# Digital Twin of a Two-Tank System: A Bond Graph Modeling Approach

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***Abstract*—Digital Twin (DT) technology has recently gained significant attention in both industry and academia. By improving efficiency, reducing costs, and enabling predictive maintenance, DT plays a crucial role in control, diagnostics, prediction, and optimization. However, at the definition level, the rapid growth of research has led to a variety of definitions and inconsistent terminology, underscoring the need for a clearer and more unified understanding of the DT concept. At the design stage, two major challenges remain. The first is to develop models that are both accurate and dynamic, capable of supporting optimization and real-time supervision, including control and diagnosis. The second lies in ensuring a reliable link to the physical system through online identification, which is particularly challenging due to the non-stationary nature of real-world processes. This study addresses the definition level and the first challenge at the design level by offering a comprehensive overview of the DT concept, including its definition, development phases, and an illustration of its design carried out in a two-tank system. The research consists of using the Bond Graph (BG) tool as a multi-physical graphical tool for designing a DT not only for dynamic modeling but also for online supervision, including diagnosis and recovery decisions. The results first show the effectiveness of BG in constructing the DT using 20Sim software for dynamic modeling and fault detection and isolation. This BG tool enables DT to monitor the state of the system and provides the health indicators necessary to reconfigure the system controller.**

***Index Terms*—Digital Twin, Digital Shadow, Bond Graph, multi-physical modeling, two-tank system, real-time monitoring, fault detection.**

## I. Introduction

Rapid advancement of information and communication technologies has led industries to embrace digitalization, automation, and smart solutions, which form the backbone of Industry 4.0 [1]. In this context, Digital Twins (DTs) [2] have emerged as powerful tools that bridge the physical and digital worlds by creating virtual replicas of real-world systems. These replicas are enriched with real-time data integration, enabling real-time monitoring, predictive maintenance, and performance optimization [3]. Although the concept of DTs is relatively new, it has its origins in different simulation and modeling disciplines. The idea was first introduced by Grieves in 2002 in the aerospace sector under Product Life-Cycle Management (PLM) [3], evolving from the "Mirrored Spaces Model" (2005) to the "Information Mirroring Model" (2006). The term 'digital twin' was officially introduced in NASA's 2010 technological roadmap, but similar concepts were employed during the 1970 Apollo program, where virtual simulations replicated spacecraft systems for maintenance purposes [3]. As technology evolved, the idea of DTs became more refined and expanded across industries, improving efficiency, reducing costs, and supporting predictive maintenance [4].

The growing importance of DTs in applications like control, diagnostics, prediction, and optimization has led to a surge in publications, resulting in a variety of definitions and inconsistent terminology across disciplines. Initially, in its first implementation by NASA, DT was defined by (Glaessgen et al.2010) [5] as " an integrated multi-physics, multiscale simulation of a vehicle or system that uses the best available physical models, sensor updates, history data, etc., to mirror the life of its corresponding flying twin". Over time, the concept has evolved to meet specific needs in research, engineering, and industry. In industrial contexts, (Garetti et al.2012) [4] described DTs as "a virtual representation of a production system that synchronizes with the real system through sensed data, smart devices, and real-time data processing". In academia, (Gabor et al.2016) [1] defined DT as " a special simulation, built based on expert knowledge and real data collected from the existing system, to realize a more accurate prediction and simulation on different scales of time and space". The lack of a unified definition complicates the understanding and distinction of DTs from related concepts, such as digital models (DMs) and digital shadows (DSs), which are distinguished by their data integration mechanisms [3], [4]. DMs are traditional computer simulations that are static in nature and depend on manual updates, while DSs enable a one-way automatic flow of data from the physical to the digital system. In contrast, true DTs offer real-time two-way interaction between the physical and digital worlds, optimizing performance throughout a product's lifecycle. Other related concepts [2], such as "product avatar" and "digital thread," are sometimes confused with DT. The "product avatar" is a static virtual replica that does not have the intelligence to influence its physical counterpart, while the "digital thread" focuses on data management and record keeping across a product's lifecycle rather than allowing real-time interaction.

The creation of a DT relies on accurately modeling the physical system—a crucial step in developing a virtual replica. The literature identifies three main modeling approaches [6]: physics-based, data-driven, and hybrid. Each offers specific features, advantages, and limitations that must be aligned with the application's objectives.

Physics-based models, grounded in first principles (e.g., physical and chemical laws), offer high accuracy by providing a theoretical representation of the system. These models are particularly well-suited for applications requiring precision—such as aerospace, manufacturing, and energy—where they yield optimal results in design, simulation, and analysis [7]. Common modeling techniques include the finite element method (FEM), computer-aided design (CAD), computational fluid dynamics (CFD), bond graphs, and analytical formulations [8], [9]. In contrast, data-driven models rely on historical or empirical data and use machine learning or statistical methods to build predictive models [10], [11]. These approaches offer greater adaptability and computational efficiency, making them ideal for applications in dynamic environments. However, they often struggle with issues such as data quality and quantity, limited physical interpretability, and reduced generalizability. Hybrid models combine the strengths of physics-based and data-driven approaches, offering accurate real-time simulations that adapt to complex and evolving conditions. These models are increasingly used in prognostics and health management, fault detection, and system optimization [12], thanks to their ability to balance accuracy, flexibility, and robust performance. However, their implementation remains challenging due to the need for precise integration and considerable effort to align data with the models.

The current state of the art shows a strong reliance on data-driven methods for digital twin design. Although these techniques provide adaptability and computational efficiency, they often face challenges due to limited data and poor physical interpretability, as the system is treated like a black box. In addition, they are usually used for either optimization or control, but rarely for both [1], [10], [11]. Among the various modeling strategies, the Bond Graph (BG) approach stands out for its ability to overcome many of these limitations, while offering a robust alternative to traditional physics-based methods. Based on first principles, BG models ensure physically consistent representations of the system under study [9]. Unlike analytical models, which can be computationally intensive and domain-specific, the BG methodology provides a unified, multi-domain framework focused on energy exchange [9], [13]. A key advantage of BG is its ability to automatically generate system dynamics, which is particularly useful for real-time parameter identification and state estimation. Moreover, its inherent causal structure facilitates the direct integration of supervisory functions such as fault diagnosis and closed-loop control [13]. Thanks to its graphical structural and causal properties, BG clearly reveals the system topology and allows diagnosis algorithms to be directly derived from the model. Moreover, the model's accuracy and robustness can be improved by simply adding new BG elements without rewriting equations. This makes BG particularly suitable for DTs, which require accurate, dynamic, and consistent modeling across multiple physical domains for real-time monitoring and supervision. However, BG has some limitations, especially in modeling systems with delays and distributed parameters, typically represented by delay or partial differential equations.

This paper aims to clarify the concept of the DT and support its design methodology by providing a comprehensive overview. It first defines DT and explains the design process, which involves creating a virtual model and updating it with sensor data to accurately reflect the physical system's state (building the digital shadow). This enables data analysis and decision-making to optimize performance and adjust the physical system, ensuring a two-way connection—a key feature of DT. Finally, it illustrates the potential of DT by presenting its development for a two-tank system using the Bond Graph (BG), a multiphysical graphical method that enables accurate dynamic modeling and integration of supervisory functions.

## II. Methodology

### A. DT Definition

Based on the literature [2], [5], a digital twin framework typically consists of three main parts, which are physical entities in the "Physical Space", the virtual models in "Virtual Space", and the connected data that links both spaces. As illustrated in "Fig. 1", the "Physical Space" represents tangible entities, corresponding to real-world products or systems that users interact with and operate. The link between the physical and virtual worlds is supported by data flows recorded by sensors that operate as the sensory organs of the digital twin, continually monitoring and collecting precise information about the physical entity. This data flows into the virtual models within the "Virtual Space," where the digital twin resides. These digital models, which are continually updated with real-time data, create a high-fidelity virtual replica of the physical asset, including its properties, conditions, and behaviors. This virtual replica's data analytics insights, such as forecasts, performance analysis, and optimization strategies, are then transmitted back to the physical space to control the corresponding physical entities, establishing a two-way data exchange. This continuous loop with bidirectional data flow is what makes digital twins (DTs) so powerful. Indeed, by providing real-time monitoring, DTs assist companies in making better decisions, lowering costs, and improving overall efficiency. Users interact with the DT through dedicated platforms or dashboards equipped with communication interfaces such as displays. These interfaces provide complete overviews of the physical system, allowing for particular applications such as fault detection, diagnostics, and predictive maintenance.

Based on this framework, a DT can be defined as follows: "A digital twin is a virtual replica of a physical asset, system, or process, designed to mirror its real-world counterpart in real time. This digital counterpart captures data from sensors and other sources to simulate physical system state with high accuracy, enabling advanced analytics and actionable insights".

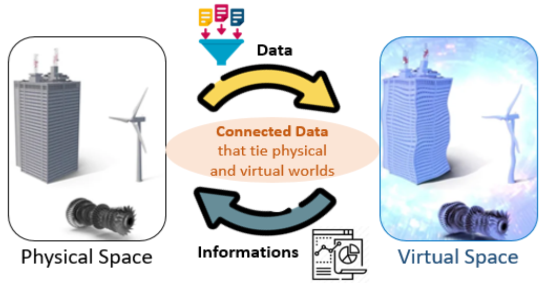



Fig. 1: Key Elements of a Digital Twinning Framework

### B. Development Phases of Digital Twin

As illustrated in "Fig. 2", a DT is developed in three phases: the Digital Model (DM), the Digital Shadow (DS), and the Digital Twin itself. Each step denotes an increase in complexity, interaction, and real-time synchronization with the physical system [4], [14]. Each step is described below.

*1)* ***Digital Model****:* The DM is the fundamental step in which a virtual representation of the physical system is built [15]. This phase focuses on physical system modeling via physics-based, data-driven, or hybrid techniques [6]. Digital models can take various forms, such as geometric models, system behavior models, or data-driven models. Key requirements at this stage include accuracy for reliable forecasting and computational efficiency for real-time simulations.
Although effective for simulating system behavior, this model does not account for unforeseen events or real-time data, resulting in limited performance for diagnostic and prognostic algorithms. Integrating real-time data into the DM marks the transition to the DS phase.

*2)* ***Digital Shadow****:* This phase links the virtual model to the physical system through real-time data, allowing the model to update itself based on sensor data and provide estimates without actively influencing the physical system.
This virtual replica must be reconfigured to reflect the actual, evolving state of the physical system. To support this, parameter identification methods under different operating conditions can be applied. However, as noted in [16], traditional identification techniques are limited to static parameters and struggle to capture dynamic changes, especially during unexpected events or system degradation. In order to represent these evolutions, a mathematical update of the virtual model is required, integrating dynamic and adaptive elements for a better understanding and prediction of system behavior. In addition, online identification remains a crucial challenge to guarantee a reliable connection between physical and virtual spaces. In this context, artificial intelligence (AI) techniques can be used to support adaptive and responsive system monitoring, as in the example of [17].
During this phase, it is essential to ensure that the model accurately represents the physical system to provide precise estimates. These estimates act as inputs for the diagnostic and prognostic layers, which are key components of the DS. The information provided by these layers is then used by the optimization and control layers, which are part of the Digital Twin (DT), the final phase of the process. The DS evolves into DT when it gains the ability to actively influence the physical system through feedback loops and control actions.

*3)* ***Digital Twin****:* The DT is the final phase, where the virtual model becomes a fully interactive and real-time representation of the physical system. Unlike the Digital Shadow, it not only updates itself using sensor data but also controls and optimizes the physical system through bidirectional interaction. This phase aims to evaluate and adjust virtual models to maintain consistency with the evolving physical system. By closing the loop between the digital and physical worlds, the Digital Twin significantly improves decision-making, operational performance, and system adaptability, making it a crucial tool for industries requiring real-time monitoring and optimization.

### C. Bond Graph (BG) theory

BG theory for diagnosis and modeling can be found in [13], where the basic elements are introduced. A BG is a unified graphical language used to model multi-physical systems. It is represented as a graph G(S, A), where the vertices S correspond to physical components, subsystems, and basic elements called junctions. The edges A, known as power bonds, represent the instantaneous power exchanged between nodes. As shown in Fig. 3(a), the power exchanged (represented by a bond) between two systems, $S_1$ and $S_2$, is the product of two conjugated power variables: an intensive variable, called effort $e(t)$ (e.g., voltage, pressure), and the derivative of an extensive variable, referred to as flow $f(t)$ (e.g., current, volume flow). The global bond graph is obtained by connecting different interacting subsystems. Since its invention in 1961, BG modeling has proven to be a powerful tool for dynamic modeling (specific software tools exist for automatic generation of dynamic models). One key structural property of the BG is its causality concept, which is crucial for control and diagnosis analysis.

Causal analysis, which determines the causes and effects within the system, is directly deduced from the graphical representation. It is indicated by the position of the cross-stroke on the bond, as shown in Fig. 3(a). By convention, the causal stroke is placed near (respectively, far from) the bond graph element $(S_i)$ for which the effort (respectively, flow) is known. For example, in Fig. 3(a), the assigned causality means that the system $S_1$ imposes efforts on $S_2$, as represented in the corresponding block diagram in Fig. 3(b). Regardless of causality, the direction of positive power is indicated by the half-arrow on the bond. The BG tool uses a set of components S={R, C, I, $S_e$, $S_f$, $D_e$, $D_f$, TF, GY, J}, where each element has a specific role. The R-element represents a passive energy dissipation element, while C and I model the passive energy storage elements. The elements $(S_e)$ and $(S_f)$ are the sources of effort and flow, respectively. Sensors are represented by flow $(D_f)$, and effort $(D_e)$ detectors. $TF$ $(\Phi_{TF}(\{e_1, e_2\}, \{f_2, f_1\}) = 0)$, and $GY$ $(\Phi_{GY}(\{e_1, f_2\}), \{e_2, f_1\}) = 0$ are the transformers and gyrators, respectively, used to represent transformations

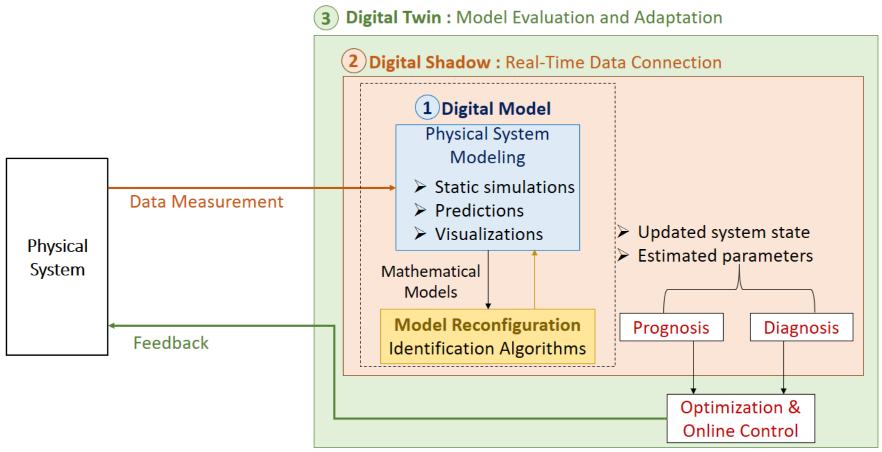



Fig. 2: Development Phases of Digital Twin

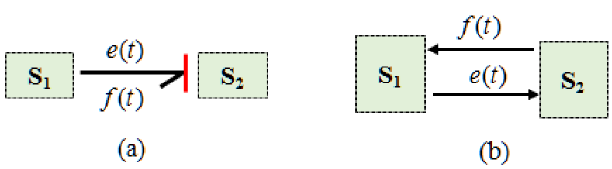



Fig. 3: Bond graph representation (a) and its corresponding block diagram (b)

between different system domains. Finally, $J$ is a junction (which can be a zero or a one junction) that connects elements having the same effort (0-junction) or flow (1-junction). These junctions give rise to conservative energy laws. Passive elements are described by generic constitutive equations. R-elements (electrical resistors, mechanical dampers, dots) are modeled using algebraic equations, expressed as $\Phi_R(e, f) = 0$. C-elements (electrical capacitors, spring stiffness, etc. etc.dots) are quantified by the following integral equation $\Phi_C(e(t), \int f(t)dt) = 0$. Also, I-elements (mechanical inertia, electrical inductance,...), are described by an integral equation in the following form $\Phi_I(f(t), \int e(t)dt) = 0$ . Based on the structural and graphical properties of BG in derivative causality, which helps avoid initial conditions, fault indicators known as analytical redundancy relations can be derived. Algorithms and specific software can be consulted in [18].

## III. Illustration: DT development process

This part illustrates the procedure of DT development. The DT illustrated here is for a pedagogical system, specifically a hydraulic two-tank system shown in Fig. 4. The role of this DT is to monitor the two-tank system. It provides fault indicators replicating the health condition of the system after processing the measurement data provided by the sensors. To illustrate feedback communication, the communication path begins with the DT and concludes at the two-tank system. Fault-tolerant control (FTC) [19] is employed, which consists of a reconfiguration control layer that uses the fault indicators to reconfigure the water level controllers of both tanks.

The DM, which is the heart of the DT, is developed using the Bond Graph (BG) modeling tool. This BG tool is very powerful in modeling multi-physical systems. Its derivative causality property allows digitalizing the model and using it for system monitoring.

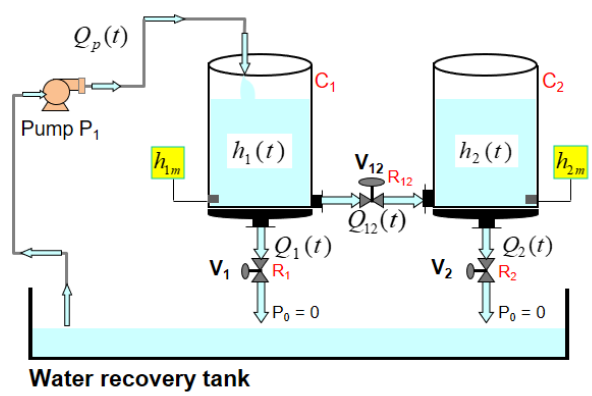



Fig. 4: Two-tank system configuration

*1) BG model of a two-tank system:* As shown in Fig. 4,the system example comprises two tanks having the capacitance $C_1$ and $C_2$. Valves $V_1$ and $V_2$ with resistances $R_1$ and $R_2$ emulate the water leakage from tank 1 and tank 2, respectively. The valve $V_{12}$ with resistance $R_{12}$ connects tank 1 to tank 2, enabling water exchange between them. The bond graph (BG) model for the 2-tank system generated under derivative causality is illustrated in Fig. 5. In this model, the pump $P_1$ that supplies water to the tanks is represented as a source of flow denoted by $S_f$. Two sensors are installed in both tanks to monitor the water levels $h_1$ and $h_2$.

*2) System model integration into Digital Twin::* In the case of the DT, where derivative causality is employed, the sensors are dualized to operate as effort signal sources, represented in the BG model by modulated effort sources$MS_e$. These dual sensors establish the primary communication link between the physical system and the digital model, allowing the DT to continuously update with measurement data and maintain

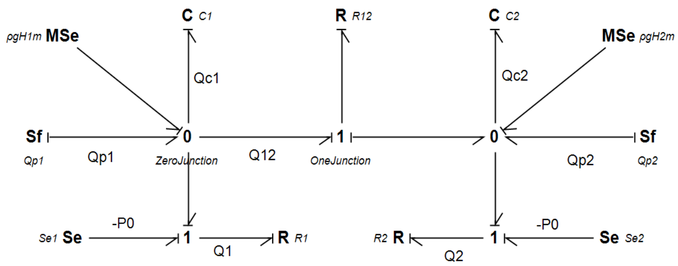



Fig. 5: Bond Graph (BG) Model of the Two-Tank System developed under Derivative Causality

synchronization with its physical counterpart. Additionally, $P_0$ can be neglected ($P_0 = 0$) as long as the water flows into a recovery tank.

The dual sensors provide the online diagnoser with relevant data about the measured variables, specifically the measured effort, to determine flows at a "0" junction where the dual sensor is attached, as illustrated in Fig. 6. This helps calculate the flow magnitude of fault impact $f_d$ generated due to fault occurrence. The calculation of flow and effort variables is achieved through a sequence of cause-and-effect relationships between different system components known as the causal path, following the causality stroke.

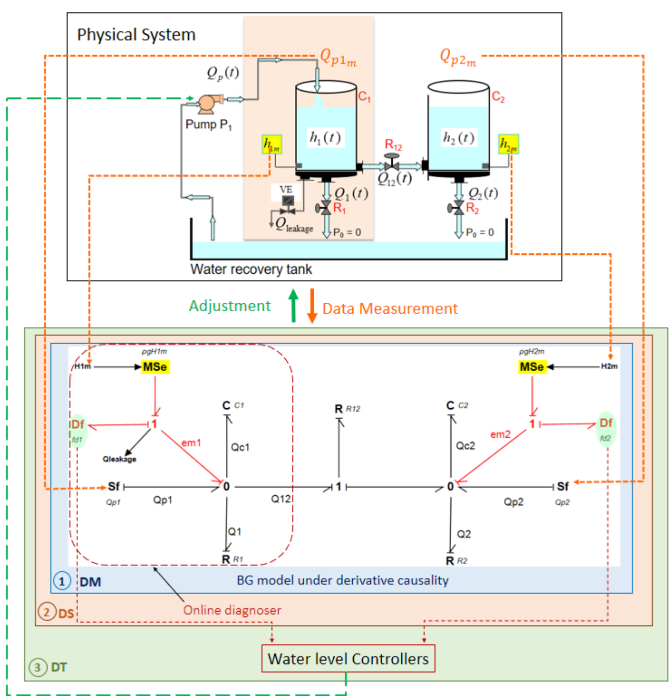



Fig. 6: Development Phases of a Digital Twin for a Two-Tank System Using the Bond Graph Approach

Under normal conditions (no fault or attack), the energy balance in junctions "0" and "1" is described by the following equations $J_0 : \sum_{i=1}^{n} a_i f_i = 0$; $J_1 : \sum_{i=1}^{n} a_i e_i = 0$. For a "0" junction, the energy balance is determined by the sum of the flows of power bonds entering the junction minus the sum of the flows of power bonds exiting it. Similarly, for a "1" junction, the energy balance is calculated by the sum of the efforts of power bonds entering the junction minus the sum of the efforts of power bonds leaving it. Here, $f_i$ represents the flow and $e_i$ represents the effort of the $i^{th}$ power bond. The coefficient $a_i$ takes the value $+1$ if the power bond enters the junction and $-1$ if it exits. The total number of power bonds connected to the junction is represented by $n$.

During a fault or attack, an additional flow is added in the case of junction "0", detected by the flow detector ($D_f : f_d$ ) connected to junction "1" (Fig. 6). $f_d$ is calculated based on the unsatisfied energy balance equation of junction "0" and is expressed in equation (1). Here, $f_{ei}$ represents the flow of the $i^{th}$ incoming power bond, $f_{sj}$ is the flow of the $j^{th}$ outgoing power bond, $n$is the number of incoming bonds, and $m$ is the number of outgoing bonds.

$$f_d = \sum_{i=1}^{n} f_{ei} - \sum_{j=1}^{m} f_{sj} \tag{1}$$

As shown in Fig. 6, the fault indicators $f_d$ are used by water level controllers, which adjust their parameters through a reconfiguration process. This helps mitigate the impact of faults on the system and ensures performance in degraded mode.

*3) Simulation setup and scenarios:* The simulation is conducted in the 20-sim software [20], as shown in Fig. 7. It presents the different parts of the digital twin with bidirectional communication and shows the digital twin's role in the supervision and fault-tolerant control (FTC) of the 2-tank system. To accurately configure the simulation environment, physical parameters, reflecting the system's geometric and hydraulic characteristics, were defined. Table I summarizes the values of these parameters used in the proposed system. One scenario is carried out, presenting a water leakage introduced in tank 1. The profile of the leakage fault is shown in Fig. 8.

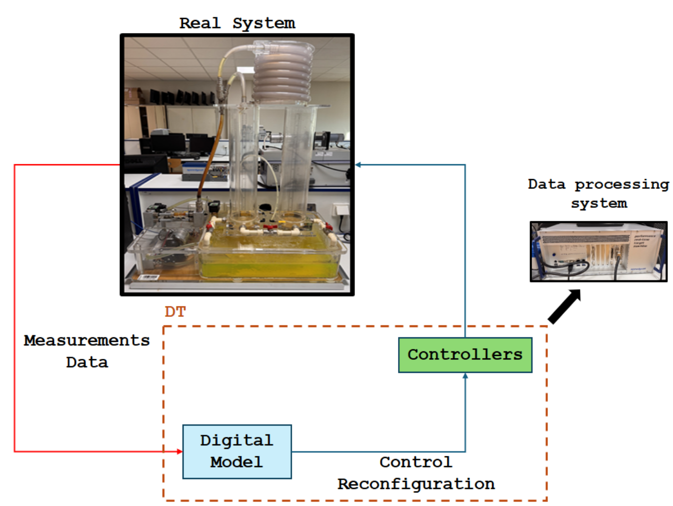



Fig. 7: Simulation conducted in 20sim software

TABLE I: Physical Parameters Used in the Two-Tank System Simulation

| Name | Value | Unit | Description |
|---|---|---|---|
| $A_1$, $A_2$ | 0.007625 | $m^2$ | Surface area of tanks 1 and 2 |
| $Q_{\max}$ | $1 \times 10^3$ | L/h | Maximum allowed flow rate |
| $CV_1$ | $2.499 \times 10^{-8}$ | $m^3/s/Pa^{0.5}$ | Discharge coefficient of tank 1 |
| $CV_2$ | $1.9 \times 10^{-8}$ | $m^3/s/Pa^{0.5}$ | Discharge coefficient of tank 2 |
| $CV_{12}$ | $1.4286 \times 10^{-8}$ | $m^3/s/Pa^{0.5}$ | Discharge coefficient between tanks |
| $\rho$ | 1000 | $kg/m^3$ | Density of water |
| $g$ | 9.809 | $m/s^2$ | Gravitational acceleration |

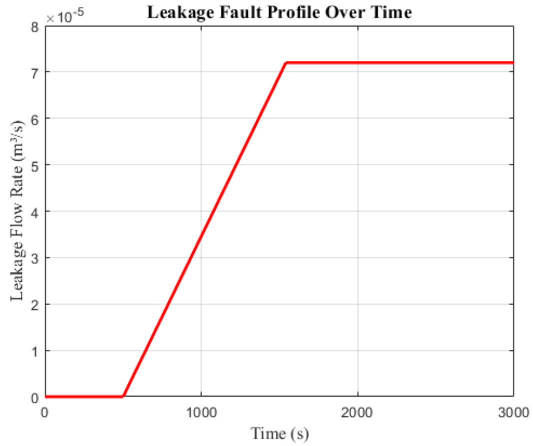
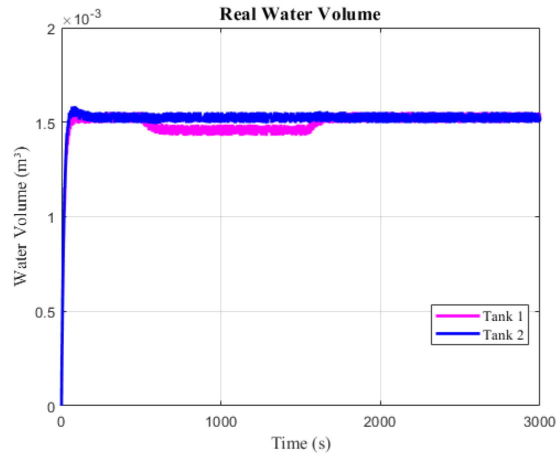



Fig. 8: Leakage Fault Profile: Ramp Growth and Stabilization

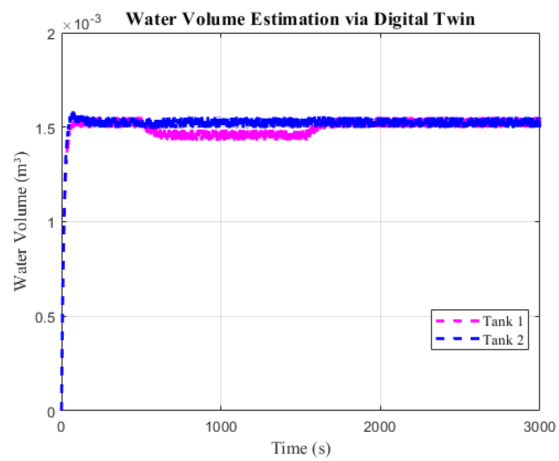
*4) Results and Discussion:* Fig.9 presents the simulation results, illustrating the system's behavior and the digital twin's performance. The first two plots (Fig. 9a and Fig. 9b) compare the estimated water volume in the tanks with the measured levels. The nearly identical curves confirm that the digital twin accurately replicates the real system's behavior, ensuring reliable state estimation. Fig. 9a shows the water levels in both tanks, reflecting system operation. Initially, the stable water levels indicate proper functioning. At 500 seconds, a leakage fault (see Fig. 8) is introduced in tank 1, impacting the water level, as shown by the deviation illustrated in Fig. 9a. Fig. 9b further illustrates this deviation, demonstrating the digital twin's ability to accurately replicate the real system's behavior, enabling effective monitoring and reliable state estimation. According to Fig. 9c, it is noted that DT is capable of estimating the fault magnitude, which is the leakage. The magnitude of the fault increases rapidly until it reaches saturation, starting from 1500 seconds. The FTC receives information regarding this leakage and reconfigures the parameters of the water level controller to mitigate the impact of the fault. Initially, the FTC is deactivated and is activated at 1500 seconds. Figures ( 9a and 9b) clearly show the fault mitigation due to the activation of the FTC. The FTC accomplishes its mission thanks to the crucial health condition information provided by DT to the reconfiguration process of the water level controllers.

## IV. Conclusion and future works

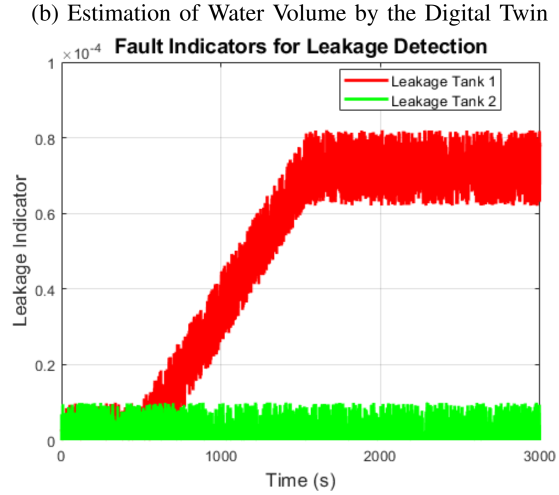
This study offers a comprehensive overview of DT technology, emphasizing its importance in system control, diagnostics,




(a) Real Water Volume in Tanks




(b) Estimation of Water Volume by the Digital Twin




(c) Fault Indicators for Leakage Detection

Fig. 9: Digital Twin Monitoring and Fault Detection Results

prediction, and optimization. By clarifying inconsistencies in DT definitions and distinguishing it from the concepts like digital shadow, digital model, and digital thread, the paper contributes to a clearer understanding of DT development. A DT design has been illustrated using a two-tank system. The BG modeling tool was employed to develop the system's digital twin, while 20sim software provided simulation results to reinforce this illustration. The simulation results confirmed the DT's ability to accurately estimate system state and detect fault, enabling control reconfiguration for ensuring the acceptable performance in the presence of the fault. These results reinforce the potential of DTs for real-time monitoring and fault detection, making them valuable tools for various engineering applications. Future work could explore extending this approach to more complex systems, integrating machine learning for enhanced predictive capabilities, and improving real-time decision-making processes.

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