Error reduction for image encoding - reconstruction for quantum photonic systems

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***Abstract*— We report a successful results in the quantum image encoding - reconstruction experiments with the usage of local phase for pixel representation. This implementation was intended for using the photonic quantum system, since the phase shift is easy to produce. Our method will be implemented in real photonic quantum computers, after further development.**

Quantum Image Representation is well studied area in quantum informatics. The first proposed algorithm was the qubit-lattice method introduced in 2003. This method was not harnessing the full potential of quantum system - using as many qubits as there were pixel in the image.

Most commonly used methods right now are Flexible Representation of Quantum Images (FRQI) proposed in 2011, and Novel Enhanced Quantum Representation (NEQR) proposed in 2013, which uses superposition to encode a monochrome image on a quantum computer using reduced number of qubits.

In this paper we want to propose the combination of Local Phase Image Quantum Encoding method and Phase Distortion Unraveling (PDU) error mitigating method as an alternative method of quantum image encoding, producing satisfying results.

I. INTRODUCTION

We want to propose the combination of the LPIQE method [1] for quantum image encoding in conjunction with PDU error correction [2], as a new way of hybrid image processing, where all calculations are done on a quantum computer, while error correction is applied on a classical machine. This method is intended for use in quantum object detection of aviation instruments, on images taken on a Microsoft HoloLens 2 device inside the flight simulator cockpit.

LPIQE method of quantum image encoding uses $\lceil log_2 X \rceil + \lceil log_2 Y \rceil + 1$ qubits, where *X* and *Y* reference the X and Y dimensions of the encoded image. The additional qubit is used for encoding color information. LPIQE method uses the local phase to encode color information. LPIQE method uses superposition to encode pixel coordinates. Encoding color information in the local phase allows the method to be combined with PDU error reduction.

The PDU method uses the PDU functions, which are the functions interpolated for the set of points (x_d, ε_d), where x_d is a value for which the function was measured and ε_d is the recorded error. This way all qubits available on quantum backends can be used for computation, without worrying about error correction.

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Currently in the area of quantum image processing, for encoding monochrome images two methods are most commonly used: **FRQI** [3] was the first method proposed in 2011, which used superposition to encode the color information of the image. Then the corresponding qubits are linked using multiple controlled R_y gates rotated by a specific angle θ, which are applied to a remaining qubit. **NEQR** [4] method was proposed in 2013. In contrary to it's predecessor it stores the image's gray scale value in basis state of a qubit sequence. One of the downsides is the larger number of qubits needed for storing color value - the NEQR algorithm uses 8 qubits to represent the value of shade of gray intensity for each pixel (values from 0 to 255). This means, that encoding just 2x2 image uses 10 qubits. NEQR has a possibility to encode color images, but then for only the color storage 24 qubits are used (3 groups of 8 qubits for each of the RGB colors). The other downside is that color storing is so involving, that it prevents any complex transformations.

Quantum error correction and reduction is also a highly researched area. Physical implementations of quantum computers are limited by current technology. Those limitations lead to occurrences of decoherence which can destroy a quantum state.

Error correction, aims to undo and prevent the changes caused by decoherence in the system, using additional qubits, and is intertwined within the circuit performing quantum computation. Two main classes of error correction methods are:

- **Quantum redundancy and measurement stabilizer** uses multidimensional Hilbert space on which the states are recorded. An example of such code is, proposed in 1996, 5-qubit error correction code [5], where 4 additional qubits were used, to correct the error of just one of them. This is also the smallest number of additional qubits needed to achieve a perfect quantum error correction for one qubit.
- **Quantum surface codes**, are the second class of error correction, most widely researched right now. These codes are implemented on 2D qubit lattices [6], and tackles the error of the whole quantum system. This approach reduces the number of qubits needed for error correction, but it also increases the complexity of quantum circuit creation, because only the qubits lying next to each other on the lattice could be directly linked by quantum gates.

Quantum error reduction, on the other hand, aims just to minimize the error. These methods could be implemented in quantum, hybrid classical-quantum, or classical systems. One of them is Richardson extrapolation error mitigation [7] . Another example could be the quasi-probability method [8], [9]. In this method the inverse process to the noise process that occurred is probabilistically implemented.

In the presented work we use the LPIQE (Local Phase Image Quantum Encoding), which was preliminary described in [1]. This method is developed for image encoding and processing in photonic quantum processors. Such quantum systems are being currently widely developed. Therefore, the LPQIE method is based on the phase shifts, since it is easy to produce in photonic devices using just one phase shifter. The encoded images can be utilized in two general ways: they can be processed inside the quantum processor (e.g. for object detection), or they can be send to another system. For the second purpose, the scheme of encoding - reconstruction is needed.

This work shows experimentally, that the encoding - reconstruction scheme after PDU based error reduction is possible in NISQ era. In the area of quantum communication its usefulness can be conserved even beyond NISQ, since the expectations of transmitted state are that their fidelity will be on a much lower level than inside the quantum computers, e.g. Tann [10].

II. Materials and methods

A. LPIQE method

Let's consider an image of height H pixels and of width W represented by a matrix $I_m = \left[\hat{p}_{r,c}\right]_{H\times W} \wedge r \in \{0,\dots H-1\} \wedge c \in \{0,\dots W-1\} \wedge \hat{p}_{r,c} \in [0,1] \cap \mathbb{R}$, where $\hat{p}_{r,c}$ is the intensity of pixel placed in r-th row and c-th column. We can execute the (vertical) vectorization and obtain

$$\overrightarrow{I_m} = [p_j]^T, s.t.: j = rW + c \tag{1}$$

Using the *Encoding the constant data* (described by us in previous works [1]) we can represent this form of an image with the state:

$$|I_m\rangle = \left[e^{ip_0}\dots e^{ip_J}, 0^{M-J}\right] = \sum_{j=0}^{J} e^{ip_j}\,|j\rangle\,, \tag{2}$$
$$J = WH-1,\quad M = 2^{\lceil log_2(WH)\rceil}$$

The first part of above state is the vector of WH exponential functions of pixels intensities, the second part is a complements to quantum state's requirement of having power of two coefficients. This exponential functions can be treated as local phases of a state, therefore we call it *Local Phase Image Quantum Encoding* LPIQE. The state can be obtained by an operator defined by matrix:

$$\tilde{\mathcal{L}}(I_m) = \mathbf{1}\overrightarrow{I_m} = \begin{bmatrix} e^{ip_0} & 0 & \dots & 0\\ 0 & e^{ip_1} & 0\dots & 0\\ \vdots & 0 & \ddots & 0\\ 0 & \dots & 0 & e^{ip_J}\end{bmatrix}$$
$$\mathcal{L}(I_m) = \left[\begin{array}{c|c} \tilde{\mathcal{L}}(I_m)_{J\times J} & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{1}\end{array}\right]_{M\times M} \tag{3}$$

where $\mathbf{1}$ is an identity matrix / operator. Hence this is operator that has on diagonal, the exponential e^{ip_j} of local phases being the intensities of pixels. It can be generated, on the low level of abstraction, using the combination of multi-controlled phase shift gates and the technique of uncomputation; however, currently available solution (e.g., qiskit) provides tools for defining the operator as matrix, and the mapping it to the set of physically implemented gates. Therefore, we can conclude that such a definition of an operator is sufficient for current implementation.

B. Image reconstruction

To reconstruct an image, we have to extract the local phases, using the following operator:

$$\mathcal{R}(I_m) = \Upsilon\cdot\mathbf{1}^M\otimes H\cdot\Theta(I_m), \text{where:} \tag{4}$$
$$\Upsilon = perm(2,1,4,3,\dots,J,J-1,J+1,J+2,\dots,2M)$$
$$\Theta(I_m) = \frac{\mathcal{L}(I_m)\otimes|0\rangle\langle 0| + \mathbf{1}\otimes|1\rangle\langle 1|}{\sqrt{2}}$$

which is *unitary* (but not Hermitian) operator and can be used as en evolution operator. In the matrix representation, the above operator has a block-matrix form, as in II-B.

$$\mathcal{R}(I_m) = \frac{1}{2}\times$$
$$\left[\begin{array}{ccccc|ccccc} e^{ip_0} & -1 & 0 & \dots & 0 & & & & & \\ e^{ip_0} & 1 & 0 & \dots & 0 & & & & & \\ \vdots & & \ddots & & & & & \mathbf{0} & & \\ 0 & \dots & 0 & e^{ip_J} & -1 & & & & & \\ 0 & \dots & 0 & e^{ip_J} & 1 & & & & & \\ \hline & & & & & 1 & -1 & 0 & \dots & 0\\ & & & & & 1 & 1 & 0 & \dots & 0\\ & & \mathbf{0} & & & \vdots & 0 & \ddots & & 0\\ & & & & & 0 & \dots & 0 & 1 & -1\\ & & & & & 0 & \dots & 0 & 1 & 1 \end{array}\right]_{2M\times 2M.} \tag{5}$$

If we act with this state on homogeneous superposition of $log_2 2M$ qubits we obtain probabilities for measuring eigenstates:

$$\mathfrak{p}\big(|ka\rangle\,|a=0\big) = \frac{1}{2M}(1-cos(p_k)) \tag{6}$$
$$\mathfrak{p}\big(|ka\rangle\,|a=1\big) = \frac{1}{2M}(1+cos(p_k))$$

In the virtue of above considerations, the formula for the reconstructed pixel $\tilde{p}_k$ in vectorized image is as follows

$$\tilde{p}_k = acos(1-2M\mathfrak{p}(|k0\rangle)), \text{or}\ \ \tilde{p}_k = acos(2M\mathfrak{p}(|k1\rangle)-1) \tag{7}$$

It would seem that the $2M$ factor before probability, will kick the argument out of $acos$ domains. Although from 6 we know, that $\mathfrak{p}(|kx\rangle) \in \left[0, \frac{1}{M}\right]$, however the pixels intensities are from the range of $[0, 1]$, hence the range of $\mathfrak{p}(|k0\rangle)$ is limited to $\left[0, \frac{1-cos(1)}{2M}\right] \sim \left[0, \frac{0.46}{2M}\right]$, and the range of $\mathfrak{p}(|k1\rangle)$ to ca. $\left[\frac{1.54}{2M}, \frac{1}{M}\right]$. Hence for both cases the argument under the $acos$ function is in the range $\left[0.54, 1\right]$, and the fact that $acos$ domain is $[0, \pi]$, gives sufficient margin for distortion of outcomes, due to noise.

C. Error reduction

The proposed method is the development of PDU (*Phase Distortion Unraveling* function method presented by Werner et al. in [2] and its adjustment to the image processing area.

Let's consider the PDU function $\varepsilon(x) = \tilde{\gamma}(x) - \gamma(x)$. It is an interpolation based on experimentally designated set of ε_k, which are the errors for arbitrary arguments x_k of γ function. These arguments are the probabilities following from local phases of qubits, extracted using phase-kickback technique. We can generalize this notion considering that x can be any object, for which we can experimentally designate the set outcomes from the quantum computation process and compare with expectation values. Therefore, we can define the general version of PDU as follows:

Definition 2.1: Let $|\psi\rangle_x$ be the the local phase e^{ix} of the eigen state $|\psi\rangle \in \mathbb{E}$ ($\mathbb{E}$ is a measurement basis) in a subspace of a n-qubit state, and the $\mathfrak{p}\,|\psi\rangle_{x_k}$ is, experimentally designated probability amplitude for state $|\psi\rangle$ with phase x_k. In that case the function $\varepsilon : \mathbb{E}\times[-\pi, \pi]\cap\mathbb{R} \longrightarrow \mathbb{R}$ is **General PDU function** if and only if its each projection on the eigen-state is smooth due to phase and fulfills following:

$$\forall k: \quad \varepsilon(|\psi\rangle, x_k) = \mathfrak{p}\,|\psi\rangle_{x_k} - \langle\psi\rangle_{x_k}$$
$$\forall\,|\psi\rangle \;\neg\exists\tilde{\varepsilon}(|\psi\rangle, x): \quad \int_{-\pi}^{\pi} dx\;\; \tilde{\varepsilon}(|\psi\rangle, x) < \int_{-\pi}^{\pi} dx\;\; \varepsilon(|\psi\rangle, x) \tag{8}$$

It is obvious, that this definition formulates the *interpolation task*, that may produce under-optimal solutions, in practice. Therefore, an interesting attempt to solve the problem of finding the proper curve for each $|\psi\rangle$ is to use B-spline curves with knots $x_1, \ldots, x_k, \ldots, x_K$.

Furthermore, we can arrange the data dividing eigen-states in a groups, as follows:

Definition 2.2: The arrangement of state $|\psi\rangle$ is a tensor product of sub-spaces of the space that it is embedded in. Formally, an n-arrangement of state $|\psi\rangle$ is the set $A_{|\psi\rangle} = \{|\psi_j\rangle, j \in [1, n]\}$, such that:

$$|\psi\rangle = \bigotimes_{j=1}^{n} |\psi_j\rangle = |\psi_1\psi_2\ldots\psi_n\rangle \tag{9}$$

Definition 2.3: General PDU function on an arrangement, is written in tensor-like notation:

$$G^{\psi_1,\psi_2,\ldots\psi_n}(x) = \varepsilon(|\psi\rangle, x) \tag{10}$$

Definition 2.4: The Error sampling for arrangement $A_{|\psi\rangle} = \{|\psi_j\rangle, j \in [1, n]\}$ and results of quantum sampling made for phases $\{x_1, \ldots, x_K\}$ is a function $R : \mathbb{E}\times \longrightarrow \mathbb{R}$ such that:

$$R^{\psi_1,\psi_2,\ldots\psi_n,k} = \mathfrak{p}\,|\psi_1\psi_2\ldots\psi_n\rangle_{x_k} - \langle\psi_1\psi_2\ldots\psi_n\rangle_{x_k} \tag{11}$$

Definition 2.5: Lets assume the arrangement $A_{|\psi\rangle} = \{|\psi_j\rangle, j \in [1, n]\}$. The calibration operator is an operator $\mathcal{C}$ defined as follows:

$$\begin{aligned}\mathcal{C}R^{\psi_1,\psi_2,\ldots\psi_n,k} =& G^{\psi_1,\psi_2,\ldots\psi_n}(x), s.t.\\ \forall\,|\psi\rangle\,\neg\exists\tilde{G}^{\psi_1,\psi_2,\ldots\psi_n}(x):&\\ \int_{-\pi}^{\pi} dx\;\; \tilde{G}^{\psi_1,\psi_2,\ldots\psi_n}(x)&\\ < \int_{-\pi}^{\pi} dx\;\; G^{\psi_1,\psi_2,\ldots\psi_n}(x)&\end{aligned} \tag{12}$$

Basing on above definition, we can say that state representing pixel is 2-arrangement $|rc\rangle$, and the image sum of such an arrangements. The reconstruction state is the sum of 3-arrangements $|rca\rangle$, where $|a\rangle$ is single ancilla. However, for reconstruction we use the subspace of the space where reconstruction occurs, since we use the results only for one ancillas eigen-state e.g, $|a\rangle = |0\rangle$. Therefore the general PDU for encoded picture P is equal to $P^{r,c}(x)$ and calibration operator creates B-spline function.

D. Experimental protocol

In the experiments we have proved that it is possible to use PDU function to reduce the error after encoding and decoding an image on a real quantum device. For quantum image encoding we have used Local Phase Image Quantum Encoding (LPIQE) method. We have improved the results for all tested images and quantum backends provided by IBM, and available from Qiskit python library. Only 7-qubit devices were used to encode and decode 8×8 black and white images - IBM Nairobi, IBM Perth and IBM Lagos, which was done to use the full potential of those 7-qubit machines. The method was also tested for larger images with the use of statevector simulator, also provided by IBM. Experiments were done using libraries Qiskit for Python, and also prepared by us for the research purposes - LPIQE and PDU Python libraries.

The one ER (Encoding-Reconstruction) experiment was conducted to check how the reconstructed image differs from the original one. Hence, we define the ER experiment as follow.

1) Parameters: τ - intensities of the pixels.
2) Generate an artificial image that has the same intensities of pixels, let's say $\tau \in [0, 1] \cap \mathbb{R}$.
3) Encode the image using quantum backend (simulator or real quantum computer).
4) Reconstruct an image.
5) Compute the basic statistics of the difference image: standard deviation, mean and MSE (Mean Square Error).

We will denote $ER(\tau)$ the one experiment with the intensities equal to τ. Using the above ER experiment, we can define the calibration algorithm for Image PDU method as follows.

1) Parameter: *granularity* g, which means the division of the range $[0,1] \cap \mathbb{R}$ into g values.
2) $j \leftarrow 0$
3) Create an empty error sampling arrangement: $R^{\psi_1,\psi_2,\ldots\psi_n,k}, k \in \{0,1,\ldots r\}$, where r is such a number that $\frac{r}{g} \leq 1$ and $\frac{r+1}{g} > 1$
4) For each $v \in \{0, \frac{1}{g}, \frac{2}{g}, \ldots, \frac{r}{g}\}$, do:
   a) execute $ER(v)$ experiment.
   b) determine the j-th "slice" of the error sampling for the arrangement $R^{\psi_1,\psi_2,\ldots\psi_n,j}$ using the original and reconstructed images where states $|\psi_m\rangle$ represents consecutive pixel of the images.
   c) increment j

The ER experiment can be also made for the real image. In that case we denote it $ER(I_m)$:

1) Parameters: I_m - input image of proper resolution.
2) Load image I_m
3) The remaining points are as in the original experiment.

After the whole arrangement is ready, the calibration operator is the interpolation spanned by its slices. In a case of this work we use the cubic interpolation.

After calibration we can reduce the errors using calibration operator. The image after reduction is given by a pixel-wise formula:

$$I_{corrected}(p_{u,v}) = I_{reconstructed}(p_{u,v}) + \mathcal{C}R^{\psi_1,\psi_2,\ldots\psi_n,k}(p_{u,v}), \wedge \\ k = I_{original}(p_{u,v}) \quad (13)$$

If k is equal to one of the slices obtained in calibration procedure its value is taken directly from the remembered slices, in opposite case it is interpolated. Here we have to emphasize, that calibration once made is applicable for the relatively long time period, which we experimentally proved in the mentioned previous work [2].

The final experiments was made in following way, assuming that the calibration was made already. We take the data set of images, that is described in the next subsection. For each image we made the ER experiment. We stored the original, reconstructed and corrected images and the basic statistics: mean, standard deviation and MSE of absolute error between original and reconstructed and original and corrected images.

Finally, we computed the correlation measures for the same configurations as above. Namely, we used Spearman, Pearson and Kendall correlation coefficient for each triple of images separately. We also computed the aggregated results determined on the sets of pixels of all images put together.

E. The parameters and dataset for our experiments

In this experiment we have proven that using PDU method in conjunction with LPIQE method produces statistically significant error reduction, while not using additional qubits. As a dataset to use in the experiments, we created images containing various shades of gray, as well as pure white and pure black, of various sizes, all of which can be found on projects GitHub page. Four of 8×8 images were created - the first one, *„named 1_original_small.png"* being only black and white, while the others are increasingly more complex. As 16×16 images we used pictures of game character faces, modified to our needs (scaled down and converted to black and white). Both those sets of images can be found inside original_images catalogue in the repository. For experiment on real quantum device we determined PDU functions for quantum backends, with granularity 6 - meaning that the output error on image was determined for 6 distinct values between 0 and 1. The granularity is a hyperparameter, that should be determined for the specific problem. Than, batch of 8×8 pixel, black and white images (40 at a time) was encoded and decoded using LPIQE method. The PDU function was than applied on the resulting image. Than the results along with the mean square error, and standard deviation of error was denoted. Also the time of calibration of PDU function and the time of applying the method was also recorded.

In addition we also tried this combination of LPIQE method for quantum image encoding, and PDU method for error reduction on larger - 16×16 images, using statevector simulator.

III. Results

A. Error reduction of 8×8 monochrome images encoding on real quantum devices

For this experiment, we collected data from 7-qubit quantum computers. Data were collected between 02.2023 and 03.2023. Over 300 distinct results were recorded during that time.

An example of PDU function result on an image could be seen on figure 1. Also the difference in mean square error (*mse*) and standard deviation of this of error for the experiment, from which those images were taken, was recorded (an experiment consists of one calibration of PDU method and 10 cycles of encoding, decoding and applying corrections) 2.

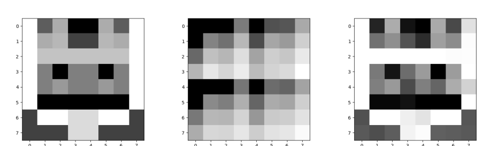

Fig. 1. Results for image collected on backend IBMQ Nairobi on 02.03.2023 before (left), after quantum computation,but before correction (center) and after applying error correction factors (right).

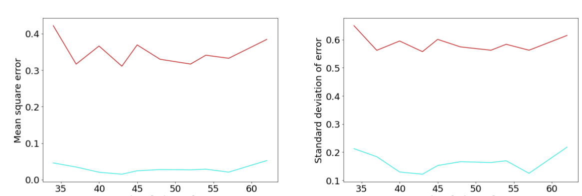

Fig. 2. Mean square error and standard of deviation change across the experiment collected on backend IBMQ Nairobi on 02.03.2023

Numerically we observed improvement in all statistical tests, which could be seen in tables 3 (partial results are available on a github repository). The statistics were measured also for mean square error before and after applying correction for all collected samples, and we observed the drop from 0.321 on average, to 0.026, which is a twelve-fold decrease. Standard deviation of error also dropped from 0.533 on average to 0.143, decreasing 3.7 times.

	original image vs image after encoding		original image vs image after error correction	
	correlation	pvalue	correlation	pvalue
Pearson's r	0,047121408	0,566920517	0,906914938	1,6445E-17
Spearman's rho	0,007733873	0,49602996	0,811492623	3,20442E-08
Kendall's tau	0,011878687	0,503352637	0,709547163	3,62586E-07

Fig. 3. The correlation coefficients, divided by images tested: Pearson product - moment, Kendall's τ and Spearmans ρ with p-values generated for examination of simplicity for result after PDU correction vs before, collected on real quantum devices.

B. *Error reduction on* 16×16 *monochrome images encoded on statevector simulator*

For this experiment, we collected data using statevector simulator provided by IBM. We have used more complex 16×16 pixel images. The results were similar to these we obtained on real quantum devices. This could be seen in image 4. This time experiment consisted of 100 runs after one calibration. The data showing mse and standard deviation of error change for the experiment, from which that image was sampled could be found on figure 5.

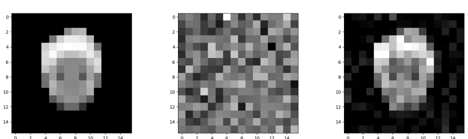

Fig. 4. Results for image collected on statevector simulator on 02.03.2023 before (left), after quantum computation,but before correction (center) and after applying error correction factors (right).

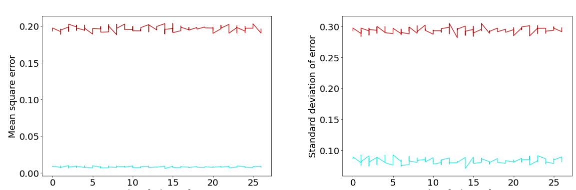

Fig. 5. Mean square error and standard of deviation change, divided by images tested, across the experiment collected using statevector simulator on 02.03.2023

In this case we observed even greater improvement in the statistical tests, which could be seen in tables 6. The MSE statistics further illustrates, that the correction method works for larger images - dropping from 0.188 on average to 0.008 which is 23.5 times a decrease. Standard deviation of error dropped from 0.278 to 0.082 - which is 3,4 times lower. We suspect such high drop occurred, because the noise in statevector simulator is deterministic, so it might be more consistent than one, on the real quantum computers. The statistical correlations (Pearson, Spearman and Kendall) are shown on the fig. 3 for the quantum computer and on fig. 5 for simulator. Columns are organized in pairs regarding correlations between different sets of images. First column of this pair presents the correlation value and the second one the statistical significance. The first pair shows the results for original and reconstructed images, the second one between the reconstructed and corrected images, while the last one consider the original and corrected images. Therefore, in the first one we see the level of image harm due to quantum errors and decoherence occurring in quantum computer. The third one shows the quality of presented error reduction method. The middle one, present for the statevector only, shows the interesting relation which shows the level of corrections made by our method.

	original image vs image after encoding		original image vs image after error correction	
	correlation	pvalue	correlation	pvalue
Pearson's r	-4,47765E-05	0,507413538	0,948853662	3,1595E-108
Spearman's rho	0,000152314	0,508479362	0,811244306	1,02643E-51
Kendall's tau	0,000153346	0,506745832	0,699094521	4,91351E-41

Fig. 6. The correlation coefficients: Pearson product - moment, Kendall's τ and Spearmans ρ with p-values generated for examination of simplicity for result after PDU correction vs before, for experiments run on statevector simulator.

From the fig. 3, concerning real quantum computers, we see that the correlation between the original and reconstructed images doesn't appear at all, since the all coefficients oscillate around zero. Furthermore, the p-values are on the level of 0.5, which means that statistical significance of the results if low. We can guess, that the results are quite strange, because the correlation has to appear, because if not, the correction wouldn't be possible. In other world - the correlation exists but is hidden by quantum effect that influences computation. Therefore, it cannot be captured by statistical tests, which is seen by very high p-value. In contrary, In a case of correlation between original and corrected images the linear correlation (measured by Pearson's r coefficient) is very high - on the level of 0.906. The monotonic correlation is a little bit poorer, but still considered as high, on the level of 0.709 (Kendall's tau) and 0.811 (Spearmann's rho). The significance of those results is high, since p-values are close to zero. It proves, firstly, that our method can better the reconstructed image significantly. Moreover it proves indirectly, that the reconstructed images are not just random noise, but holds the information about the original image. Which is the experimental prove that LPIQE methods is proper for image encoding in quantum systems.

The results obtained from simulator are higher, in case of original vs corrected images and Pearson's r coefficient: $0.94, +0.04$. However in a case of Spearman's rho and Kendall's tau are very similar, in case of rho, the same to the 3rd digit after point and for tau the difference is on the level of 0.01. Maybe it shows the relatively high quality of the simulator? In a case of original image vs reconstructed, there is also no significant differences between simulator and real quantum computer in a perspective of statistical results.

The case of reconstructed vs corrected images brings the interesting results with high statistical significance (p-value for r is below 0.02, and for rho and tau close to 0). The Pearson's correlation is on the level of 0.19 and Kendall's

0.25, so boths are slim. However Spearman's is weak (0.33). Certainly, we cannot say that reconstructed and corrected images are correlated, but they have much better results than the original vs reconstructed case. In our opinion it might be the place, where the last one correlation arise in data.

IV. Discussion & Conclusions

In this paper we have presented that LPIQE quantum image encoding method in conjunction with PDU error reduction, which we find useful for implementation in the photonic quantum solution. We proved experimentally that the proposed methods corrects the reconstructed images statistically significant. This is confirmed by the fact that the corrected images are visually similar to original ones, while the reconstructed images, before error reduction resembles the noise. Moreover the statistical measures shows the lack of the correlation in case of reconstructed images and the much higher correlations after the correction with very high statistical significance. The Pearson coefficients are on the level of 0.90 with p-value near zero, which proves very high linear correlation between original and corrected images. The monotonic correlations are lower, but still reasonable: The Kendall tau is on the level of 0.70, and Spearman rho on the level of 0.81, both with p-value near zero. It proves that on the corrected images there still appears non-linear distortions. Recapitulating, we can say that proposed method is promising and proper for implementation in a photonic circuit.

The method should also be scalable to larger images, as the main limiting factor is the space needed to store all of the collected values, and - if there is a need - some calculated values of the interpolated functions. But even for high-resolution photography, the space requirements for the PDU table shouldn't be a limiting factor, as the size of the table is the same as the size of the image, and current computers have no problem handling that. We plan to develop our method in the several directions. Firstly we would like to check how the granularity influences the quality of the error reduction. Basing on the very preliminary experiments, we expect the obvious relation that the increasing of granularity betters the quality. However, we would like to check if there is the optimal granularity, such that its greater values doesn't improve the error reduction quality. Furthermore, we would like to check the method on other quantum computers and compare the results in the context of reconstructed and corrected images qualities. The other research area is to evaluate this method for the bigger images resolution. We expect that the method will decrease its quality, due to the exponentially growing measurement eigen-state number. Therefore we plan to develop our reconstruction method for the possibility of partial image reconstruction, using the entanglement and uncomputation methods. In this area the important think is to elaborate the method of finding the maximum resolution of the image, which can be reconstructed and corrected with sufficient quality.

It is worth to say that we focused in the area of image processing, since we are interested in developing methods from this area. However our method of data encoding on quantum computers, its processing, reconstruction and correction can be applied to any massive data, in general. Therefore, it can be treated as the method for decreasing the level of the bottleneck appearing in the data encoding phase of quantum computation.

Concluding, in this paper we have presented The error reduction method for quantum encoded images using the local phase as the resource for this encoding. It will be utilized in the image processing in photonic quantum solutions.

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