Analysis of Sensitivity Function-Based Robustness Constraints in Decentralized PID Controller Design for TITO Systems

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***Abstract*—In the design of Multiple-Input-Multiple-Output (MIMO) control systems, one of the most critical considerations is system stability. This is typically addressed in the controller design process through the incorporation of a stability index, often based on frequency domain analysis, to quantify system robustness after controller implementation. This work focuses on analyzing the design of Decentralized Proportional-Integral-Derivative (PID) controllers for Two-Input-Two-Output (TITO) processes, which are among the most common multivariable systems. The proposed approach evaluates stability margins using the maximum of the sensitivity function, despite robustness also being measurable through the maximum of the singular values. By utilizing the equivalent sensitivity function for multivariable systems, the method provides a direct means of assessing robustness once decentralized controllers are implemented. The feasibility of this approach is examined for a specific level of robustness and is validated employing two benchmark processes.**

***Index Terms*—Performance Analysis, FOPID Tuning, Fractional Control, Integrating Model, Optimal Control.**

I. Introduction

In the field of multivariable control, Two-Input-Two-Output (TITO) systems represent a widely used modeling framework across various industrial sectors [1]. These systems are capable of capturing complex dynamics and varying levels of interaction between variables [2], making them a versatile basis for controller design. As a result, they are frequently used in the development of control strategies for industrial processes.

Numerous methodologies have been proposed in the literature for the identification of industrial TITO systems. Techniques based on relay feedback experiments, as presented in [3], [4], and approaches incorporating time-delay modeling [5], have proven effective for capturing system dynamics. In parallel, the control of TITO systems has long been a subject of research. As system dimensionality increases, so does the complexity of the associated control algorithms. Decoupling-based techniques have been explored to simplify control design [6], [7]. Nonetheless, decentralized PID controllers remain one of the most widely applied strategies in industry due to their simplicity, scalability, and cost-effectiveness [8], [9].

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This popularity has led to the development of various tuning methodologies tailored to decentralized PID controllers [8], [10]. Additionally, extensions into fractional-order control theory have shown promise in further improving performance [11], [12]. However, robustness in the context of decentralized PID control has not been as thoroughly investigated. Although several methods have been proposed to evaluate or improve robustness in TITO systems—such as multi-objective optimization frameworks [13], [14], singular value-based robustness estimation [15], [16], and frequency response analysis techniques [17], most of these focus on general system assessment rather than informing the tuning process of decentralized PID controllers.

In particular, despite the recognized importance of robustness metrics, the direct integration of the maximum of the sensitivity function as a design constraint in decentralized PID tuning has not been explicitly addressed. This study aims to address that gap by investigating the use of this criterion to guide the design process and enhance robustness in TITO control systems.

To this end, the present work proposes a constrained optimization-based design methodology for decentralized PID controllers, extending principles from robust SISO design to multivariable systems. The approach is validated on standard benchmark processes, enabling a detailed analysis of performance-robustness trade-offs. The results contribute to the understanding of robustness-constrained decentralized PID tuning and suggest potential paths for advancing the design of decentralized controllers in complex multivariable settings.

II. Problem about Performance Degradation

A. Control Scheme Configuration

The control scheme under consideration is illustrated in Fig. 1, which depicts the two principal blocks that comprise it. The notation $C_1(s)$ and $C_2(s)$ represents the two decentralized PID controllers, while the sub-processes $P_{11}(s)$, $P_{12}(s)$, $P_{21}(s)$ and $P_{22}(s)$ constitute the TITO process. Then, r_1 and r_2 represent the two set-point signals, e_1 and e_2 refer to the error signal associated with each input and finally, u_1 and u_2 denote the two control variables, while y_1 and y_2 represent the two process variables that are feedback.

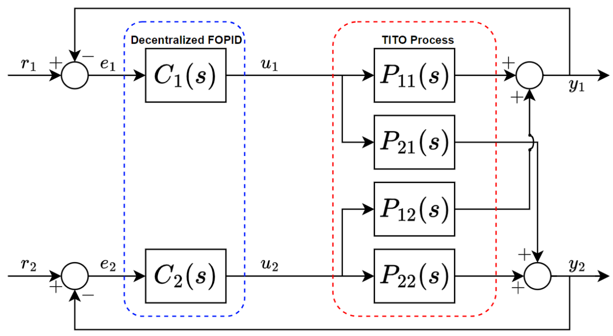

Fig. 1. TITO Decentralized Control System Diagram.

The transfer functions associated with the two PID controllers are defined as:

$$C_j(s) = K_{p,j}\left(1 + \frac{1}{T_{i,j}s} + \frac{T_{d,j}s}{\frac{T_{d,j}}{N}s + 1}\right) \quad j = 1, 2 \tag{1}$$

where $K_{p,j}$ is the proportional gain, $T_{i,j}$ denotes the integral time constant, and $T_{d,j}$ denotes the derivative time constant. The parameter N defines the filter time constant for the derivative action. As previously studied for PID controllers [18], it is typical for this value to be set to 10. Therefore, $N = 10$ will be considered from this point forward.

In general, the TITO process can be represented in matrix form as follows:

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \tag{2}$$

In this context, the decentralized control system features only the main control loops, which are composed of the controllers $C_j(s)$ and the sub-processes $P_{11}(s)$ and $P_{22}(s)$. However, due to the intrinsic interaction of the system, the control action in these loops will also affect the opposite variable. Consequently, the processes $P_{12}(s)$ and $P_{21}(s)$ describe these effects, which, for analytical purposes, can be considered in a similar way to an external disturbance to the control loops.

B. Optimization Framework

As previously stated, the design procedure of this work does not employ decouplers. However, it is essential to consider the interaction of the sub-processes and its impact on the control action. Consequently, the optimal parameters of the decentralized PID controller will be identified by a minimisation of an objective function that can consider these effects. This approach will ensure an effective mitigation of the aforementioned effects. This section presents the methodology employed in the construction of this index and the basis for the optimization process. In a similar way to the tuning procedure of controllers for Single-Input-Single-Output (SISO) systems, it is feasible to define the integrated absolute error (IAE) index in relation to either the error signal or the set-point and the process variable. For TITO processes, it must be generalized to encompass any of the four potential responses ($i, j = \{1, 2\}$).

$$IAE_{i,j} = \int_0^\infty |e_i(t)|dt = \int_0^\infty |r_i(t) - y_i(t)|dt \tag{3}$$

However, the issue of coupling remains, if this index serves as an objective function. Consequently, a controller configuration will emerge and its performance will be dependent on the level of interaction between the TITO process. In order to avoid this issue, it is recommended that the four potential IAE indices be considered, with a single index and an extra parameter serving to weigh the importance or degree of decoupling that can be achieved with a decentralized controller:

$$J_T(\alpha) = \alpha(\mathrm{IAE}_{12} + \mathrm{IAE}_{21}) + (1 - \alpha)(\mathrm{IAE}_{11} + \mathrm{IAE}_{22}) \tag{4}$$

The employment of $J_T(\alpha)$ as the objective function to be minimised by the optimization process allows for the derivation of decentralized PID controllers with the capacity to account for the coupling effect of the TITO process, while simultaneously maintaining optimal performance levels on the control task. In the conditions described, the additional parameter α describes the level of weighting towards the coupling effect and the performance of the main control loops. In order to facilitate the development of the design proposed in this work, values within the full operational range for α ($0 \leq \alpha \leq 1$) will be considered. Additionally, step-type inputs will be employed in the set-point for the calculation of the IAE indices, thereby enabling a comprehensive evaluation of the performance of the designed PID controllers in the set-point tracking task.

III. Robust Control Design Approach

The methodology proposed in this paper is based on the same principle of robust design of PID controllers for SISO systems based on the sensitivity function. This approach is extended to TITO systems and decentralized PID controllers. In this case, the robustness requirement will be part of the design, imposing itself as a constraint in the optimization procedure for the search of the optimal parameters, as described in the previous section.

In the first instance, one may consider the case of a classical SISO system, where $C(s)$ represents the controller and $P(s)$ is an arbitrary process. In this context, the sensitivity function and its complementary can be expressed according to the following expression:

$$S(s) = \frac{1}{1 + P(s)C(s)} \tag{5}$$

In light of this premise, it is common practice to consider the maximum of the frequency response of the sensitivity function M_s as an index representing the inverse of the maximum distance from the Nyquist plot to the critical point $(-1, 0)$. This is on the understanding that a lower M_s^t value indicates greater robustness of the system due to its gain and phase margins. It is defined as:

$$M_s \doteq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + P(j\omega)C(j\omega)|} \tag{6}$$

As widely evidenced in the literature [19], it is a common practice to design for M_s values between 1.4 and 2. For $M_s = 2$, the minimum value of robustness is identified as the

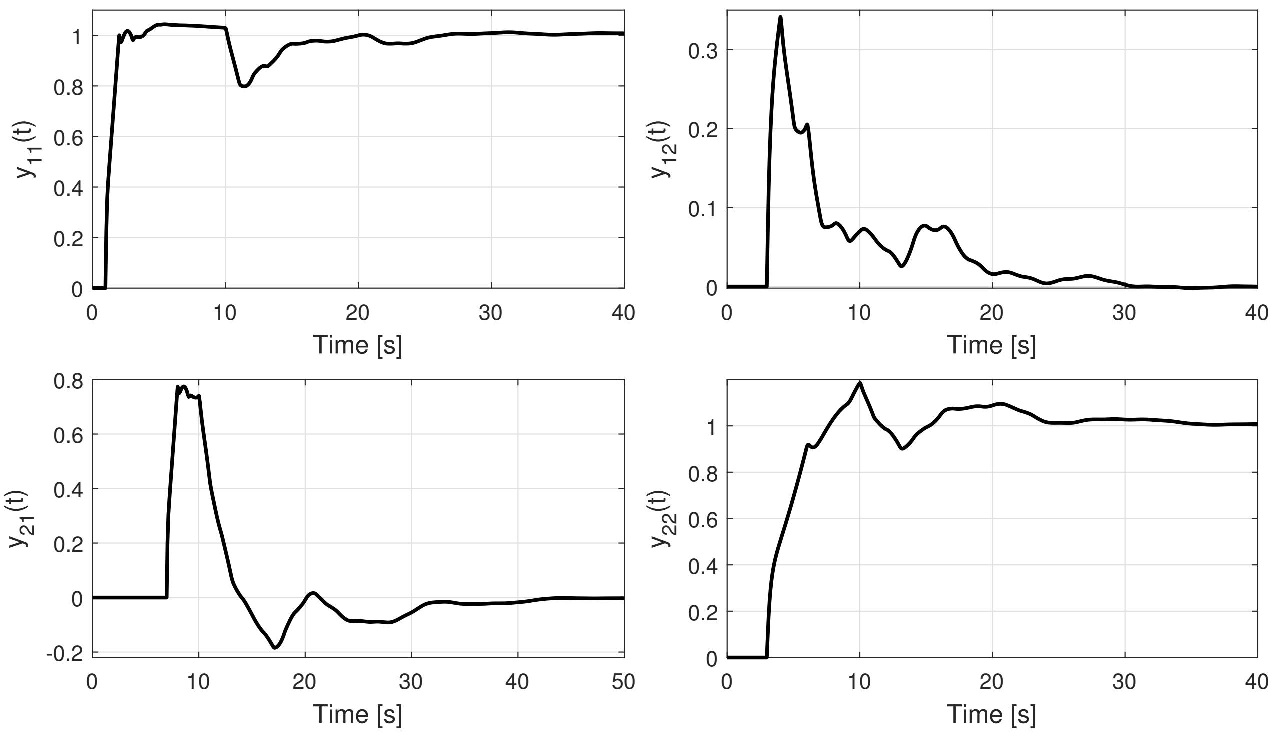

Fig. 2. $P_1(s)$ step response with the Decentralized PID Controller implemented for the case $\alpha = 0.50$.

minimum value of robustness required to obtain phase and gain margins that ensure reasonable stability margins of the system. Conversely, for $M_s = 1.4$, the control system is considered more robust and stable since the margins obtained in these cases are higher. As previously stated, robust controllers can be implemented to guarantee the stable operation of the control system. With regard to TITO systems, a generalization of the sensitivity function can be made with a view to employing the same index of robustness (M_s) that is already in common use. The sensitivity function for a TITO control system can be expressed as shown below:

$$S(s) = (I + P(s)C(s))^{-1} \tag{7}$$

where I corresponds to the 2×2 identity matrix. Although it is accurate to state that previously there was only one sensitivity function, in the case of 2×2 systems there will be four sensitivity functions. Thus, if it were desired to define the equivalent of equation 6, the following expression would be obtained:

$$M_s^o = \begin{bmatrix} M_{s11} & M_{s12} \\ M_{s21} & M_{s22} \end{bmatrix} \tag{8}$$

It should be remarked that these M_{sij} are obtained from the frequency response of the functions $S_{ij}(s)$ which can be expressed in terms of the decentralized PID controllers $C_1(s)$ and $C_2(s)$ and all the $P_{ij}(s), (i, j = \{1, 2\})$, sub-processes of the TITO process under consideration. However, for the sake of brevity, such equivalences will not be presented explicitly. At last, the robustness constraint to which the design will be subject can be defined. There are various approaches that can be taken with regard to this constraint. One option would be to consider it in relation to the two main control loops. However, in this instance, this approach will be set aside in favour of adopting the equivalent constraint that is typically used for SISO systems and adapting it to suit this case.

Thus, we first define:

$$M_s^t \doteq \max_{\omega} |M_{sij}|, \quad i, j = \{1, 2\} \tag{9}$$

Thus, the value of M_s^t represents the maximum value-equivalent of all entries of the matrix M_s^o, such value will be restricted to a specific target of robustness and it defines the constraint to which the optimization will be subject. In general terms, the optimization problem can be formulated as follows:

$$\bar{\theta}_{opt} := \{K_{pj}, T_{ij}, T_{dj}\} = \arg \left[\min_{\bar{\theta}} |J_T(\bar{\theta}; \alpha)| \right] \tag{10}$$

subject to

$$M_s^t \doteq \max |M_{sij}(\bar{\theta}; \alpha)| \leq 2, \quad i, j = \{1, 2\}$$

In where $\bar{\theta}$ will be the parameters of the decentralized PID controller that will be implemented in the TITO process in order to comply with the stated requirements.

IV. Numerical Studies

A. Example 1: Wood and Berry Distillation Column

The distillation column process presented in [20] will be used as a first example for the robust decentralized PID controllers design.

$$P_1(s) = \begin{bmatrix} \frac{12.8e^{-s}}{1+16.7s} & \frac{-18.9e^{-3s}}{1+21s} \\ \frac{6.6e^{-7s}}{1+10.9s} & \frac{-19.4e^{-3s}}{1+14.4s} \end{bmatrix} \quad (11)$$

Table I presents the parameters of the decentralized controllers designed with different values of the α weight factor, while Table II shows the performance indices for all the desings. Fig. 2 illustrates the system response to step-type inputs for $\alpha = 0.50$ and Fig. 3 presents a detailed analysis of the M_s^t values for this case.

TABLE I
Decentralized PID controller parameters in regard to $P_1(s)$.

α	K_{p1}	T_{i1}	T_{d1}	K_{p2}	T_{i2}	T_{d2}
0	0.927	12.177	0.417	-0.125	3.976	1.969
0.10	0.930	12.634	0.399	-0.119	3.535	2.351
0.20	0.924	11.343	0.418	-0.123	3.849	2.091
0.30	0.923	10.979	0.417	-0.123	3.817	2.135
0.40	0.902	10.501	0.435	-0.124	3.913	2.099
0.50	**0.911**	**10.248**	**0.430**	**-0.124**	**4.021**	**2.040**
0.60	0.892	9.809	0.443	-0.123	3.948	2.145
0.70	0.882	9.434	0.452	-0.123	4.015	2.155
0.80	0.791	7.846	0.538	-0.127	4.651	2.097
0.90	0.719	7.069	0.621	-0.132	5.324	2.010
1.00	0.579	5.920	0.777	-0.142	6.264	2.007

By analysing the system response, it is possible to determine the impact of the coupling effect on the two main loop responses $y_{11}(t)$ and $y_{22}(t)$. It should be noted that the analysis considers the error caused by the set-point tracking and the regulation of the system coupling effects on $y_{12}(t)$ and $y_{21}(t)$ as equally important.

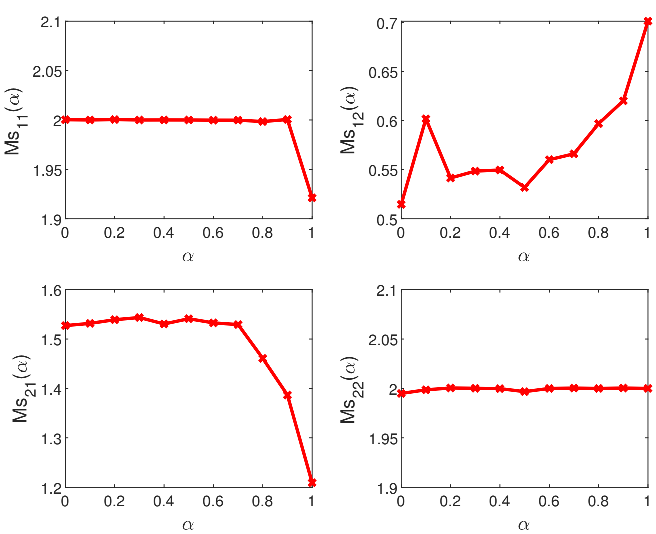

Fig. 3. Robustness Index according to α for the case of process $P_1(s)$

A further crucial aspect to be considered is the correspondence between the obtained values of $M_s^t(\alpha)$ and the anticipated ones. Given that the robustness restriction merely stipulates that the maximum value of the M_s^o matrix should be 2, it is of interest to examine the two principal control loops. In the majority of cases involving α values, the obtained value of $M_s^t(\alpha)$ is equal to 2. This indicates that the system strives to achieve balance between the two main loops and the secondary ones, thereby fulfilling the robustness restriction and optimising performance.

TABLE II
Performance index values for $P_1(s)$.

α	IAE_{11}	IAE_{22}	IAE_{12}	IAE_{21}	$J_T(\alpha)$
0	2.553	5.732	1.968	4.566	8.285
0.10	2.593	5.702	2.034	4.537	8.123
0.20	2.566	5.723	1.842	4.588	7.917
0.30	2.571	5.729	1.791	4.604	7.728
0.40	2.591	5.722	1.766	4.601	7.534
0.50	**2.568**	**5.764**	**1.718**	**4.635**	**7.342**
0.60	2.605	5.742	1.706	4.623	7.136
0.70	2.619	5.757	1.684	4.630	6.932
0.80	2.773	5.848	1.636	4.598	6.711
0.90	2.889	5.884	1.635	4.533	6.428
1.00	3.129	5.818	1.712	4.321	6.033

With regard to the minimized values of the objective function, a decrement can be observed as analysed in Table II, when the weighting factor is close to 1. However, upon examination of the four IAE values, it becomes evident that there is no discernible trend in these values, as the system strives to achieve a balance between the performance attained in the set-point tracking tasks and the regulation of the coupling effect.

B. Example 2: Polymerization Reactor

The second example considers the polymerization reactor TITO process presented in [21].

$$P_2(s) = \begin{bmatrix} \frac{22.89e^{-0.2s}}{1+4.572s} & \frac{-11.64e^{-0.4s}}{1+1.807s} \\ \frac{4.689e^{-0.2s}}{1+2.174s} & \frac{5.80e^{-0.4s}}{1+1.801s} \end{bmatrix} \quad (12)$$

Similarly to the previous example, Table III shows the parameters of the decentralised controllers designed from different values of the weight factor, while Table IV illustrates the performance indices and the objective function.

TABLE III
Decentralized PID controller parameters in regard to $P_2(s)$.

α	K_{p1}	T_{i1}	T_{d1}	K_{p2}	T_{i2}	T_{d2}
0	0.707	2.950	0.087	0.280	1.525	0.254
0.10	0.637	3.539	0.102	0.286	1.566	0.261
0.20	0.591	3.768	0.113	0.300	1.593	0.266
0.30	0.682	3.794	0.093	0.279	1.556	0.239
0.40	0.704	3.691	0.089	0.276	1.565	0.225
0.50	**0.709**	**3.718**	**0.087**	**0.273**	**1.554**	**0.246**
0.60	0.745	3.683	0.080	0.266	1.550	0.227
0.70	0.767	3.658	0.076	0.255	1.557	0.221
0.80	0.794	3.542	0.071	0.241	1.565	0.233
0.90	0.828	3.346	0.064	0.236	1.878	0.231
1.00	0.848	3.037	0.062	0.217	2.253	0.193

The analysis of the achieved robustness can be performed from Fig. 4. Finally, in Fig. 5 it can be analyzed the system responses to step inputs for the case of $\alpha = 0.50$.

Fig. (5) shows the system responses, highlighting the coupling effect in the main control loops. Unlike the previous example, where both loops aimed for maximum robustness, here only the $C_1(s)$ control loop consistently reamain on the 2.00 value for the M_s. Meanwhile, the $C_2(s)$ control loop begins near 1.80 and decreases as α increases.

TABLE IV
DECENTRALIZED PID CONTROLLER PARAMETERS FOR $P_2(s)$.

α	IAE$_{11}$	IAE$_{22}$	IAE$_{12}$	IAE$_{21}$	$J_T(\alpha)$
0	0.430	0.702	0.400	0.350	1.132
0.10	0.429	0.689	0.396	0.327	1.079
0.20	0.439	0.676	0.422	0.311	1.039
0.30	0.416	0.702	0.367	0.342	0.996
0.40	0.411	0.713	0.353	0.351	0.956
0.50	**0.411**	**0.710**	**0.357**	**0.345**	**0.912**
0.60	0.412	0.724	0.335	0.359	0.871
0.70	0.411	0.751	0.319	0.365	0.828
0.80	0.415	0.793	0.306	0.367	0.781
0.90	0.423	0.970	0.290	0.343	0.708
1.00	0.422	1.270	0.246	0.335	0.572

The graphical analysis of the system responses can be carried out in Fig. (5), which again demonstrates the coupling effect in the main control loops. As with the previous example, we can compare the level of $M_s^t(\alpha)$ obtained by the system for the cases considered. In contrast to the previous example, however, the two main control loops are not seeking to attain the maximum possible value for this process. Instead, the control loop of the $C_1(s)$ controller is the one that always remains at the robustness target value, while the $C_2(s)$ controller loop starts with a value close to 1.80 and decreases as α increases.

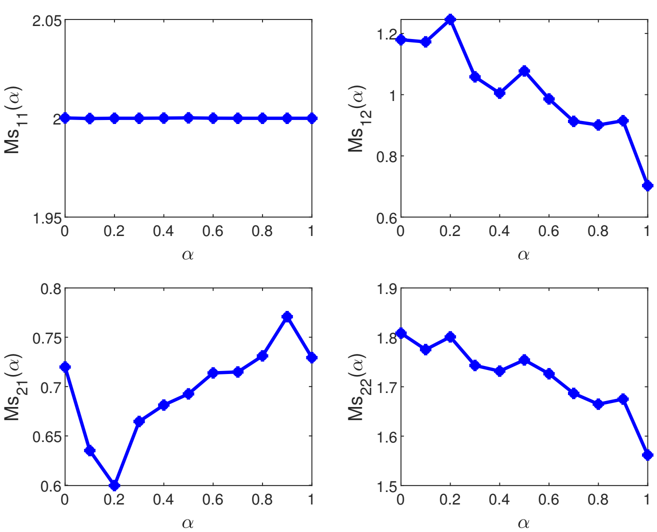

Fig. 4. Robustness Index according to α for the case of process $P_2(s)$

Additionally, Fig. (6) illustrates the maximum singular value in contrast to the frequency response of the two primary components of the sensitivity function for the proposed example in the $\alpha = 0.50$ case. It is evident by analize the graphs that the singular value $\sigma(j\omega)$ is greater than the sensitivity functions $S_{11}(j\omega)$ and $S_{22}(j\omega)$. This observation suggests that when designing based on M_s, the $\sigma(j\omega)$ is not constrained; however, if the opposite is considered, the singular value would also constrained the M_s. However, typically when designing TITO control systems, specific values for the peak of the maximum singular value are not targeted instead of that a small value is desired. On the other hand, by employing the M_s index, it is possible to anticipate more accurate system behaviors and responses and attribute certain levels of robustness to the system.

V. CONCLUSIONS AND FUTURE WORK

This study investigated the use of the maximum of the sensitivity function as a robustness constraint in the design of decentralized PID controllers for TITO processes. While robustness in multivariable control is typically evaluated using singular values or frequency-domain techniques, this work proposed an alternative formulation based on the sensitivity function. Applied to benchmark TITO systems, the method demonstrated how this constraint shapes the balance between performance and stability in decentralized control. The results showed that incorporating the sensitivity function constraint captures interaction effects through a weighted objective function, offering design flexibility based on robustness requirements. An inverse relationship between the objective function and weighting factors was observed, similar to the trends seen in robust SISO design. This highlights the trade-off between performance and robustness and the importance of selecting suitable robustness constraints for a given system. Future work will extend this approach to more complex MIMO systems and explore hybrid robustness strategies that integrate sensitivity and singular-value-based measures to enhance controller performance in highly interactive processes.

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REFERENCES

[1] T. Hägglund, S. Shinde, A. Theorin, and U. Thomsen, "An industrial control loop decoupler for process control applications," *Control Engineering Practice*, vol. 123, p. 105138, 3 2022.

[2] S. Skogestad and I. Postlethwaite, *Multivariable feedback control*. Wiley-Interscience, 11 2005.

[3] J. Berner, K. Soltesz, T. Hägglund, and K. J. Åström, "Autotuner identification of TITO systems using a single relay feedback experiment," *IFAC-PapersOnLine*, vol. 50, pp. 5332–5337, 7 2017.

[4] M. Hofreiter, "Shifting Method for Identification of TITO Systems using Decentralized Relay Feedback," *IFAC-PapersOnLine*, vol. 55, pp. 2115–2120, 1 2022.

[5] S. Pandey, "Limit cycle based identification of two-input two-output systems with time delay," *IFAC-PapersOnLine*, vol. 57, pp. 421–426, 1 2024.

[6] R. H. Naik, D. A. Kumar, and P. Sujatha, "Independent controller design for MIMO processes based on extended simplified decoupler and equivalent transfer function," *Ain Shams Engineering Journal*, vol. 11, pp. 343–350, 11 2019.

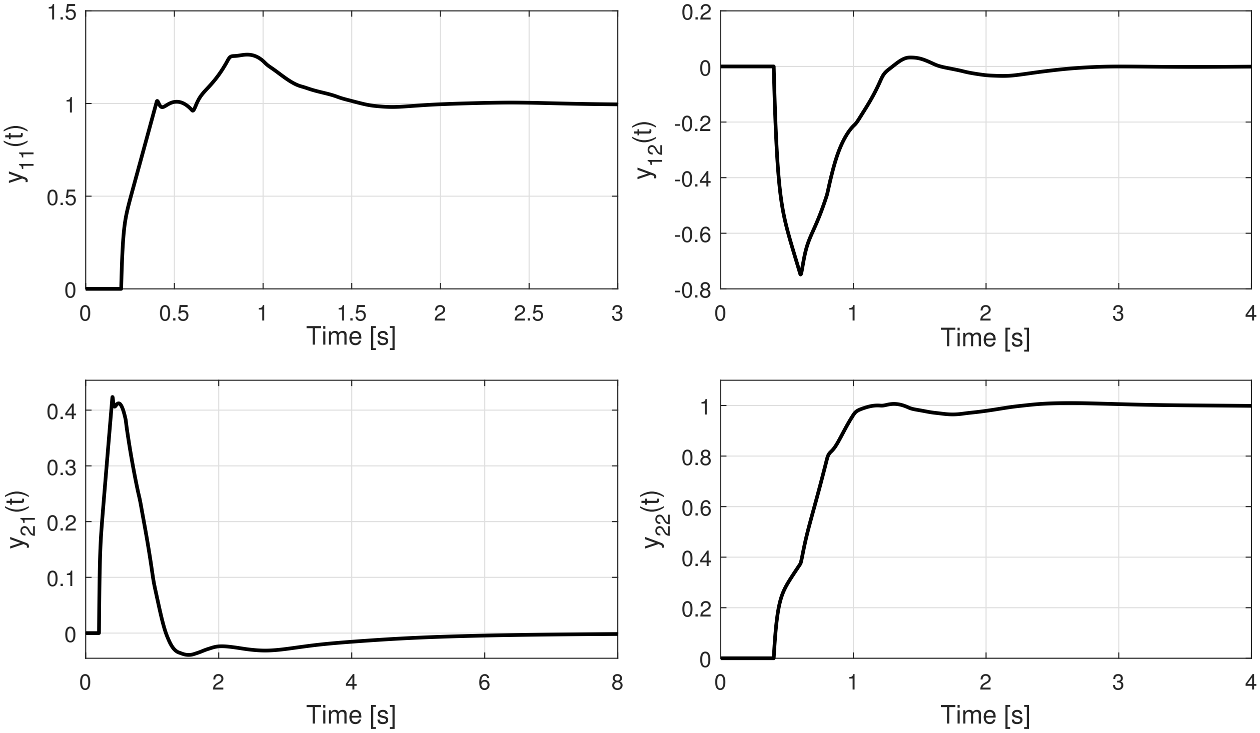

Fig. 5. $P_2(s)$ step response with the Decentralized PID Controller implemented for the case $\alpha = 0.50$.

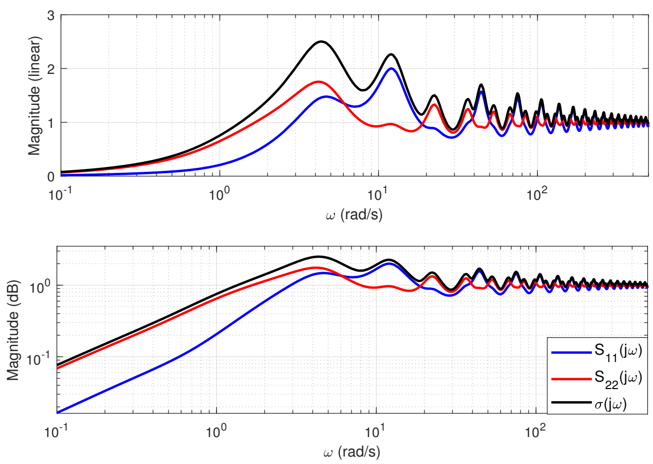

Fig. 6. Maximum Singular Value compared with the two main components Sensitivity Function for the case $\alpha = 0.50$

[7] B. T. Jevtović and M. R. Mataušek, "PID controller design of TITO system based on ideal decoupler," *Journal of Process Control*, vol. 20, pp. 869–876, 6 2010.

[8] E. C. Goud, S. R. A, and M. Chidambaram, "Improved Decentralized PID Controller design for MIMO Processes," *IFAC-PapersOnLine*, vol. 53, pp. 153–158, 1 2020.

[9] S. Tavakoli, I. Griffin, and P. J. Fleming, "Tuning of decentralised PI (PID) controllers for TITO processes," *Control Engineering Practice*, vol. 14, pp. 1069–1080, 8 2005.

[10] V. Hajare and B. Patre, "Decentralized PID controller for TITO systems using characteristic ratio assignment with an experimental application," *ISA Transactions*, vol. 59, pp. 385–397, 11 2015.

[11] S. Lakshmanaprabu, M. Elhoseny, and K. Shankar, "Optimal tuning of decentralized fractional order PID controllers for TITO process using equivalent transfer function," *Cognitive Systems Research*, vol. 58, pp. 292–303, 7 2019.

[12] O. Arrieta, A. Barbieri, H. Meneses, F. Padula, R. Vilanova, and A. Visioli, "On the Use of Fractional-Order PID Controllers for TITO Processes," *IFAC-PapersOnLine*, vol. 56, pp. 3284–3289, 1 2023.

[13] A. Govind and S. Mahapatra, "Improving precision and robustness in level control of coupled tank systems: A tree seed optimization and μ-analysis approach," *Results in Control and Optimization*, vol. 15, p. 100410, 3 2024.

[14] A. Javadian, N. Nariman-Zadeh, and A. Jamali, "Evolutionary design of marginally robust multivariable PID controller," *Engineering Applications of Artificial Intelligence*, vol. 121, p. 105938, 2 2023.

[15] J. S. Freudenberg, D. P. Looze, and J. B. Cruz, "Robustness analysis using singular value sensitivities," in *1981 20th IEEE Conference on Decision and Control including the Symposium on Adaptive Processes*, pp. 1158–1166, 1981.

[16] C. A. Devia and G. Giordano, "Topology-Independent Robust Stability for Networks of Homogeneous MIMO Systems," *IFAC-PapersOnLine*, vol. 53, pp. 3379–3384, 1 2020.

[17] Y.-J. Wang, "Tuning Optimal and Robust Decentralized PID Controllers for Two-Input Two-Output Time Delay Systems with a Decoupler Based on Frequency Domain Robustness Specifications," *Franklin Open*, vol. 6, p. 100072, 2 2024.

[18] A. Visioli, *Practical PID Control*. Springer-Verlag London Limited 2006, 2006.

[19] K. J. Åström and T. Hägglund, *Advanced PID control*. ISA-The Instrumentation, Systems and Automation Society, 2006.

[20] R. Wood and M. Berry, "Terminal composition control of a binary distillation column," *Chemical Engineering Science*, vol. 28, pp. 1707–1717, 9 1973.

[21] I.-L. Chien, H.-P. Huang, and J.-C. Yang, "A Simple Multiloop Tuning Method for PID Controllers with No Proportional Kick," *Industrial & Engineering Chemistry Research*, vol. 38, pp. 1456–1468, 3 1999.