Advancements in Decentralized FOPID Control for TITO Systems via Reduced-Order Model-Based Design: A Case Study

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***Abstract*—The design of effective controllers for complex Two-Input-Two-Output (TITO) systems presents a significant challenge, particularly in the context of implementing a design methodology suited to this specific type of process. In the majority of cases, if the system in question exhibits highly complex dynamics or a high order, the procedure to be performed is an order reduction. This is done in order to describe the essential dynamic characteristics of the original system in a way that allows for the reliable design of controllers. This work presents a practical approach to the design of decentralized Fractional-Order Proportional-Integral-Derivative (FOPID) controllers for TITO systems. The methodology comprises a reduced-order representation of the original system and an optimization-guided design. The intention is to quantify the performance loss in the decentralized FOPID controller design procedure when reduced-order models are employed. This will facilitate a validity analysis of the fractional controller selection. To enable a fair comparison in the validation example, an equivalent procedure for a decentralized PID controller will be presented in parallel.**

***Index Terms*—Fractional Control, FOPID, Model-based Control, MIMO Processes, Optimization.**

I. Introduction

The advancement of fractional control algorithms has enabled the introduction of this concept into the industry over the past few years [1]. In this context, studies have been conducted to quantify the improvement in performance, both in terms of control and modelling of dynamic systems [2]. In fact, a considerable number of studies have concentrated on the development of diverse methodologies for the construction of Fractional-Order PID (FOPID) controllers, with a particular emphasis on both stable systems, [3], [4] and processes exhibiting more intricate dynamics [5], such as inverse response [6], or the incorporation of diverse criteria for the optimization of the control system [7].

However, one area where their application is yet to be fully explored is in Multiple-Input-Multiple-Output (MIMO) systems. In this context, the majority of studies investigating fractional control for this type of system have considered Two-Input-Two-Output (TITO) systems that employ decentralized controllers [8]. This is unsurprising, as TITO systems are the most commonly encountered among multivariable processes. Decentralized controllers based on PID control for TITO systems have been the subject of extensive study [9], [10], as they represent a highly attractive alternative due to their cost-effectiveness in economic terms, particularly in comparison to many other algorithms. It can be stated that the PID controller is often sufficient for the complexity of the process to be controlled [11].

However, the current research landscape reveals a notable gap in the investigation of the decentralized FOPID controllers applicability to TITO systems. While the design procedures for Single-Input-Single-Output (SISO) systems that exhibit more complex dynamics have been subjected to rigorous examination across a spectrum of different control algorithms [12], the same cannot be said for TITO systems. While some proposals have been put forth, they have not been specifically oriented towards the controllers design and validation [13].

This paper analyses the typical design procedure based on low-order model identification but in the case of FOPID controllers for TITO processes, using these reduced-order model design to assess the benefits of decentralized Fractional-Order PID controllers over traditional decentralized PID controllers. To assess the viability of this design approach for decentralized FOPID controllers, a case study will be conducted. This case study will present the design for a high-order TITO system, with a comparative analysis of the results obtained for the designed controllers.

II. Problem Formulation

A. Control Scheme Configuration

The control scheme is shown in Fig. 1, highlighting its two main components. The labels $C_1(s)$ and $C_2(s)$ correspond to the two decentralized FOPID controllers, while the sub-processes $P_{11}(s)$, $P_{12}(s)$, $P_{21}(s)$, and $P_{22}(s)$ represent the TITO system. The signals r_1 and r_2 indicate the two set-point of each main sub-process, whereas e_1 and e_2 denote the error signals associated with each respective input. Finally, u_1 and u_2 are the control variables, and y_1 and y_2 denote the process variables that will be managed through feedback.

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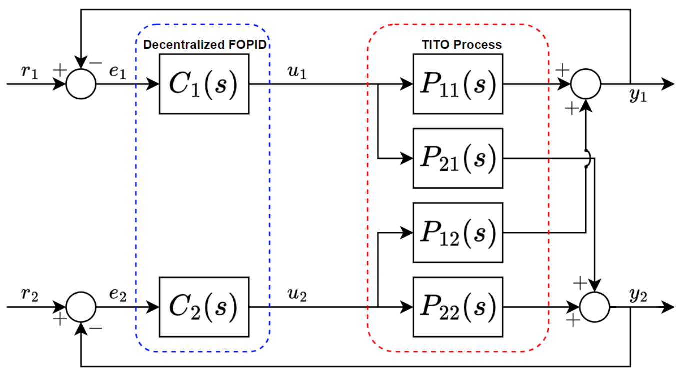

Fig. 1. TITO Decentralized Control System Diagram.

The transfer functions of the two FOPID controllers are described as follows:

$$C_j(s) = K_{p,j}\left(1 + \frac{1}{T_{i,j}s^{\lambda_j}} + \frac{T_{d,j}s^{\mu_j}}{\frac{T_{d,j}}{N^{\mu_j}}s^{\mu_j}+1}\right) \quad j = 1,2 \quad (1)$$

Where the term $K_{p,j}$ is used to denote the proportional gain, $T_{i,j}$ is the integral time constant, $T_{d,j}$ represents the derivative time constant, while λ and μ the fractional order of the integral action and of the derivative action, respectively. It should be noted that when $\lambda = 1$ and $\mu = 1$, an integer order controller is obtained, which is equivalent to a PID controller. Finally, the constant N defines the time constant for the derivative action filter. As it has been extensively studied previously [14], for PID controllers it is typical to find this value fixed at $N = 10$, for this study, we employ this fix value for all cases.

In order to implement the FOPID controller, it is necessary to employ an approximation for the non-integer terms. In this paper, the procedure is based on the Oustaloup approximation, which is a well-established technique in the field of fractional control. A high-order transfer function is employed to approximate the dynamics of the fractional operator. According to [15], the fractional term is equivalent to the following expression:

$$s^{\nu}_{[\omega_l,\omega_h]} \cong K\prod_{k=1}^{n}\frac{1+\frac{s}{\omega_{z,k}}}{1+\frac{s}{\omega_{p,k}}}, \quad \nu > 0 \quad (2)$$

In where

$$\begin{aligned}\omega_{z,1} &= \omega_l\sqrt{\eta} \\ \omega_{p,n} &= \beta\omega_{z,n} \qquad n = 1,\ldots,\bar{n}, \\ \omega_{z,n+1} &= \eta\omega_{p,n} \qquad n = 1,\ldots,\bar{n}-1, \\ \beta &= \left(\frac{\omega_h}{\omega_l}\right)^{\frac{\nu}{\bar{n}}} \qquad \eta = \left(\frac{\omega_h}{\omega_l}\right)^{\frac{1-\nu}{\bar{n}}}\end{aligned}$$

The value of K is chosen to ensure that the gain of the approximation is equal to 1 at the logarithmic midpoint of the interval under consideration, which is defined by the limits ω_l and ω_h. In this study, the frequency limits of the interval are set to $\omega_l = 0.001$ and $\omega_h = 1000$, which are appropriate values for the considered cases given that the crossover gain of the frequencies is always within the established frequency interval. Finally, to achieve a balance between the computation time of the solution and the performance of the approximation, $n = 8$ will be used.

In general, the TITO process can be represented in matrix form as follows:

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s)\end{bmatrix} \quad (3)$$

In this context, the decentralized control system consists solely of the main control loops, each formed by controllers $C_j(s)$ and sub-processes $P_{11}(s)$ and $P_{22}(s)$. However, because of the system inherent interactions, the control actions within these loops impact the opposing variables. In this way, the processes $P_{12}(s)$ and $P_{21}(s)$ characterize these interactions, which, for analysis purposes, can be treated similarly to external disturbances affecting the main control loops.

If one were to consider representing the decentralized FOPID controller in its matrix form, we have:

$$C(s) = \begin{bmatrix} C_{11}(s) & 0 \\ 0 & C_{22}(s)\end{bmatrix} \quad (4)$$

where $C_{11}(s) = C_1(s)$ and $C_{22}(s) = C_2(s)$. In this manner, the feedback control system can be more readily depicted in accordance with its corresponding equation, as the decentralized controller will be in series with the TITO process. Consequently, the open-loop transfer function matrix will be equivalent to the following equation:

$$L(s) = P(s)C(s) \quad (5)$$

Finally, the overall feedback system can be defined by:

$$T(s) = [I + L(s)]^{-1}L(s) \quad (6)$$

I being the identity matrix of 2×2.

B. Optimization Framework

As previously stated, the design approach employed in this work does not utilise decouplers. Nevertheless, it is imperative to consider the interaction between sub-processes and its impact on control actions. In order to mitigate these effects, the optimal parameters of the decentralized FOPID controller will be determined by minimizing an objective function that considers these interactions.

This strategy ensures the effective mitigation of the aforementioned effects and was studied in [16]. In particular, we employ an optimization strategy where a performance index is minimized. In this context, as with the tuning of controllers in single-input-single-output (SISO) systems, the integrated absolute error (IAE) index can be defined in terms of either the error signal or the relationship between the set-point and the process variable. For TITO systems, the approach is generalized to encompass any of the four potential responses in order to consider the cross-coupling indexes too.

$$\mathrm{IAE}_{ij} = \int_0^{\infty}|e_{ij}(t)|dt = \int_0^{\infty}|r_{ij}(t) - y_{ij}(t)|dt \quad (7)$$

Where $e_{ij}(t)$, $r_{ij}(t)$ and $y_{ij}(t)$ are the error signal, the reference and the feedback signal of the $\{i, j\}$ loops respectively, It should be noted that the issue of coupling persists when this index is employed as an objective function. Consequently, the configuration of the controller is dependent upon the degree of interaction within the TITO process, which in turn affects its performance. To address this issue, it is recommended that all four possible IAE indices be incorporated.
This should result in the creation of a unified composite index, which should be augmented with an additional parameter. This parameter would serve to weigh the importance or degree of achievable decoupling with a decentralized controller. This approach allows a more detailed evaluation of the effectiveness of the controller in managing interaction effects. Thus, the index can be defined as:

$$J_T(\alpha) = \alpha(\text{IAE}_{12} + \text{IAE}_{21}) + (1-\alpha)(\text{IAE}_{11} + \text{IAE}_{22}) \quad (8)$$

The employment of $J_T(\alpha)$ as the objective function to minimise in the optimization process facilitates the design of decentralized FOPID controllers that effectively address the coupling effects inherent in the TITO process, while also preserving optimal performance in the control task. In this context, the additional parameter α represents the balance between weighting the regulation of the coupling effects and prioritising the performance of the primary control loops.
In order to complete the development of the design proposed in this work, values within the full operational range for α ($0 \leq \alpha \leq 1$) will be considered. Note that in the case of $\alpha = 0$ no coupling effects are considered and, on the other hand, when $\alpha = 1$ the performance of the main control loops is not taken into account. Additionally, step-type inputs will be employed in the set-point for the calculation of the IAE indices, thereby enabling a comprehensive evaluation of the performance of the designed FOPID controllers in the set-point tracking task.

III. Decentralized Controller Design

In control system design, a common challenge is the uncertainty in the dynamic behavior of the plant. This uncertainty arises unless an explicit representation of the system is available, such as a mathematical model or specific parameterization. In most cases, controller design is based on an identified reduced-order model.
In this section, we review the specific design procedure under consideration. We assume a multivariable control system design framework, where the goal is to implement a decentralized fractional-order PID controller. This controller is designed using a Two-Input Two-Output (TITO) reduced-order model that represents the plant dynamics, along with an optimization procedure to determine the controller parameters.
As previously mentioned, a direct comparison of the performance of decentralized FOPID and PID controllers is presented in [8], providing an initial approach to quantifying performance improvements. However, in this case, the design focuses on a decentralized fractional control system for a

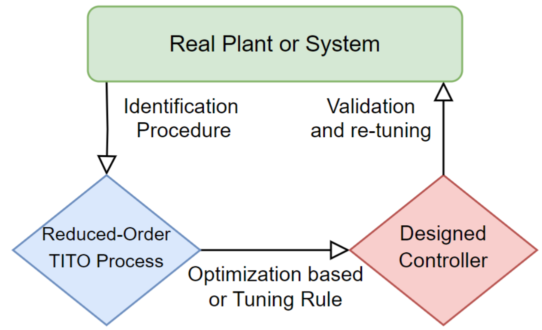

Fig. 2. Classical Controller Design Approach Adapted for TITO Processes Based on Reduced-Order Model Identification.

plant represented by a reduced-order model. Consequently, this model does not perfectly capture the actual behavior of the plant, leading to an inherent performance loss associated with model-based design or, in the worst case, system unstability once the controller is implemented.
This performance loss has not been quantified in previous studies. The present work addresses a fundamental question: under the given design conditions, *is it preferable to adopt a decentralized control system based on integer-order or fractional-order controllers, considering performance loss due to possible modeling inaccuracies.*
The diagram shown in Fig. (2) illustrates the design procedure for decentralized FOPID-PID controllers, considering the case of an identified reduced-order TITO process. The following section provides a brief explanation of each step in the proposed methodology:

- **Identification Procedure:** This stage involves simplifying a complex system to make it easier to manage. This is useful when there is no model for the plant and its behaviour can be described by low-order transfer functions. It is also useful for process models with a large number of poles and zeros that render them intractable. It can be given from identification procedures based on step-input testing or even based on relay testing [17]. Optimization-based identification methods can also be considered [18].
- **Design Stage: Optimization or Tuning Rule:** This stage pertains directly to the point at which the controllers are designed. A considerable number of the methodologies currently in use are based on model-based optimization methods, which is precisely the approach that we intend to adopt in this case. It may also be considered appropriate to apply controller tuning rules.
- **Validation and re-Tuning:** The validation stage of the designed controller is of particular significance, as it represents the first instance of the controller being directly implemented in the real plant. Prior to implementation, it is essential to consider the sensitivity of the system in terms of stability. This is typically achieved through the definition of several criteria, which serve to either confirm the implementability of the controller or, alternatively, to

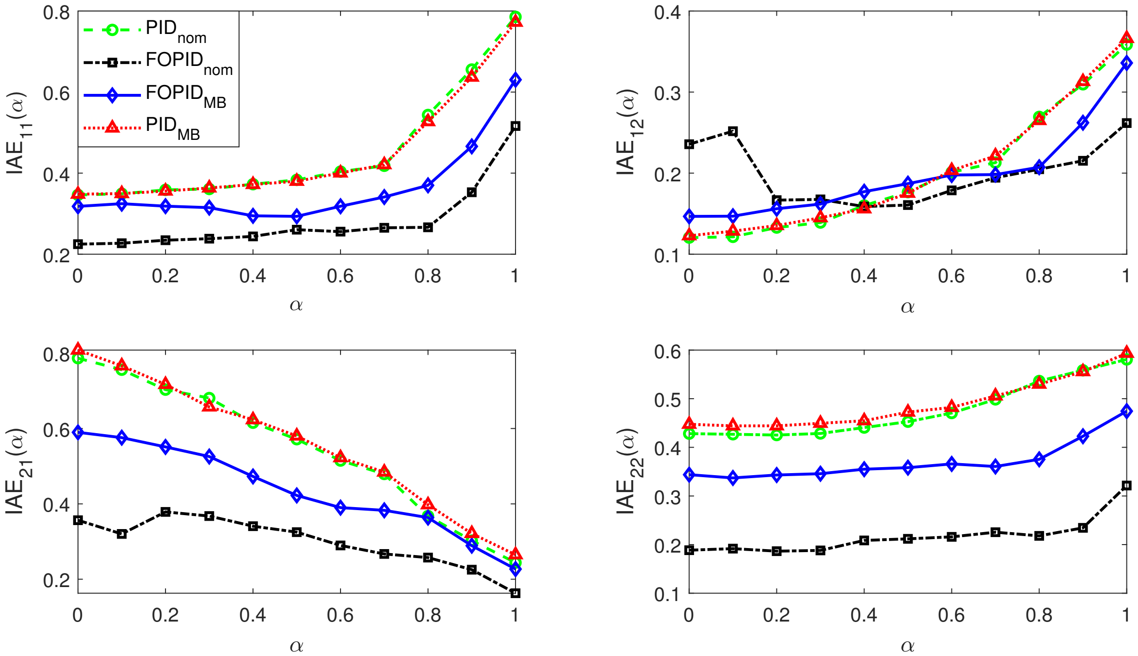

Fig. 3. Comparison of IAE Index Between the Nominal (nom) and Model-Based (MB) Controllers.

discard it or initiate a re-tuning procedure.

In the present study, a Second-Order-Plus-Dead-Time (SOPDT) model is employed, identified through optimization-based techniques. In this case, it is important to note that more analytical or simplified identification methods, such as the Half-Rule [19], [20], can be used. However, when dealing with a highly interactive multivariable process, the identified models may not accurately capture the actual system dynamics. As a result, they may not be suitable for the design of a decentralized control system, as explored in this study.

IV. Case Study: High-Order TITO Process

To numerically and graphically analyze the actual performance degradation that occurs when using a reduced-order system model for designing a decentralized FOPID controller, this study examines the case of the TITO process generated for research purposes in [21]. In this context, we refer to this process as the nominal process $P_{nom}(s)$, assuming it represents the actual plant of the system. Consequently, the controllers are designed based on a reduced-order SOPDT model for each input of the TITO process. These models were identified from the system step response, providing a basis for controller design and performance evaluation.

$$P_{nom}(s) = \begin{bmatrix} \frac{0.50}{(1+0.1s)^2(1+0.2s)^2} & \frac{-1}{(1+0.1s)(1+0.2s)^2} \\ \frac{1}{(1+0.1s)(1+0.2s)^2} & \frac{2.40}{(1+0.1s)(1+0.2s)^2(1+0.5s)} \end{bmatrix} \quad (9)$$

In this way, $P_{ro}(s)$ is presented as the identified reduced-order TITO model:

$$P_{ro}(s) = \begin{bmatrix} \frac{0.50e^{-0.1430s}}{(0.2328s+1)^2} & -\frac{1.0e^{-0.0699s}}{(0.2173s+1)(0.2170s+1)} \\ \frac{1.0e^{-0.0699s}}{(0.2172s+1)^2} & \frac{2.40e^{-0.1814s}}{(0.41273s+1)(0.41363s+1)} \end{bmatrix} \quad (10)$$

From this point onward, the previously mentioned procedure will be followed to design a decentralized fractional control system. In this context, two types of controllers will be identified for comparison:

- **Nominal Controller:** This controller is assumed to be the optimal designed under the assumption of full knowledge of the plant dynamic behavior. In our analysis, this controller will be designed based on $P_{nom}(s)$.
- **Model-Based Controller**: These controllers will be optimally designed based on the reduced-order model of the plant $P_{ro}(s)$.

This distinction allows for a systematic evaluation of performance differences between an idealized control strategy and one constrained by model reduction.

TABLE I
Nominal and Model-Based Controller Parameters for the Analized Case $\alpha = 0.70$

Parameter	PID_{nom}	PID_{MB}	$FOPID_{nom}$	$FOPID_{MB}$
K_{p1}	1.289	1.301	6.606	2.806
T_{i1}	0.492	0.501	1.211	0.908
T_{d1}	0.272	0.238	0.033	0.078
λ_1	-	-	1.080	1.047
μ_1	-	-	1.595	1.488
K_{p2}	0.656	0.662	2.661	0.866
T_{i2}	0.627	0.653	1.062	0.716
T_{d2}	0.193	0.174	0.083	0.197
λ_2	-	-	1.063	1.007
μ_2	-	-	1.470	1.051

The full range of $\alpha = [0, 1]$ was considered for the design of the controller parameters in each case, resulting in the computation of the IAE error indices for each input of the

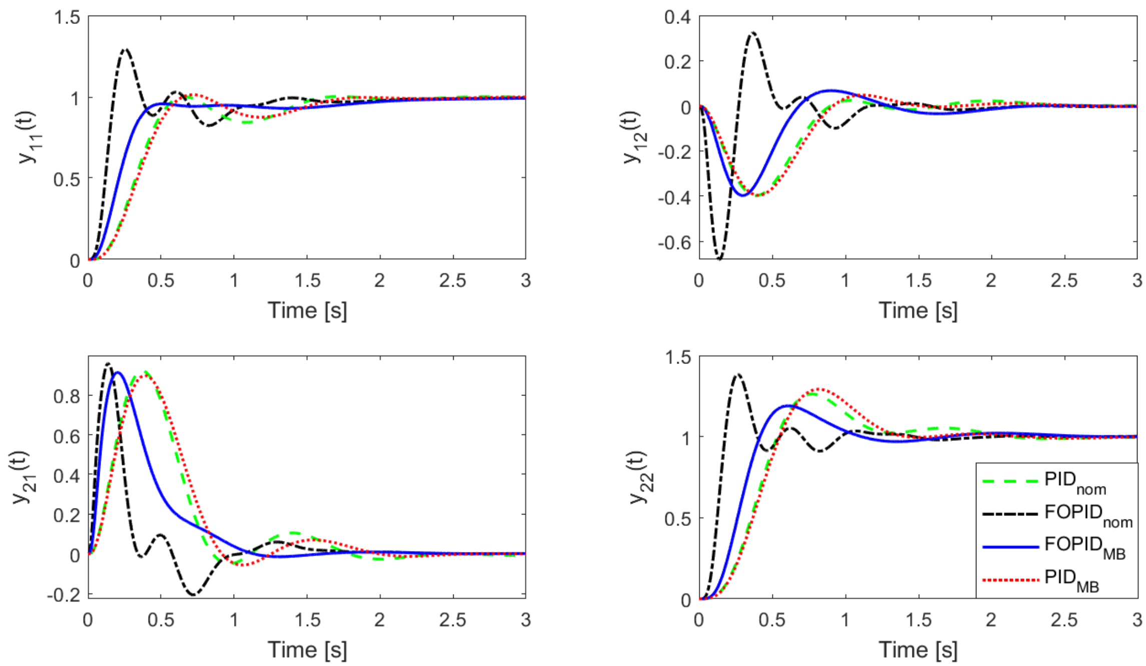

Fig. 4. System Responses for the specific $\alpha = 0.70$ case.

TITO process. As analyzed in Fig. (3), a clear difference in values can be observed between the nominal controller and the model-based controller designed using a reduced-order model. To conduct a more detailed performance analysis and quantify the intrinsic loss of the model-based methodology, a focused examination of the case $\alpha = 0.70$ is proposed. In this context, the parameters of the decentralized PID and FOPID controllers, both nominal and model-based are presented in Table I. Additionally, the error index values associated with the implementation of these controllers in the nominal process $P_{nom}(s)$ are provided in Table II. The response of the nominal process after implementing the four controllers is presented in Fig. (4), providing a graphical analysis. Finally, a direct comparison of the error indices obtained from the decentralized FOPID and PID controllers using the model-based approach is presented in Fig. (5), clearly demonstrating the significant performance improvement achieved when implementing a fractional-order controller instead of an integer-order controller in the system.

TABLE II
ERROR INDEX VALUES CORRESPONDING TO EACH CONTROLLER TYPE FOR THE ANALIZED CASE $\alpha = 0.70$

Error Index	PID_{nom}	PID_{MB}	$FOPID_{nom}$	$FOPID_{MB}$
IAE_{11}	0.418	0.421	0.265	0.341
IAE_{12}	0.213	0.221	0.194	0.198
IAE_{21}	0.479	0.484	0.267	0.382
IAE_{22}	0.499	0.506	0.225	0.360

Graphically, as shown in Fig. (4), the FOPID controller designed using the reduced-order model-based methodology achieves the best overall performance. The nominal decentralized FOPID controller exhibits significant overshoot in set-point responses and oscillations in load-disturbance regulation, compromising system robustness despite achieving the lowest error indices. Conversely, both decentralized PID controllers yield similar responses, consistent with expectations from error index analysis and the controller parameters. Furthermore, the model-based FOPID controller outperforms its integer-order counterpart, as can be seen in Fig. (5), improving performance by up to 30%, except in disturbance regulation for $\alpha < 0.50$, where the model-based PID controller is superior, as indicated by the IAE_{12} index.

V. CONCLUSIONS AND FUTURE WORK

When implementing reduced-order models of the system for controller design, a certain degree of performance loss is inevitable. In the case of decentralized FOPID controllers for TITO process control, the observed performance degradation due to model reduction is lower than experienced by an PID controller. Additionally, a PID controller designed using a model-based approach closely approximates the performance of the optimal PID for the nominal system, suggesting that, in some applications, a model-based decentralized PID controller may achieve performance closer to optimal than an FOPID controller designed using the same methodology. Despite this, the fractional-order control approach demonstrates a significantly higher percentage of improvement for TITO processes with complex dynamics. However, it is important to acknowledge that our analysis is based on a single example, which limits the generalizability of our conclusions. Future work should extend this investigation to a broader set of high-order or more complex TITO systems to assess the consistency

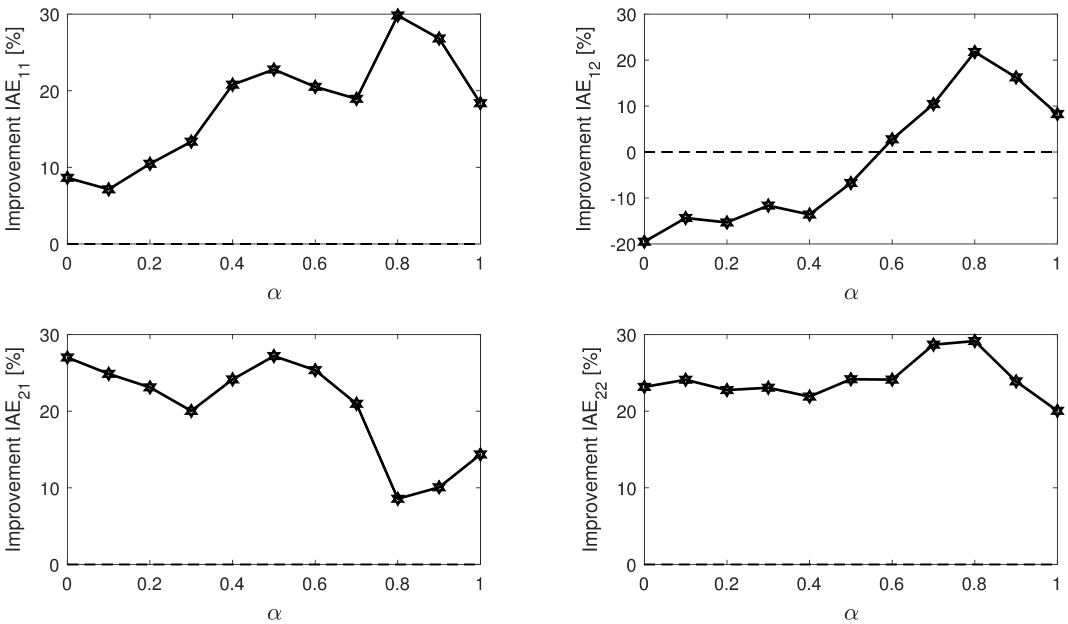

Fig. 5. Percentage Improvement of the Model-Based Decentralized FOPID Controller Over the Model-Based PID.

of the observed trends and further validate the advantages of fractional-order control in diverse scenarios.

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