A two-part pricing mechanism for demand side management

Silvia Cianchi, Anibal Sanjab, Sergio Grammatico

***Abstract*— The electrical grid is experiencing a significant restructuring to face the challenges posed by the green transition. A key aspect in this restructuring process is the increased engagement of consumers as active players, who, through the participation in demand side management schemes, can deliver services to the grid. In this paper, a novel pricing mechanism composed of two dynamic volumetric parts is introduced which can be employed by a central coordinator (e.g., a DSO) to simultaneously: i) prevent congestions in the grid, and ii) recuperate its investment costs. To capture the interconnected objective functions, and coupling constraints between the central coordinator and the consumers, the problem is formulated as a generalized Nash equilibrium problem, which is solved using a distributed and privacy-friendly hybrid best-response plus dual-ascent algorithm with convergence guarantees. Moreover, the work considers that the consumers' decision-making processes can deviate from full economic rationality and be driven by personal preferences about the tradeoff between their induced comfort and their economic savings. Numerical simulations corroborate the convergence of the proposed algorithm, and show the effectiveness of the proposed two-part tariff in eliminating grid congestions. The results highlight the negative impact of a reduced user price sensitivity on the collective costs of all consumers.**

I. Introduction

The proliferation of renewables and electrification of the consumer space (with the introduction of, e.g., heat pumps and electric vehicles) at the end-user level, are increasingly yielding stressed operational conditions to the grid [1]. In fact, on one side, this transformation contributes to achieving the goals of sustainability, reliability, and carbon neutrality of the energy system, as highlighted in the European Green Deal [2]. On the other hand, this calls for an electricity system that is more flexible and better placed to integrate a greater share of renewables and ever increasing but flexible demand. In this context, the engagement of end-consumers as active players in the grid is crucial [3]. In fact, consumers, empowered by digitalization and electrification, are increasingly gaining the ability to dynamically generate, store, and consume energy in response to varying prices and tariffs [4]. Such ability is defined in the literature as *flexibility* [5], and can be leveraged by the consumers for the delivery of services to the grid, such as congestion management [6], thereby reaping financial benefits. This mechanism is commonly referred to as Demand Side Management (DSM) [7]–[11]. The electrical grid is, hence, increasingly shifting towards an architecture governed by distributed decisions autonomously taken by its constituents, each pursuing potentially different objectives [12]–[15]. In particular, in this paper, we propose a novel two-stage price-based DSM mechanism which can be employed by a central coordinator (CC) – e.g., an energy service company (ESCO) or the distribution system operator (DSO) – with the double aim of ensuring the safe operation of the grid and recuperating its investment costs (typically referred to for DSOs as the regulated budget) [16]. Since the effectiveness of the DSM mechanism depends on the aggregative consumption behavior of the consumers and shared constraints, a generalized Nash equilibrium game-theoretic framework is adopted to model the interconnected strategic setting – i.e., to allow the CC to take into account the users' reaction in designing the tariff, and for the users to optimally schedule their consumption when subjected to the tariff.

Silvia Cianchi is with Delft University of Technology, and VITO and EnergyVille `scianchi@tudelft.nl`. Anibal Sanjab is with VITO and EnergyVille. Sergio Grammatico is with Delft University of Technology.

Related work. In recent years, DSM mechanisms have been widely investigated in the literature, where, e.g., [9], [10] provide an overview of the topic. Many of these works, such as [17]–[20], employ bilevel models to represent the interaction between i) a price-setting agent who designs the network tariffs in order to achieve its own specific goal, and ii) the consumers, who react to such tariffs aiming at minimizing their bills. In particular, in [18], [20], the goal of the price-setting agent concerns the cost recovery. The work in [19], on the other hand, aims at shifting the load to off-peak hours to guarantee the safe operation of the grid. In most of them, e.g. [21], the equilibrium of the problem is achieved via ADMM-based algorithms, which enjoy favorable properties such as separability and fast convergence, and employ best response dynamics for the consumers. However, what all these works have in common is that the price-setting agent has a unique goal. In fact, in the literature, price-based schemes designed by the CC to achieve multiple goals are limited, and they typically involve structures that face practical implementation challenges. For instance, the work in [22] proposes a tariff that is composed of an ex-ante component and an ex-post component, which faces regulatory barriers, as it assumes the consumers not to know the full electricity price before making their consumption decisions.

Paper Contribution. This paper proposes a novel two-part pricing structure which allows a CC to meet two goals: i) ensuring grid safety by resolving anticipated congestions, and ii) recuperating its regulated budget. In particular, this pricing structure is made up of two dynamic volumetric components which can adaptively change to reflect grid loading: i) the base dynamic tariff, and ii) an additional dynamic price adder, which is activated, when needed, to resolve congestions. The main contribution is to show that the tuning of these tariff variables follows from a hybrid best-response plus dual-ascent algorithm that solves a specific max-min problem reformulation of the initial generalized Nash equilibrium problem. Specifically, our proposed algorithm: i) relies on consumers' best responses, ii) has a distributed structure, and iii) preserves consumers' privacy, as the CC does not have visibility on their private constraints and objective functions, which can deviate from full economic rationality and take into account subjective preferences and priorities. In this regard, we consider the users' decisions to be driven by an individual *sensitivity to the applied electricity prices*, which reflects their willingness to deviate from a desired consumption profile (captured using a comfort function), and, in turn, impacts the design of the pricing structure. The convergence of the proposed algorithm and the effectiveness of the pricing structure are corroborated using a numerical case study under different scenarios of price sensitivity.

Paper Organization. The paper is organized as follows. Section II introduces the problem formulation, while Section III presents the game equilibrium derivation and the proposed algorithm therefor. Section IV presents the simulation results, and Section V concludes the paper.

Notation. We let $\mathbf{x} = [x_1, ..., x_n] \in \mathbb{R}^n$ denote a vector, and x_t be its t-th component. Moreover, we let $\mathbf{0}_n = \text{zeros}(n, 1)$. We denote by proj_C the Euclidean projection operator onto a convex set C, i.e., $\text{proj}_C(\mathbf{x}) := \underset{\mathbf{y}\in C}{\text{argmin}}||\mathbf{x} - \mathbf{y}||^2$.

II. Generalized Nash Equilibrium Game Formulation

Consider a set of N households, hereafter referred to as agents, aiming at scheduling their electricity demand on a given time horizon T. Let $\mathbf{x}_i = [x_{i,1}, ..., x_{i,T}] \in \mathbb{R}^T$ be the vector containing the consumption level of the i-th agent at each time slot. Each household schedules their consumption in order to optimize their individual objective function in (1a), that is a weighted tradeoff between discomfort and costs. For each agent, we define the discomfort as the Euclidean distance from a desired consumption profile $\mathbf{r}_i \in \mathbb{R}^T$ which reflects the preferences set by the agent. In addition, each agent can also specify its own price sensitivity, $0 < \omega_i \leq 1$, indicating its willingness to deviate from $\mathbf{r}_i$ (increasing discomfort) to achieve economic savings. Moreover, each household, besides the dynamic retail price $\mathbf{p} \in \mathbb{R}^T$, is also subject to a network tariff $\mathbf{x}_0 \in \mathbb{R}^T$ that is set by the CC, whose aim is to recover a certain investment budget, as in (1b). Furthermore, the scheduling problem of each household is subject to the coupling constraint in (1c), which aims to guarantee the safe operation of the grid. For privacy reasons, each agent does not have access to the consumption profiles of the other agents. In addition, the agents have no direct incentive to adjust their consumption behavior just to satisfy the grid capacity limits. It is, indeed, the responsibility of the CC to ensure grid safety (i.e., feasibility of the overall decision problem). As such, we model the interconnected decision making process of the consumers as a generalized aggregative game, whose players are the households (*regular agents*) and the central coordinator:

(regular agents, $i \in \{1, ..., N\}$):

$$\min_{\mathbf{x}_i \in \mathcal{X}_i} \frac{1-\omega_i}{\omega_i}||\mathbf{x}_i - \mathbf{r}_i||^2 + (\mathbf{p} + \mathbf{x}_0)^\top A_i \mathbf{x}_i, \quad \text{(1a)}$$

(central coordinator, $i = 0$):

$$\max_{\mathbf{x}_0 \in \mathcal{X}_0} \mathbf{x}_0^\top \sum_{i=1}^{N} A_i \mathbf{x}_i, \quad \text{(1b)}$$

$$\text{subject to:} \quad \sum_{i=1}^{N} A_i \mathbf{x}_i \leq \mathbf{b}. \quad \text{(1c)}$$

The matrix A_i is such that $A_i\mathbf{x}_i$ is the total energy offtake of agent i. In basic scenarios where households are only allowed to consume energy from the grid, A_i is the identity matrix. In more elaborate scenarios (e.g., when energy injection is also considered), A_i can have a different structure to select the energy offtake from the main grid. For simplicity of notation, we introduce the following

$$J_i(\mathbf{x}_i) := \frac{1-\omega_i}{\omega_i}||\mathbf{x}_i - \mathbf{r}_i||^2 + \mathbf{p}^\top A_i \mathbf{x}_i. \quad (2)$$

As a solution concept, we adopt the notion of generalized Nash equilibrium (GNE):

Definition 1. ***Generalized Nash equilibrium:*** *A solution* $[\mathbf{x}_1^*, ..., \mathbf{x}_N^*, \mathbf{x}_0^*]$ *of the game in* (1) *is a generalized Nash equilibrium (GNE) if*

$$\mathbf{x}_i^* \in \underset{\mathbf{x}_i \in \mathcal{X}_i}{\arg\min}\, J_i(\mathbf{x}_i) + {\mathbf{x}_0^*}^\top A_i \mathbf{x}_i \quad \text{(3a)}$$

$$\textit{s.t. } A_i \mathbf{x}_i + \sum_{j=1, j\neq i}^{N} A_j \mathbf{x}_j^* \leq \mathbf{b},$$

$$\mathbf{x}_0^* \in \underset{\mathbf{x}_0 \in \mathcal{X}_0}{\arg\max}\, \mathbf{x}_0^\top \sum_{i=1}^{N} A_i \mathbf{x}_i^*. \quad \square \quad \text{(3b)}$$

In particular, we focus on a specific subset of generalized Nash equilibria, that is the set of *variational* generalized Nash equilibria (v-GNE), in view of their well-establisthed interpretation as the solution set of a variational inequality [23]:

Definition 2. ***Variational generalized Nash equilibrium:*** *A solution* $[\mathbf{x}_1^*, ..., \mathbf{x}_N^*, \mathbf{x}_0^*]$ *of the game in* (1) *is a variational generalized Nash equilibrium (v-GNE) if it is a solution of the following variational inequality: VI(F, Ω), where* $F = \text{col}((\nabla_{\mathbf{x}_i}(J_i(\mathbf{x}_i) + {\mathbf{x}_0}^\top A_i \mathbf{x}_i))_{i\in\{1,...,N\}}, \nabla_{\mathbf{x}_0}(\mathbf{x}_0^\top \sum_{i=1}^{N} A_i \mathbf{x}_i))$, *and* $\Omega = \mathcal{X}_1 \times ... \times \mathcal{X}_N \times \mathcal{X}_0 \bigcap \{\mathbf{x} | A\mathbf{x} \leq \mathbf{b}\}$. $\square$

Since the CC does not have access to the decision making process of the individual agents, it cannot directly guarantee the satisfaction of the coupling constraint. Therefore, the coupling constraint can be dualized for each agent $i \in \{1, ..., N\}$ with dual variables $\boldsymbol{\lambda}_i \in \mathbb{R}^T$. Under the assumption that $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}$ for all i, namely searching for a variational equilibrium as defined in Definition2, we obtain the following reformulation of the game:

$$\forall i: \quad \min_{\mathbf{x}_i \in \mathcal{X}_i} J_i(\mathbf{x}_i) + (\mathbf{x}_0 + \boldsymbol{\lambda})^\top A_i \mathbf{x}_i, \tag{4a}$$

$$\max_{\mathbf{x}_0 \in \mathcal{X}_0} \mathbf{x}_0^\top \sum_{i=1}^N A_i \mathbf{x}_i, \tag{4b}$$

$$\max_{\boldsymbol{\lambda} \geq \mathbf{0}} \boldsymbol{\lambda}^\top \left(\sum_{i=1}^N A_i \mathbf{x}_i - \mathbf{b} \right). \tag{4c}$$

Interestingly, $\boldsymbol{\lambda}$ appears in the objective functions of the households summed to $\mathbf{x}_0$, meaning that in practice each household should react to an augmented network tariff ($\mathbf{x}_0 + \boldsymbol{\lambda}$). As such, the dual variable $\boldsymbol{\lambda}$ can be interpreted as a price adder set by the CC with the aim of guaranteeing the safe operation of the grid, which is only activated when the coupling constraint is binding (i.e., when congestion occurs). Furthermore, we introduce the following assumption:

Assumption II.1. *The sets $\mathcal{X}_1, ..., \mathcal{X}_N \in \mathbb{R}^T$ are convex and compact.* □

III. GNE derivation

In this section, we propose Alg.1 to seek a $v-$GNE of the game in (1), in which the pricing components $\mathbf{x}_0$, $\boldsymbol{\lambda}$, and the corresponding optimal consumption schedules of the users $\mathbf{x}_i$ are updated in a semi-decentralized way. Differently from most GNE seeking algorithms, e.g. [24], [25], we impose that regular agents adopt a best response update, which reflects user behaviors and supports the natural integration of human users in the decision-making process. Specifically,

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Algorithm 1: hybrid best-response plus dual-ascent algorithm to seek a $v-$GNE of (1)

Parameters: step size $\left(\alpha^k\right)_{k \in \mathcal{K}}$.
Initialization: $k \leftarrow 0, \mathbf{x}_i^k \in \mathcal{X}_i, \mathbf{x}_0^k \in \mathcal{X}_0, \boldsymbol{\lambda}^k \in \mathbb{R}^m_{\geq 0}$.
Iterate until convergence:
$\forall i: \mathbf{x}_i^{k+1} = \arg\min_{\mathbf{x}_i \in \mathcal{X}_i} \quad J_i(\mathbf{x}_i) + (\mathbf{x}_0^k + \boldsymbol{\lambda}^k)^\top A_i \mathbf{x}_i$;
$\mathbf{x}_0^{k+1} = \mathrm{proj}_{\mathcal{X}_0}(\mathbf{x}_0^k + \alpha^k \sum_{i=1}^N A_i \mathbf{x}_i^{k+1})$;
$\boldsymbol{\lambda}^{k+1} = \mathrm{proj}_{\geq 0}(\boldsymbol{\lambda}^k + \alpha^k (\sum_{i=1}^N A_i \mathbf{x}_i^{k+1} - \mathbf{b}))$;
$k \leftarrow k+1$.

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at each iteration $k \in \mathcal{K}$, each agent i reacts with its best response, while the CC updates the pricing factors $\mathbf{x}_0$ and $\boldsymbol{\lambda}$ by executing one step of gradient ascent. We now provide a technical observation on the special structure of our game equilibrium problem.

Lemma III.1. (max-min characterization of the Nash equilibria of the game) *The game in* (4) *can be reformulated as a* 2*-player Nash game, whose Nash equilibrium solutions are a subset of the solutions of the following max-min problem:*

$$\max_{\boldsymbol{\mu} \in M} \quad \min_{\mathbf{x} \in \mathcal{X}} \sum_{i=1}^N J_i(\mathbf{x}_i) + \boldsymbol{\mu}^\top (\bar{A}\mathbf{x} - \bar{\mathbf{b}}), \tag{5}$$

where $\boldsymbol{\mu} = [\mathbf{x}_0, \boldsymbol{\lambda}]^\top$, $\mathbf{x} = [\mathbf{x}_1, ..., \mathbf{x}_N]^\top$, $A = [A1, ..., A_N]$, $\bar{A} = [A; A]$, $\bar{\mathbf{b}} = [\mathbf{0}_m, \mathbf{b}]^\top$. □

We are now in the position to show the convergence guarantee of our proposed algorithm by stating the following theorem:

Theorem III.2. *If Assumption II.1 holds, then Algorithm 1 globally converges to a variational generalized Nash equilibrium ($v-$GNE) of the game in* (1). □

Proof: In view of Lemma III.1, Algorithm 1 reads as a gradient ascent algorithm applied to the max-min problem in (5).

IV. Numerical Results

As a numerical test case, we consider a network composed of N households and a DSO. Each household i can have:

- A load $\mathbf{x}_i \in \mathbb{R}^T$, which is partially shiftable and flexible. In particular, $\underline{\mathbf{x}}_i$ represents the non flexible consumption profile, while $\underline{d}_i$ and $\bar{d}_i$ the lower and upper bounds on the cumulative demand to be consumed throughout the time horizon.
- A battery, which can be charged and discharged at rates of, respectively, $\mathbf{x}_i^{\text{BAT}} \in \mathbb{R}^T$ and $\mathbf{a}_i^{\text{BAT}} + \mathbf{i}_i^{\text{BAT}} \in \mathbb{R}^T$ with efficiency η_i^{BAT}. In particular, $\mathbf{a}_i^{\text{BAT}}$ is the power used by the consumer for its own demand, and $\mathbf{i}_i^{\text{BAT}}$ is the power that is injected into the grid. For each t the state of charge of the battery evolves as follows:

$$s_{i,t}^{\text{BAT}} = s_{i,0}^{\text{BAT}} + \eta_i^{\text{BAT}} \sum_{h=1}^t x_{i,h}^{\text{BAT}} - \frac{1}{\eta_i^{\text{BAT}}} \sum_{h=1}^t (a_{i,h}^{\text{BAT}} + i_{i,h}^{\text{BAT}}). \tag{6}$$

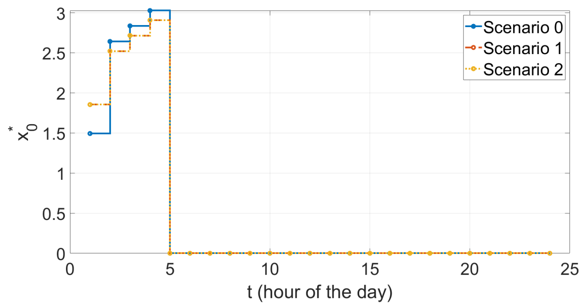
  Moreover, C_i^{BAT} denotes the capacity of the battery, while P_i^{BAT} its maximum charging and discharging power.
- An electric vehicle (EV), which is charged at a rate of $\mathbf{x}_i^{\text{EV}} \in \mathbb{R}^T$ during a specific time window $[\underline{t}, ..., \bar{t}]$. For each $t < \bar{t}$, its state of charge evolves as follows:

$$s_{i,t}^{\text{EV}} = s_{i,\underline{t}}^{\text{EV}} + \eta_i^{\text{EV}} \sum_{h=\underline{t}}^t x_{i,h}^{\text{EV}}. \tag{7}$$

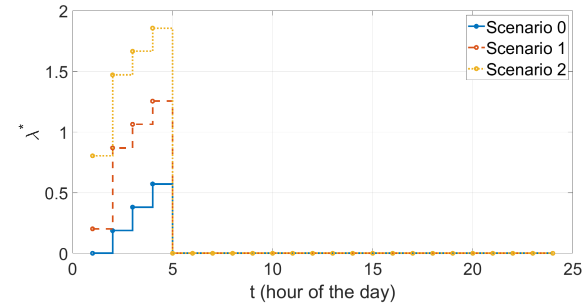
  The capacity of its battery is indicated by C_i^{EV}, and its maximum charging power is denoted by P_i^{EV}. Furthermore, we assume that each user can specify the minimum level of charge $\underline{d}_i^{\text{EV}}$ which has to be reached by the end of the charging session.
- A solar panel (PV), whose generated power can be either used ($\mathbf{a}_i^{\text{PV}} \in \mathbb{R}^T$), or injected ($\mathbf{i}_i^{\text{PV}} \in \mathbb{R}^T$), or curtailed. Also, $\mathbf{P}^{\text{PV}} \in \mathbb{R}^T$ is the power production profile throughout the time horizon.

Finally, $\bar{\mathbf{O}}_i$ and $\bar{\mathbf{I}}_i$ are, respectively, the upper and lower bounds on the total power offtake and injection of user i. As such, the state of user i can be expressed by the vector

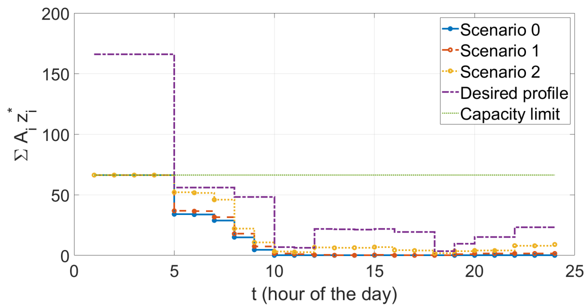
$$\mathbf{z}_i = [\mathbf{x}_i, \mathbf{x}_i^{BAT}, \mathbf{x}_i^{EV}, \mathbf{a}_i^{BAT}, \mathbf{a}_i^{PV}, \mathbf{i}_i^{\text{BAT}}, \mathbf{i}_i^{\text{PV}}]^\top, \tag{8a}$$



(a) Network tariff $\mathbf{x}_0^*$ reached throughout the day.

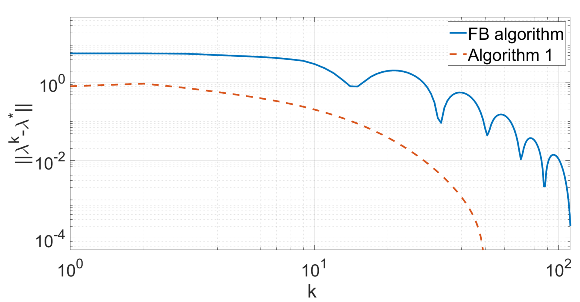


(b) Price adder $\boldsymbol{\lambda}^*$ reached throughout the day.

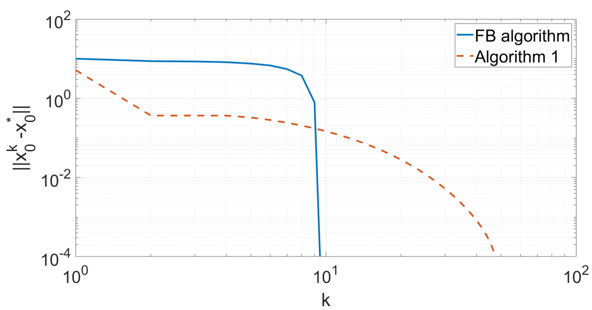


(c) Aggregative consumption $\sum_{i=1}^{N} A_i \mathbf{z}_i^*$ reached throughout the day.

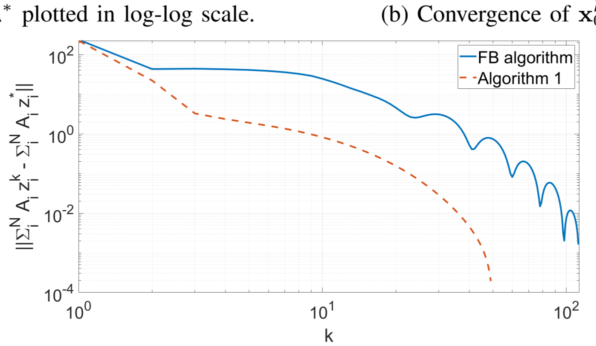
Fig. 1: $v-$GNE solution achieved by Algorithm 1 under different scenarios of sensitivity to the price.



(a) Convergence of $\boldsymbol{\lambda}^k$ to $\boldsymbol{\lambda}^*$ plotted in log-log scale.



(b) Convergence of $\mathbf{x}_0^k$ to $\mathbf{x}_0^*$ plotted in log-log scale.



(c) Convergence of the aggregative consumption $\sum_{i=1}^{N} A_i \mathbf{z}_i^k$ to $\sum_{i=1}^{N} A_i \mathbf{z}_i^*$ plotted in log-log scale.

Fig. 2: Convergence performances exhibited by Algorithm 1 and by the Forward-Backward algorithm.

and the total power offtake and injection of consumer i are respectively formulated as:

$$A_i\mathbf{z}_i = \mathbf{x}_i + \mathbf{x}_i^{\text{BAT}} + \mathbf{x}_i^{\text{EV}} - \mathbf{a}_i^{\text{BAT}} - \mathbf{a}_i^{\text{PV}}, \tag{9a}$$
$$\tilde{A}_i\mathbf{z}_i = \mathbf{i}_i^{\text{BAT}} + \mathbf{i}_i^{\text{PV}}. \tag{9b}$$

Furthermore, we consider that each household can specify a desired profile $\mathbf{r}_i$ for its consumption and injection behavior as well as its own sensitivity to the price ω_i, respectively reflecting its subjective preferences and the tradeoff between its comfort and its cost savings resulting from the usage of its appliances in reaction to the dynamic prices. The presence of this reference profile, and the subjectively weighted resistance to deviate therefrom, is an important indicator of what is known as the bounded rationality of the consumers (i.e., deviation from full economic rationality assumptions). It, indeed, takes into account the fact that humans adopt complex and subjective decision-making processes rather than simply maximizing their economic objective [26]. The goal of each household is to schedule their consumption (and injection) behavior on a day-ahead basis in order to optimize the weighted tradeoff between the discomfort (that is computed as the deviation with respect to $\mathbf{r}_i$) and the achieved economic benefit (in terms of cost reduction).
In particular, each household, besides the offtake and injection dynamic retail prices $\mathbf{p_O}, \mathbf{p_I} \in \mathbb{R}^T$, is also subject to the network tariffs $\mathbf{x}_0$ and $\boldsymbol{\lambda}$ that are set by the DSO. Specifically, the DSO separately sets such network tariffs with the double aim of recovering its regulated budget and guaranteeing the safe operation of the grid. In view of (4), this setup can be formulated as the following game:

$\forall\, i$:

$$\min_{\mathbf{z}_i \in \mathcal{Z}_i} \frac{1-\omega_i}{\omega_i}||\mathbf{z}_i - \mathbf{r}_i||^2 + (\mathbf{p_O} + \mathbf{x}_0 + \boldsymbol{\lambda})^\top A_i\mathbf{z}_i + \mathbf{p_I}^\top \tilde{A}_i\mathbf{z}_i, \tag{10a}$$

DSO:
$$\max_{\mathbf{x}_0 \in \mathcal{X}_0} \mathbf{x}_0^\top \sum_i A_i\mathbf{z}_i, \tag{10b}$$
$$\max_{\boldsymbol{\lambda} \geq 0} \boldsymbol{\lambda}^\top (\sum_i^N A_i\mathbf{z}_i - \mathbf{b}). \tag{10c}$$

In particular, $\mathcal{Z}_i$ is the feasibility set of the devices:

$$\underline{\mathbf{x}}_i \leq \mathbf{x}_i, \tag{11a}$$
$$x_{i,t}^{\text{BAT}} \leq P_i^{\text{BAT}} \quad \forall t \in [1,...,T], \tag{11b}$$
$$a_{i,t}^{\text{BAT}} + i_{i,t}^{\text{BAT}} \leq P_i^{\text{BAT}} \quad \forall t \in [1,...,T], \tag{11c}$$
$$x_{i,t}^{\text{EV}} \leq \begin{cases} P_i^{\text{EV}} & \forall t \in [\underline{t},...,\bar{t}] \\ 0 & \text{else} \end{cases}, \tag{11d}$$
$$\underline{d_i} \leq \sum_{t=1}^{T} x_{i,t} \leq \bar{d}_i, \tag{11e}$$
$$0 \leq s_{i,t}^{\text{BAT}} \leq C_i^{\text{BAT}}, \tag{11f}$$
$$0 \leq s_{i,t}^{\text{EV}} \leq C_i^{\text{EV}}, \tag{11g}$$
$$\underline{d}_i^{\text{EV}} \leq s_{i,\bar{t}}^{\text{EV}}, \tag{11h}$$
$$i_{i,t}^{PV} + a_{i,t}^{PV} \leq P_t^{\text{PV}} \quad \forall t \in [1,...,T], \tag{11i}$$
$$\mathbf{0}_T \leq \mathbf{x}_i + \mathbf{x}_i^{\text{BAT}} + \mathbf{x}_i^{\text{EV}} - \mathbf{a}_i^{\text{BAT}} - \mathbf{a}_i^{\text{PV}} \leq \bar{\mathbf{O}}_i, \tag{11j}$$
$$\mathbf{i}_i^{\text{BAT}} + \mathbf{i}_i^{\text{PV}} \leq \bar{\mathbf{I}}_i, \tag{11k}$$

while $\mathcal{X}_0$ defines the constraints related to the budget recovery problem of the DSO, constraining the maximum value, $\bar{\mathbf{x}}_0$, of the tariff for each time step, and the average value throughout the day, $\tilde{\mathbf{x}}_0$, as follows :

$$\mathbf{0}_T \leq \mathbf{x}_0 \leq \bar{\mathbf{x}}_0, \tag{12a}$$
$$\sum_{h=1}^{T} x_{0,h} = \tilde{\mathbf{x}}_0. \tag{12b}$$

The lower bounds on $\mathbf{x}_i$ and $\sum_{t=1}^{T} \mathbf{x}_{i,t}$ are greater than 0 to take into account the non shiftable and not flexible portion of the load. The upper bound on $\mathbf{x}_i^{\text{EV}}$ is equal to 0 outside the time window $[\underline{t}, \bar{t}]$, that is when the EV is not plugged in. We run the simulations for $N = 20$ households and over a time horizon $T = 24$. The dynamic offtake and injection retail prices are computed based on dynamic retail contracts in Belgium [27], considering the wholesale day-ahead prices $\mathbf{p}$ registered by ENTSOE [28] on 07/02/2025, as follows:

$$\mathbf{p_O} = 0.1\mathbf{p} + 0.204, \tag{13a}$$
$$\mathbf{p_I} = 0.1\mathbf{p}. \tag{13b}$$

We employed real data of random days for the PVs' generation profiles available through [29]. Batteries and electric vehicles are considered to be alike for all the households and have the following parameters: $C^{\text{BAT}} = 6$KWh, $P^{\text{BAT}} = 2.2$KW, $\eta^{\text{BAT}} = 0.9$, $C^{\text{EV}} = 60$KWh, $P^{\text{EV}} = 22$KW, $\underline{d}^{\text{EV}} = 80\% C_i^{\text{EV}}$, and $\eta^{\text{EV}} = 0.95$. Moreover, for each household, the remaining parameters are randomly selected among the following ranges, considering average households in Belgium [30]: $\underline{x}_{i,t} \in [0.2, 0.4]\,\forall\, t$, $\underline{d}_i \in [100\%, 120\%] \sum_{t=1}^{m} \underline{x}_{i,t}$, $\bar{d}_i \in [120\%, 140\%] \sum_{t=1}^{m} \underline{x}_{i,t}$. We run the simulations for three different scenarios of users' price sensitivity. In Scenario 0, that is used as the reference one, all the users are considered to be neutral, with $\omega_i = 0.5\,\forall\, i$. In Scenario 1 and in Scenario 2, a minority of users (i.e., 25% of them) and a majority of users (i.e., 75% of them) are, respectively, considered to exhibit a low sensitivity to the price, with $\omega_i = 0.1$.
In Fig. 1, the $v-$GNE solutions that are reached by Algorithm 1 under these 3 scenarios are shown. In particular, from Fig. 1a we can observe that $\mathbf{x}_0^*$ is slightly affected by users' low sensitivity to the price, with no variations between Scenario 1 and Scenario 2. On the other side, in Fig. 1b, we can see that an increasing number of users having low sensitivity to the price leads to higher values of $\boldsymbol{\lambda}^*$. Indeed, the presence of users that are reluctant to adjust their consumption profiles makes it harder for the DSO to steer the aggregative consumption profile of the network below the capacity limits of the grid, as shown in Fig.1c. Note that in all the scenarios, $\boldsymbol{\lambda}^*$ differs from 0 only at times t when the coupling constraint is active, i.e., when the users' desired profiles – with no tariff intervention – would have exceeded the grid capacity limit. Since all the users react to the same network tariffs ($\mathbf{x}_0^*$, $\boldsymbol{\lambda}^*$), the presence of users

with low sensitivity to the price affects the entire network, and all the users end up paying more, regardless of their own sensitivity to the price. Moreover, in Fig. 1c, we can observe that the more users exhibit a low sensitivity to the price, the closer the final aggregative consumption, $\sum_{i=1}^{N} A_i \mathbf{z}_i^*$, is to the desired one, $\sum_{i=1}^{N} A_i \mathbf{r}_i$, without violating the coupling constraint.

Fig. 2 in log-log scale shows the convergence performances of our algorithm compared to those obtained when employing the well-known Forward-Backward (FB) algorithm [25]. The numerical simulation shows that Algorithm 1 converges faster than the FB algorithm, due to the fact that users adopt the best response dynamics.

V. Conclusion

In this paper, we have proposed a novel two-part pricing mechanism which allows a CC (such as a DSO) to simultaneously achieve the double goal of: i) guaranteeing the safe operation of the grid by preventing congestions, and ii) recuperating its investment budget. The interconnected CC and consumers problem is formulated as a generalized Nash equilibrium problem, where we have shown that the pricing structure can be derived from a hybrid best-response plus dual-ascent algorithm, with convergence guarantees, that solves a specific max-min problem reformulation. Moreover, we have considered the practical case that consumers' decisions can be driven by subjective preferences, and, hence, deviate from full economic rationality. The numerical results have showcased the convergence of the proposed algorithm and the effectiveness of the proposed tariff in steering the consumption behavior of the consumers to abide by the grid limitations. The simulations have also highlighted the impact of the price sensitivity of the users on the equilibrium tariff, where a lower sensitivity can drive the tariffs and, hence, the resulting costs for all the users, raising fairness concerns.

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