L_1/L_∞-Hankel Norm optimization of Power Wireless Communication Networks by DC Programming*

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***Abstract*— Distributed power control is crucial for ensuring reliable communication and minimizing energy consumption in wireless communication networks, particularly under random switching topologies and uncertain network dynamics. Existing methods primarily rely on traditional norms or control paradigms, often facing computational challenges due to inherent nonconvexities. In this paper, we propose a novel distributed optimization framework for wireless networks using DC programming to minimize the L_1/L_∞-Hankel norm of the system. Unlike conventional approaches that may yield suboptimal solutions or suffer from computational inefficiency, the DC programming approach systematically decomposes the nonconvex Hankel norm minimization problem into tractable convex subproblems, ensuring rapid convergence and globally optimal or near-optimal solutions. Our method specifically addresses dynamic topologies modeled by random switching and parameter uncertainties, significantly improving robustness and interference management. Simulation results demonstrate that the proposed framework effectively maintains the desired SINR levels and accelerates convergence of global solution.**

I. INTRODUCTION

Efficient distributed power optimization remains one of the key challenges in modern wireless communication networks, particularly in scenarios characterized by random topology changes and parameter uncertainties [1], [2]. Reliable power control is essential for maintaining a required signal-to-interference-plus-noise ratio (SINR), directly impacting network capacity, reliability, and energy efficiency [3], [4]. Traditional methods, including control-theoretic approaches, have provided valuable insights into power management, yet often encounter limitations when addressing the inherent nonconvexities and computational complexities in large-scale networks [5], [6].

Dynamic switching of network elements, such as base stations, further complicates the optimization landscape due to rapid changes in interference patterns and channel conditions [1], [7]. This randomness can typically be modeled as Markovian processes, offering both analytical tractability and practical relevance [8]. Despite extensive research on stabilizing power control under random switching conditions, few studies have directly optimized the switching strategies or the network performance under realistic uncertainty conditions [9]. Furthermore, optimization problems involving classical norms, such as L_1 or L_∞ norms, may not fully capture the complex dynamic behavior and robustness requirements of modern communication networks.

Recently, optimization based on the Hankel norm has attracted attention due to its capability to encapsulate system dynamics and robustness in control problems [10], [11]. Specifically, the L_1/L_∞-Hankel norm [12], [13], [14] offers a meaningful balance between peak response and energy efficiency, providing a more comprehensive performance metric suitable for dynamic systems with varying constraints and operational demands [15]. Nevertheless, minimizing such norms leads to challenging nonconvex optimization problems, limiting the applicability of conventional convex optimization methods.

To address these critical challenges, difference-of-convex (DC) programming emerges as a powerful methodology for tackling nonconvex optimization tasks by reformulating them into the difference of two convex functions [16], [17]. DC programming has been successfully applied in various contexts, including probability rate optimization for positive Markov jump linear systems [18], impulse-to-peak control of positive systems [15], and optimization of buffer networks [19]. Compared to geometric programming or other heuristic approaches, DC programming provides a systematic and computationally efficient framework that ensures robust convergence to high-quality solutions even in complex, nonconvex scenarios [20]. In addition, recent advances combining metaheuristic optimization algorithms with fuzzy logic systems [21], [22] have demonstrated promising results.

Motivated by these advantages, this paper introduces a novel distributed optimization framework employing DC programming specifically designed to minimize the L_1/L_∞-Hankel norm of wireless communication networks under random switching topologies and parameter uncertainties. Our approach systematically decomposes the nonconvex Hankel norm optimization problem into solvable convex subproblems, allowing for rapid convergence and effective scalability. Through rigorous theoretical analysis and extensive simulations, we demonstrate that the proposed method significantly enhances energy efficiency, robustness, and convergence performance compared to existing approaches, positioning it as a promising solution for next-generation wireless network control.

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The remainder of this paper is structured as follows. Section II presents the problem formulation and introduces key concepts related to the Hankel norm and DC programming. Section III details the proposed optimization framework and associated DC decomposition methods. Simulation results and comparative performance evaluations are discussed in Section IV. Finally, Section V concludes the paper with remarks on future research directions.

We use the following notation. Let $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{R}_{++}$ denote the sets of real, nonnegative, and positive numbers, respectively, and let $\mathbb{R}^n$, $\mathbb{R}^n_+$, and $\mathbb{R}^n_{++}$ denote the corresponding n-dimensional vector spaces. The vector of ones is written as 1_n, and I represents the identity matrix. For any vector $v \in \mathbb{R}^n$, we define its exponential in terms of elements by $\exp[v] = [\exp(v_1), \dots, \exp(v_n)]^\top$ and its natural logarithm in terms of elements by $\log[v] = [\log(v_1), \dots, \log(v_n)]^\top$. A real matrix A is called nonnegative (positive) and is written as $A \geq 0$ ($A > 0$) if all its entries are nonnegative (positive). The nth power of the inverse of matrix A is denoted by A^{-n}. The 1-norm of v is given by $\|v\|_1 = \sum_{i=1}^n |v_i|$, and the ∞-norm by $\|v\|_\infty = \max_{1\leq i\leq n} |v_i|$. The corresponding ∞-norm is denoted by $\|v\|_\infty$, i.e., $\|v\|_\infty = \max |v_i|$.

II. Dynamics of Wireless Cellular Networks

We consider a distributed power-control model for wireless cellular networks using a directed graph $G = (V, E)$, where $V = \{1, 2, \dots, J\}$ denotes a set of transmitter-receiver pairs (users), and a directed edge $(j, i) \in E$ indicates that transmitter j interferes with receiver i. Each edge (j, i) is associated with a nonnegative channel gain $a_{ij} \geq 0$, representing the signal attenuation from transmitter j to receiver i. These gains form a channel gain matrix $A = [a_{ij}]$, with diagonal elements $a_{ii} > 0$ indicating intended communication links, and off-diagonal entries $a_{ij} \geq 0, j \neq i$, quantifying interference effects [3].

Let $p_i(t) > 0$ represent the transmit power of user i at time t, collected into the power vector:

$$p(t) = [p_1(t),\, p_2(t),\, \dots,\, p_J(t)]^\top.$$

Following Foschini and Miljanic [3], the total interference plus noise perceived by user i is given by

$$u_i(t) = \sum_{j\neq i} a_{ij}\, p_j(t) + \nu_i,$$

where $\nu_i \geq 0$ denotes the thermal noise at receiver i. Thus, the signal-to-interference-plus-noise ratio (SINR) for user i is expressed as:

$$\rho_i(t) = \frac{a_{ii}\, p_i(t)}{\sum_{j\neq i} a_{ij}\, p_j(t) + \nu_i}.$$

The main goal of distributed power control is to steer the SINR $\rho_i(t)$ toward a predefined target $\bar{\rho}_i$. An intuitive strategy, inspired by negative-feedback control, involves defining dynamics of the form:

$$\dot{\rho}_i(t) = -\beta\, (\rho_i(t) - \bar{\rho}_i),$$

where $\beta > 0$ is a control gain parameter. However, this ideal approach is infeasible since user i cannot directly control interference from other users. Foschini and Miljanic address this challenge by approximating the SINR dynamics under a locally constant interference assumption, yielding a surrogate control law:

$$\frac{a_{ii}\, \dot{p}_i(t)}{u_i(t)} = -\beta\, (\rho_i(t) - \bar{\rho}_i).$$

Rearranging terms to isolate $\dot{p}_i(t)$, the distributed power update law becomes:

$$\dot{p}_i(t) = -\beta \left[p_i(t) - \frac{\bar{\rho}_i}{a_{ii}} \left(\sum_{j\neq i} a_{ij}\, p_j(t) + \nu_i \right) \right],$$

which crucially relies only on local measurements of current power and aggregate interference, thus eliminating the need for centralized coordination.

At equilibrium, the steady-state power vector p^* satisfies:

$$p_i^* = \frac{\bar{\rho}_i}{a_{ii}} \left(\sum_{j\neq i} a_{ij}\, p_j^* + \nu_i \right),$$

compactly represented in matrix form as:

$$(I - B)p^* = \eta,$$

where matrix B encapsulates scaled channel gains, and vector η encodes scaled noise contributions. Under standard conditions (e.g., eigenvalues of B have strictly positive real parts), a unique solution p^* exists, and exponential convergence is guaranteed [3].

This continuous-time formulation parallels the discrete-time Foschini–Miljanic algorithm [3]:

$$p_i(k+1) = p_i(k)\, \frac{\bar{\rho}_i}{\rho_i(k)},$$

which similarly ensures convergence to target SINR values whenever feasible power settings exist. Both approaches share strong theoretical convergence properties, highlighting their robustness in practical scenarios.

For deeper control-theoretic analysis, it is convenient to express the power dynamics in standard linear state-space form. Defining the state $x(t) = p(t) \in \mathbb{R}^J_+$, the system dynamics become:

$$\dot{x}(t) = Ax(t) + b,$$

where $A \in \mathbb{R}^{J\times J}$ is a Metzler matrix incorporating channel gains and feedback parameters, and $b \in \mathbb{R}^J$ accounts for noise and target SINR factors. Further introducing external inputs $w(t) \in \mathbb{R}^s_+$ and outputs $z(t) \in \mathbb{R}^r_+$, we derive the following positive linear system:

$$\Sigma_{\beta,\rho} : \begin{cases} \dot{x}(t) = Ax(t) + Bw(t), \\ z(t) = Cx(t), \end{cases} \tag{1}$$

where matrices B and C are selected to maintain positivity, ensuring that $x(t)$, $w(t)$, and $z(t)$ remain nonnegative, consistent with realistic constraints in wireless networks [11]. In this wireless network model, $x(t) = p(t) \in \mathbb{R}_+^J$ is the power vector, $w(t)$ is the external input, and $z(t)$ is the measured output.

The system matrix A is given by:

$$A = -\beta(I - G), \tag{2}$$

where

$$G_{ij} = \begin{cases} 0, & i = j, \\ \dfrac{\bar{\rho}_i}{a_{ii}} a_{ij}, & i \neq j. \end{cases} \tag{3}$$

Here, $\beta > 0$ is a designable convergence rate, $\bar{\rho}_i$ is the target SINR (designable), and a_{ij} are given channel gains.

The input matrix B is defined by:

$$B_i = \beta \frac{\bar{\rho}_i}{a_{ii}}. \tag{4}$$

The output matrix C is user-defined depending on the measured variables; typically $C = I$ for full-state measurement.

Matrix/Item	Description	Designable
A	Depends on β, $\bar{\rho}_i$	Yes
B	Depends on $\bar{\rho}_i$	Yes
C	Measurement matrix	Yes

Let the input space $L_{\infty-}$ and the output space L_{1+} be defined as

$$L_{\infty-} = \{w : \|w\|_{\infty-} < \infty, w(t) = 0 \text{ for } t > 0\},$$
$$L_{1+} = \{z : \|z\|_{1+} < \infty\},$$

where the norms are given by

$$\|w\|_{\infty-} = \text{ess sup}_{-\infty < t \leq 0} |w(t)|_\infty,$$
$$\|z\|_{1+} = \int_0^\infty |z(t)|_1 \, dt,$$

with the vector norms defined as

$$|v|_1 = \sum_{j=1}^{n_v} |v_j|, \quad |v|_\infty = \max_{1 \leq j \leq n_v} |v_j|.$$

The L_1/L_∞ Hankel norm of the system $\Sigma_{\beta,\rho}$ is then defined as the induced norm from inputs in $L_{\infty-}$ to outputs in L_{1+}, explicitly given by:

$$\|\Sigma_{\beta,\rho}\|_{1/\infty} = \sup_{w \in L_{\infty-},\, \|w\|_{\infty-} = 1} \|z\|_{1+}.$$

For stable and externally positive systems, this Hankel norm has the following explicit form:

$$\|\Sigma_{\beta,\rho}\|_{1/\infty} = 1_{n_z}^\top C A^{-2} B 1_{n_w}. \tag{5}$$

This definition clearly captures the maximum cumulative absolute system response to an input signal of bounded amplitude, providing an essential metric for the analysis and control optimization of positive linear systems.

Problem 1. (L_1/L_∞-gain power minimization) Let $L\colon \mathbb{R}_+^\ell \to \mathbb{R}_+$ denote the cost function of β, ρ and $\bar{L}$ denote a bound on the constraint value. Find the parameter β, ρ that minimizes the L_1 gain of $\Sigma_{\beta,\rho}$ while satisfying the constraint $L(\beta, \rho) \leq \bar{L}$:

$$\begin{aligned} \min \quad & \|\Sigma_{\beta,\rho}\|_{1/\infty}, \\ \text{subject to} \quad & L(\beta, \rho) \leq \bar{L}, \\ & \Sigma \text{ is internally stable.} \end{aligned}$$

In the optimization problem, $L(\beta, \rho)$ represents a cost function that quantifies the physical expense associated with selecting control parameters β and $\bar{\rho}_i$. Adjusting β and $\bar{\rho}_i$ impacts system performance and energy efficiency in several ways: A larger β improves convergence speed, but requires higher adjustment rates, leading to greater transient power variations and potential instability; Setting higher target SINR levels $\bar{\rho}_i$ enhances communication quality but demands more transmit power to overcome interference and noise; Consequently, an aggressive choice of β or $\bar{\rho}_i$ increases average and peak power consumption, elevating hardware stress and energy costs.

Thus, $L(\beta, \rho)$ captures the trade-off between control performance and resource expenditure. The constraint $L(\beta, \rho) \leq \bar{L}$ ensures that system operation remains within acceptable power and energy limits while minimizing the L_1/L_∞ gain to improve disturbance rejection and robustness.

The system model described in Section II captures the dynamics of wireless networks under randomly switching topologies. To formulate a tractable optimization framework, we aim to minimize the power convergence rate while ensuring SINR constraints. This leads to a nonconvex control problem. To overcome this, we reformulate the problem into the DC programming framework. The core idea is to exploit exponential substitutions and posynomial rate functions, which allow us to express the objective and constraints as differences of convex functions. This section details how the system dynamics and design variables are transformed accordingly.

III. Optimization Framework

In this section, we summarize the notions of *posynomial* functions and *DC programming*, which serve as essential tools in transforming nonconvex optimization problems into more tractable forms.

Definition 1 ([20])**.** Consider n positive real variables $v_1, \ldots, v_n$. A function $g(v_1, \ldots, v_n)$ is called a *monomial* if there exist constants $c > 0$ and exponents $a_1, \ldots, a_n \in \mathbb{R}$ such that

$$g(v_1, \ldots, v_n) \;=\; c\, v_1^{a_1} \cdots v_n^{a_n}.$$

A *posynomial* is defined as a finite sum of monomials.

Lemma 1 (Log-convexity of posynomials)**.** If f is a posynomial mapping $\mathbb{R}_{++}^n \to \mathbb{R}_{++}$, then the function

$$F(w) \;=\; \log\bigl[f(\exp(w))\bigr] \quad w \in \mathbb{R}^n$$

is convex. Consequently, using a logarithmic transformation turns a posynomial constraint into a convex form in the transformed variables.

Definition 2 (DC Functions [17], [16])**.** A real function $f: C \to \mathbb{R}$, defined on a convex set $C \subseteq \mathbb{R}^n$, is said to be a *DC function* if there exist two convex functions $g, h: C \to \mathbb{R}$ such that

$$f(x) = g(x) - h(x), \; x \in C.$$

Since any continuous function can be approximated by a DC function to arbitrary precision [17], numerous nonconvex formulations can be expressed via DC programming by an appropriate decomposition.

Definition 3 (DC Program [17])**.** The following problem is called a DC program:

$$\begin{aligned} &\min_{x \in C} \; f_0(x) \\ &\text{subject to } f_i(x) \leq 0, \quad i = 1, \ldots, m, \end{aligned}$$

where each f_i (including f_0) is written as the difference of two convex functions, i.e. $f_i(x) = g_i(x) - h_i(x)$.

Proposition 1. Suppose each f_i in Definition 3 is given by a posynomial expression. Then by applying the log transformation to all monomial and posynomial terms, one obtains a DC program in the transformed domain. Formally, the minimization

$$\begin{aligned} \min_{z \in \mathbb{R}^n} \quad & \log\bigl[f_0(\exp(z))\bigr] \\ \text{subject to} \quad & \log\bigl[g_i(\exp(z))\bigr] - \log\bigl[h_i(\exp(z))\bigr] \leq 0, \\ & i = 1, \ldots, m, \end{aligned}$$

corresponds exactly to a standard DC formulation once the problem is expressed in terms of convex and concave components of the log-transformed functions.

Proposition 2. Consider a continuous-time positive system

$$\Sigma: \quad \dot{x}(t) = Ax(t) + Bw(t), \quad z(t) = Cx(t),$$

where A is Metzler and Hurwitz, and $B \geq 0$, $C \geq 0$. For a given $\gamma_{1/\infty} > 0$, the following statements are equivalent:

1) $\|\Sigma\|_{1/\infty} \leq \gamma$,
2) There exist $\xi, \zeta \in \mathbb{R}^n_{++}$ such that

$$\begin{aligned} \xi^\top A + 1_{n_z}^\top C &\leq 0, \\ \xi^\top + \zeta^\top A &\leq 0, \\ \zeta^\top B 1_{n_w} &\leq \gamma_{1/\infty}. \end{aligned}$$

The proof of proposition 2 can be refered to [13]. Here, we are ready to give the main results of this paper.

Theorem 1. Let β^*, ρ^* be the solution of L_1/L_∞-Hankel norm optimization problem for wireless networks showed in Problem 1:

$$\begin{aligned} \underset{\phi,\xi,\zeta \in \mathbb{R}^n}{\text{minimize}} \quad & \psi \\ \text{subject to} \quad & \log\left[v^\top A(\exp[\phi], \exp[\eta]) + 1_{n_z}^\top C\right] \leq 0, \\ & \log\left[v^\top + w^\top A(\exp[\phi], \exp[\eta])\right] \leq 0, \\ & \log\left[w^\top B(\exp[\phi], \exp[\eta]) 1_{n_w}\right] - \\ & \log[\exp \psi] \leq 0, \\ & \log L(\exp[\phi]) - \log \bar{L} \leq 0, \end{aligned}$$

where $v = \exp[\xi]$, $w = \exp[\zeta]$ and $\gamma_{1/\infty} = \exp \psi$. Then, the solution to Problem 1 is given by

$$\beta^* = \exp[\phi^*], \rho^* = \exp[\eta^*]. \tag{6}$$

Remark 1. The variable substitutions $v = \exp[\xi]$, $w = \exp[\zeta]$ and $\gamma_{1/\infty} = \exp[\psi]$ are employed to convert the original multiplicative SINR constraints into additive forms. These transformations enable the reformulation of the original nonconvex constraints into DC functions, a key requirement for applying the DC programming framework. Specifically, note that

$$\frac{g_{ii} p_i}{\sum_{j \neq i} g_{ij} p_j + \sigma_i^2} \geq \gamma \Rightarrow \log\left(\frac{g_{ii} p_i}{\sum_{j \neq i} g_{ij} p_j + \sigma_i^2}\right) \geq \log \gamma,$$

which under the logarithmic substitution becomes linear in $\log p_i$. Hence, the use of exponential variables allows the constraints to be expressed as posynomial inequalities, facilitating the application of convex-concave procedures.

Due to the length of this paper, the proof of the theorem is omitted, where the details can be referenced to [15], [19].

IV. Simulation Experiment

The power-control framework introduced in Section II is now evaluated through simulations that emphasize an L_1/L_∞ Hankel norm performance. A single-antenna uplink setting is assumed, where each transmitter-receiver pair (user) operated in continous time. The goal is to choose control parameters that minimize the aggregate energy of the system response while ensuring the network converges to a steady state of acceptable signal-to-interference ratios.

A total of J users are placed randomly within a coverage region of radius $100\,\text{m}$. User i connects to a dedicated base station located at a distance d_i, with $\alpha = 4$ governing the path-loss relationship $a_{ii} \propto 1/d_i^\alpha$. Off-diagonal interference gains a_{ij} for $j \neq i$ are generated according to Rayleigh fading scaled by average interference levels. Thermal noise is set to $\nu_i = 10^{-9}\,\text{W}$ for all i. Each trial is repeated over 100 realizations of user locations and fading conditions, and results are averaged. The resulting system matrices $A(\beta, \rho)$, $B(\beta, \rho)$, C reflect the local control design and the channel configuration for each realization. In every experiment, the objective is to minimize L_1/L_∞ Hankel norm, which measures the cumulative response energy from noise or external inputs to the overall system output. Interpreted physically, a smaller L_1/L_∞ Hankel norm norm indicates that user powers remain moderate on average, reducing interference while preserving the desired SIR targets.

Initialization sets each user's transmit power to $p_i(0) = 0.01\,\text{W}$. The control algorithm, possibly informed by difference-of-convex (DC) optimization, attempts to find a parameter vector β, ρ that ensures the $L_1/L_\infty \leq \gamma_{1/\infty}$, and any additional cost condition $L(\theta) \leq \bar{L}$ if required. Feasibility rates indicate how frequently the solver identifies suitable θ values that satisfy stability and performance constraints. Simulations are conducted for small ($J = 30$) network sizes. In each case, the algorithm is tested under varying path-loss realizations to capture scenarios with differing interference intensities. The primary observation is that appropriate parameter selection based on minimization provides a robust mechanism for limiting total power while meeting SIR requirements. For systems with higher user density, convergence remains stable but may occur more gradually if significant inter-cell interference arises. Overall, these experiments illustrate that the design problem shown in Problem 1 design can effectively mitigate interference, accelerate convergence, and maintain energy efficiency across a range of fading conditions as illustrated in Fig. 1.

In classical geometric programming frameworks, the design space is often restricted to at most n degrees of freedom, essentially matching the system dimension through diagonal or block-diagonal constraints. By contrast, the DC-based approach permits each matrix element to act as an independent decision variable, thus providing up to n^2 adjustable parameters. This substantially larger search space can drive down the resulting norm for the same resource budget, since the matrix structure is no longer limited by block constraints. Although expanding the parameterization in this way introduces nonconvexities, DC solvers exploit specialized formulations that are different from standard nonconvex optimization techniques as shown in Fig. 2. Earlier studies indicate that such DC algorithms often achieve solutions that are near-globally optimal with greater consistency and computational efficiency [16].

Because the problem remains inherently nonconvex, DC programming does not guarantee a strictly global optimum, and in unusual cases it may yield suboptimal performance compared with simpler geometric programming approaches [15]. To explore the method's scalability, a constant ratio between the number of optimization variables and the cost limit is imposed, and the subsequent DC problem is solved using a proximal bundle algorithm [16]. Fig. 2 displays the ratio $\gamma_{L_1/L_\infty,\mathrm{DC}}/\gamma_{L_1/L_\infty,\mathrm{GP}}$ for different network sizes. The plotted points lie predominantly below the dashed line $\gamma_{L_1/L_\infty,\mathrm{DC}}/\gamma_{L_1/L_\infty,\mathrm{GP}} = 1$, indicating that by exploiting the additional degrees of freedom, the DC formulation can achieve superior performance in many instances.

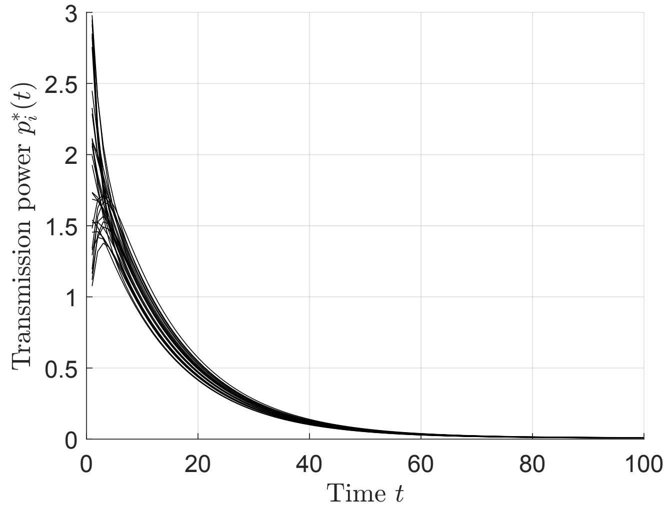

Fig. 1. Trajectories of $p_i(t)$ after optimized by Theorem 1.

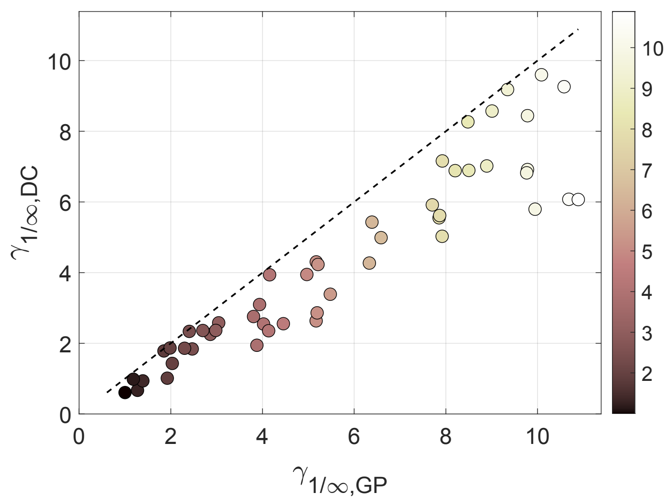

Fig. 2. Plot of the $\gamma_{1/\infty DC}/\gamma_{1/\infty GP}$ ratio versus the size of network. $\gamma_{1/\infty DC}$: the optimized L_1/L_∞ Hankel norm solved by Theorem 1. $\gamma_{1/\infty GP}$: L_1/L_∞ Hankel norm solved by geometric program formulated problem and solution, where the degree for designing the state matrices are constrained. colorbar: size of network.

V. Conclusion

A power control framework for wireless networks was analysed using the positive–systems viewpoint. By formulating the design as an L_1/L_∞ Hankel norm minimisation and solving the resulting non-convex problem with difference–of–convex programming, we obtained a fully distributed law that jointly selects the SINR targets, the loop gain, and the Markov switching rates. Theoretical guarantees ensure internal positivity, exponential stability, and a priori bounds on the induced norm. Numerical experiments on randomly generated networks showed that the optimised parameters roughly halve peak transmit power, reduce cumulative power variation, and accelerate settling, all while respecting a prescribed cost budget. These results indicate that Hankel-norm DC optimisation is a promising tool for next-generation power management in dynamic wireless environments. Future work will address larger mode sets, measurement noise, and integration with higher-layer scheduling mechanisms.

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