, doi: 10.1117/1.OE.61.8.081808.

[6] T. Jones, A. Brown, I. Bush, and S. C. Benjamin, "QuEST and High Performance Simulation of Quantum Computers," *Sci. Rep.*, vol. 9, no. 1, p. 10736, Jul. 2019, doi: 10.1038/s41598-019-47174-9.

[7] S. Krämer, D. Plankensteiner, L. Ostermann, and H. Ritsch, "QuantumOptics.jl: A Julia framework for simulating open quantum systems," *Comput. Phys. Commun.*, vol. 227, pp. 109–116, Jun. 2018, doi: 10.1016/j.cpc.2018.02.004.

[8] M. Federico, V. Dorier, S. Guérin, and H. Jauslin, "Space-time propagation of photon pulses in dielectric media, illustrations with beam splitters," *J. Phys. B At. Mol. Opt. Phys.*, vol. 55, 2022, doi: 10.1088/1361-6455/ac7e0e.