Optimal Circular Impact Time Guidance

Qindong Hu, Jiang Wang, Hongyan Li*, Shipeng Fan and Jianchuan Ye

***Abstract*— To address the guidance problem of intercepting targets at the desired impact time, an optimal circular impact time guidance law is proposed in this paper. The proposed approach is designed by augmenting circular guidance with the well-justified predictor-corrector concept. Different from the existing circular impact time guidance (CITG) laws, the proposed method exactly predicts the impact time of circular guidance without any linearization, and corrects it by minimizing a quadratic integral index proportional to the total control effort. Hence, the proposed approach is more efficient in regulating the impact time. Theoretical analysis is provided to reveal the convergence properties of the predicted impact time and heading error. Comparative simulations are performed to validate the availability and superiority of the proposed method.**

I. INTRODUCTION

The fundamental objective of guidance laws is to guide missiles for perfect interception [1]. However, with the increasing enhancement of enemy's defense systems, single missile combat encounters significant limitations in modern warfare. To enhance the penetration probability and damage effect of missiles, salvo attack is an efficient strategy to quickly saturate the enemy's anti-missile firepower system. Therefore, guidance methods utilized for controlling impact time have be extensively researched by many scholars in recent years [2-3].

Due to the simple implement and validity, proportional navigation guidance (PNG) and its modified forms are extensively adopted as the baseline term of impact time control guidance (ITCG) approaches [4-7]. The authors in [4] first designed an ITCG containing the well-known PNG baseline term and a biased term for achieving salvo attack in 2006. Based on the preceding studies, a generalized and rigorous solution of ITCG problem was derived leveraging pure proportional navigation guidance and high-order time-to-go estimation [8-9]. To avoid the large forecast deviation of time-to-go caused by small angle consumption, a varying-gain form of PNG is utilized for the design of ITCG methods [10-12]. Based upon the approximately predicted time-to-go under PNG laws, the authors in [13] designed an optimal ITCG command to nullify the time error leveraging optimal error dynamics and this work were further expanded to the three-dimensional form in [14]. However, the design and application of the aforementioned methods necessitate online estimation of time-to-go that predicted based on small angle assumption, numerical integration or other principles [15-18], and the forecast deviation seriously degrades the working performance of guidance laws.

For the exact prediction of time-to-go, circular guidance is introduced for derivation of impact time control guidance laws. Circular impact time guidance (CITG) was investigated based upon the geometric principle that restricts missiles to hit the designated target along a circular trajectory [19-21]. Based on predictor-corrector concept, the impact time control command derived utilizing sliding-mode theory or finite-time convergence concept was added to circular guidance for nullifying the time error [22-23]. For guidance problems considering multiple constraints, the authors in [16] proposed a generalized form of circular impact time guidance, deeply researching the circular guidance [24]. However, the preceding CITG methods were derived based on principles including geometry relationship, sliding-mode control theory or tracking concept, not possessing energy optimality.

Motivated by analysis of the current research status of ITCG problem, this paper proposes an optimal circular impact time guidance law. The main contribution of the finished work is embodied in the following two aspects. 1) The proposed approach is designed adopting circular guidance and time-to-go for online guidance can be accurately obtained without any linearization assumptions, thus avoiding the deficiency of decreased control accuracy caused by time forecast bias. 2) The proposed impact-time control command is derived utilizing optimal control theory. Compared with other similar circular impact time guidance laws in [18-22], the proposed method causes less energy expenditure, simultaneously maintaining the high regulating efficiency of impact time.

The remaining content and structure scheme of this paper can be summarized as follows. Model derivation and guidance problem to be addressed are introduced in Section II. The concept of circular guidance and derivation of the proposed approach are revealed in Section III and Section IV provides related theoretical analysis of convergence properties. In Section V, the availability and superiority of the proposed algorithm is verified via comparative simulations.

II. PRELIMINARY

A. Model Derivation

For the missile hitting a stationary target, the planar homing guidance geometry is exhibited in Fig.1, where M and T symbolize the missile and target, respectively. The missile's speed is denoted by V, assuming that it maintains consistency during the engagement process. The variables a_M, θ and η indicate the missile's lateral acceleration, flight-path angle and velocity lead angle, respectively. The relative

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range between the missile and target is denoted by r and λ is the line-of-sight (LOS) angle.

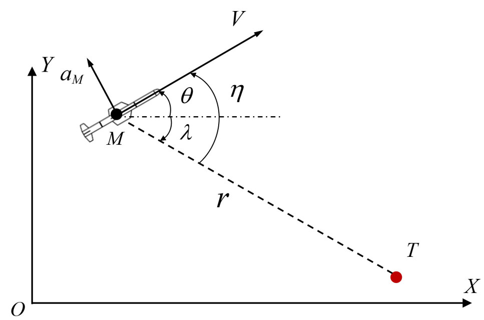


Fig. 1. The two-dimensional engagement geometry

The kinetic equations that represent the above engagement geometry are formulated as [23]

$$\frac{dr}{dt} = -V\cos\eta \tag{1}$$

$$\frac{d\lambda}{dt} = \frac{-V\sin\eta}{r} \tag{2}$$

$$\frac{d\theta}{dt} = \frac{a_M}{V} \tag{3}$$

The geometry relationship among the angles η, θ and λ can be concluded as

$$\eta = \theta - \lambda \tag{4}$$

Differentiating η with respect to t and combing with Eqs.(2)(3), the kinetic equation of lead angle is derived as

$$\frac{d\eta}{dt} = \frac{a_M}{V} + \frac{V\sin\eta}{r} \tag{5}$$

Based on the above dynamic model (1-5), the detailed derivation and related theoretical analysis of the proposed approach will be introduced subsequently.

B. Problem Formulation

In this subsection, the guidance problem with impact time constraint will be formulated. The basic purpose of guidance laws is guiding the missile to intercept the designated target. For successful interception, zero miss-distance must be realized, i.e.

$$r\left(t_f\right) = 0 \tag{6}$$

where t_f represents the instant of interception.

Based on the predictor-corrector concept, define the impact time error ε_t as the difference between the desired impact time and its predictive value, i.e.

$$\varepsilon_t = t_d - \hat{t}_f \tag{7}$$

where t_d and $\hat{t}_f$ represent the desired and predictive values of impact time, respectively.

Based on predictor-corrector concept, the guidance problem with impact time constraint can be transformed into the tracking problem nullifying the impact time error. Hence, the terminal time constraint is rewritten as

$$\varepsilon_t(t_f) = 0 \tag{8}$$

Therefore, this paper aims to design an ITCG method to address the guidance problem with constraints (6) and (8) based on the two-dimensional model (1-5).

III. DESIGN OF PROPOSED GUIDANCE LAW

The proposed method composed of two terms will be derived analytically in this section. For perfect interception, the baseline term is designed based on circular guidance to satisfy constraint (6). Then, time-to-go and final time are predicted accurately. Based on predictor-corrector concept and optimal control theory, the bias command nullifying the predicted error will be derived to cater for constraint (8). Therefore, the proposed algorithm is concluded as

$$a_M = a_c + a_b \tag{9}$$

where the first term a_c is designed to guarantee precise strike and the command a_b can satisfy the impact time constraint by regulating the error between the desired impact time and its predictive value.

A. Circular Navigation Guidance

Before designing the optimal guidance command for regulating the predicted time error, the concept of circular guidance is introduced briefly in this subsection. Just as its name implies, the missile guided by circular guidance will intercept the target following a circular trajectory. The engagement geometry under circular guidance is depicted in Fig.2, where O_c and R respectively represent the centre and radius of the circular acr. The notation S_{go} stands for the length of the remainder trajectory.

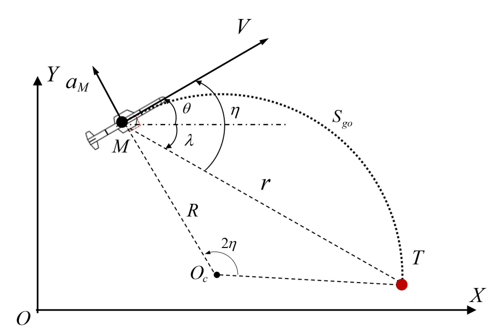


Fig. 2. Engagement geometry of circular guidance

As indicated in Fig.2, the geometry relationships of circular guidance are readily formulated as

$$R = \frac{r}{2\left|\sin\eta\right|} \tag{10}$$

$$S_{go} = R \cdot \angle MO_cT = \frac{r\eta}{\sin\eta} \tag{11}$$

Without any linearization assumptions, the accurate time-to-go of constant-speed missiles under circular guidance can be obtained as

$$t_{go} = \frac{S_{go}}{V} = \frac{r\eta}{V\sin\eta} \tag{12}$$

For intercepting stationary targets, the acceleration command of circular guidance for successful interception, i.e., satisfying constraint (6) is given as [20]

$$a_c = 2V\dot{\lambda} = -\frac{2V^2\sin\eta}{r} \tag{13}$$

B. Optimal Guidance Command for Impact-time Constraint

Based on prediction-correction and optimal control theory, the biased command a_b will be derived in this subsection to nullify the error between t_d and $\hat{t}_f$. Combing Eqs.(5)(9) with Eq.(13), the lead angle kinetics determined by a_M is formulated as

$$\dot{\eta} = -\frac{V\sin\eta}{r} + \frac{a_b}{V} \tag{14}$$

Considering the effect of biased term a_b, the predicted time-to-go $\hat{t}_{go}$ can be obtained by Eq.(12) and the predictive value of terminal time is derived as

$$\hat{t}_f = \hat{t}_{go} + t = \frac{r\eta}{V\sin\eta} + t \tag{15}$$

Differentiating Eq.(15) to time t and combining with Eqs.(1)(12)(14), we can obtain

$$\dot{\hat{t}}_f = \frac{\hat{t}_{go}}{V}\frac{\sin\eta - \eta\cos\eta}{\eta\sin\eta}a_b \tag{16}$$

Define the virtual control input u as

$$u = \frac{\sin\eta - \eta\cos\eta}{\eta\sin\eta}a_b \tag{17}$$

For the derivation of a_b, the optimal guidance problem can be formulated as

$$\begin{array}{ll} \min & J = \frac{1}{2}\int_t^{t_f} \frac{u^2}{\hat{t}_{go}^n}d\tau \\ s.t. & \dot{\hat{t}}_f = \frac{\hat{t}_{go}}{V}u \\ & \varepsilon_t\left(\hat{t}_f\right) = 0 \end{array} \tag{18}$$

where $n \geq 0$ is the weighting coefficient to be designed. The performance index can be considered as the energy minimization weighted by $1/\hat{t}_{\mathrm{go}}^n$. It can be clearly observed that the solution of the above problem with $n = 0$ can realize energy optimality. The weight function with $n > 0$ will reach infinity as $\hat{t}_{\mathrm{go}} \to 0$ guaranteeing that $u = 0$ at the instant of interception [25].

For the preceding optimization problem, the analytical command satisfying the impact-time constraint can be readily derived based on optimal control theory as

$$a_b = \frac{(n+3)V\varepsilon_t}{\hat{t}_{go}^2}\frac{\eta\sin\eta}{\sin\eta - \eta\cos\eta} \tag{19}$$

For the detailed derivation of the optimal command a_b, please refer to Appendix.

Accordingly, the optimal circular guidance method satisfying terminal time constraint developed in this paper can be summarized as

$$a_M = -\frac{2V^2\sin\eta}{r} + \frac{(n+3)V\varepsilon_t}{\hat{t}_{go}^2}\frac{\eta\sin\eta}{\sin\eta - \eta\cos\eta} \tag{20}$$

IV. ANALYSIS OF PROPOSED GUIDANCE LAW

The necessary condition for precise strike is the quasi-parallel approach principle, which means that the look angle must converge to zero guaranteeing missiles will fly straightly towards the target finally. Additionally, the predicted impact-time error also requires zero convergence property to satisfy terminal time constraint. In this section, the theoretical analysis for convergence properties of predicted error and look angle will be provided.

A. Convergence Analysis of Impact-time Error

Define the moment $\hat{t}_{go} = 0$ as the final time. Differentiating Eq.(7) and combining with Eqs.(16)(19), we can obtain the error dynamics determined by a_M as

$$\dot{\varepsilon}_t = -\frac{(n+3)\varepsilon_t}{\hat{t}_{go}} \tag{21}$$

The analytical solution of ε_t can be readily derived as

$$\varepsilon_t = \varepsilon_{t,0}\left(\frac{\hat{t}_{go}}{\hat{t}_{f,0}}\right)^{n+3} \tag{22}$$

where $\varepsilon_{t,0}$ represents the initial value of the predicted error and the predicted terminal time $\hat{t}_{f,0}$ at the initial moment has a bounded value. The analytical solution indicates that the impact-time error can decrease to zero with $\hat{t}_{\mathrm{go}} \to 0$, satisfying constraint (8) and its convergence speed relies on the weight coefficient n.

B. Convergence Analysis of Lead Angle

Combing Eqs.(7), (15) and (22), one can obtain

$$\frac{\sin\eta}{\eta} = \frac{r\left[1 + \frac{\varepsilon_{t,0}}{\hat{t}_{f,0}}\left(\frac{\hat{t}_{go}}{\hat{t}_{f,0}}\right)^{n+2}\right]}{V\left(t_d - t\right)} \tag{23}$$

where the term $\frac{\sin\eta}{\eta}$ keeps bounded with the limited domain $\eta \in (-\pi, \pi)$. Hence, the relative range r must converge to zero with $\hat{t}_{\mathrm{go}} \to 0$ to ensure that Eq.(23) holds true, satisfying perfect interception constraint (6).

Utilizing L'Hôpital's rule and dynamic equations, Eq.(23) can be reformulated as

$$\begin{aligned} \lim_{\hat{t}_{go}\to 0}\frac{\sin\eta}{\eta} &= \lim_{\hat{t}_{go}\to 0}\frac{r\left[1 + \frac{\varepsilon_{t,0}}{\hat{t}_{f,0}}\left(\frac{\hat{t}_{go}}{\hat{t}_{f,0}}\right)^{n+2}\right]}{V\left(t_d - t\right)} \\ &= \lim_{\hat{t}_{go}\to 0}\cos\eta \end{aligned} \tag{24}$$

Denote the terminal value of the lead angle as the variable η_f. Then, concluding from Eq.(24), we can obtain the condition as

$$\frac{\sin\eta_f}{\eta_f} - \cos\eta_f = 0 \tag{25}$$

With the limited domain of $\eta \in (-\pi, \pi)$, the unique solution of Eq.(25) is readily derived as $\eta_f = 0$. Therefore, the lead angle determined by (20) with $n > 0$ will convergence to zero at the instant of interception with $\hat{t}_{\mathrm{go}} = 0$.

V. NUMERICAL SIMULATION

In this section, two different simulations are provided to show the feasibility and advantages of the method (20) developed in this paper. In the first scenario, various impact time constraints are imposed to evaluate the performance of the proposed algorithm. The comparative simulation among the proposed approach and two existing ITCG methods is offered in the second scenario. The initial values of parameters used for simulations are indicated in Table I. In all simulations, the maximum acceleration command is designed as 50 m/s^2 and the simulation will end if the relative distance is less than 1m.

TABLE I

INITIAL CONDITIONS FOR SIMULATIONS

Parameter	Value
Position of the target, (x_t, y_t), km	$(10, 0)$
Position of the missile, (x_{m0}, y_{m0}), km	$(0, 0)$
Velocity of the missile, V, m/s	300
Flight-path angle of the missile, θ, deg	60

A. Different Impact-time Constraints

In scenario 1, the proposed approach (20) with $n = 1$ is employed to verify its availability. The desired impact time t_d is constrained from $50s$ to $90s$ at $10s$ interval. The simulation results are shown in Fig.3.

The results indicate that the proposed algorithm with the designed parameters can guide the missile to intercept the target successfully with different impact time constraints. With the increasing of t_d, the missile will fly along a more curved trajectory, generating a greater maximum value of the lead angle that converges to zero at the instant of strike. Additionally, the initial acceleration command will increase with a larger predicted error but it always keep bounded during all the engagement process, as shown in Fig.4(c). Under the biased term of (20), the zero terminal impact-time error can be achieved with various t_d that agrees with the convergence proof in the preceding section. Therefore, missiles guided by the proposed approach (20) can realize perfect strike at the prescribed impact time.

B. Comparsion with Existing Impact-time Control Methods

To show its superiority, the proposed guidance law is compared with the fix-time circular impact time guidance (FCITG) in [23] and the optimal impact time control guidance law (OITCG) in [13] for impact time constraint. For the fairness of comparison, the parameters involved in the following commands is designed as values of original references.

The acceleration command of FCITG is given as [23]

$$a_M = -\frac{2V^2\sin\eta}{r} + \frac{pV^2\sin\eta}{r(1-\eta\cot\eta)}\Xi(\varepsilon_t) \tag{26}$$

where $\Xi(\varepsilon_t) = |\varepsilon_t|^{1-q}\mathrm{sign}(\varepsilon_t) + \varepsilon_t + |\varepsilon_t|^{1+q}\mathrm{sign}(\varepsilon_t)$ with $q = 1/3$ and the guidance gain is designed as $p = 0.2$.

The acceleration command of OITCG is given as [13]

$$a_M = \frac{NV^2}{r}\sin\eta + \frac{K(2N-1)V^2\psi(\eta)}{r\sin\eta t_{go,d}}\varepsilon_t \tag{27}$$

$$\psi(\eta) = \begin{cases} 1, |\eta| \geq \eta_m \\ \frac{1}{2} - \frac{1}{2}\cos\left(\pi\left|\frac{\eta}{\eta_m}\right|^{\mu}\right), |\eta| < \eta_m \end{cases} \tag{28}$$

where $N = 3$, $K = 4$, $t_{go,d} = t_d - t$ is the desired time-to-go and the auxiliary function $\psi(\eta)$ avoiding the command singularity problem is design with $\mu = 2$ and $\eta_m = 0.1°$.

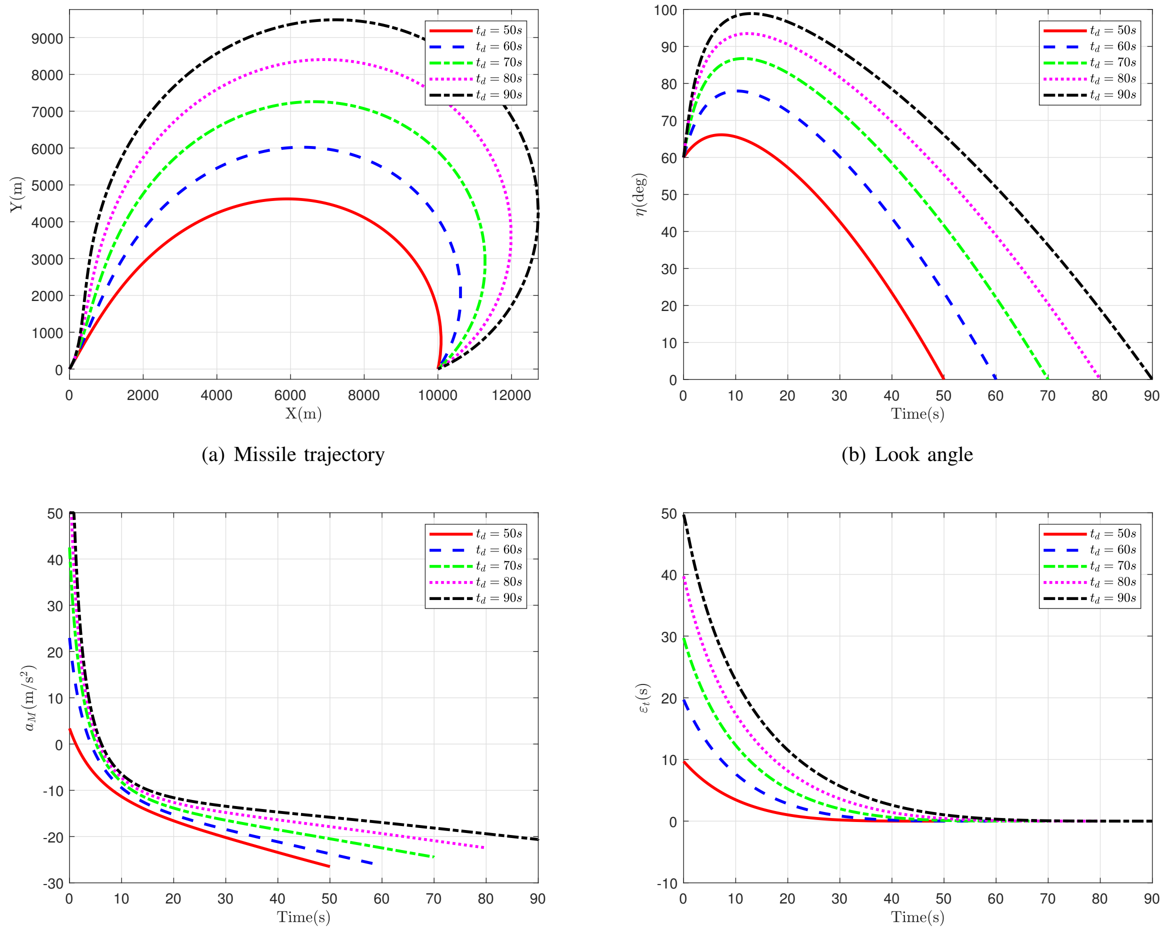
To compare the performance of these three different methods, the energy cost function $J = \int_0^{t_f} a_M^2 dt$ and the final impact-time error $\varepsilon_{tf} = |t_d - t_f|$ is introduced for evaluating both the control expenditure and accuracy of the missile. The desired impact time is designed as $t_d = 60s$. The relevant simulations and index comparison are indicated in Fig.4 and Table II, respectively.

TABLE II

COMPARISON OF ENERGY COST AND FINAL IMPACT TIME ERROR

Guidance law	Energy cost J(m^2/s^3)	Final impact-time error ε_{tf}(s)
Proposed	20489	0.001
FCITG	24333	0.004
OITCG	23641	0.056

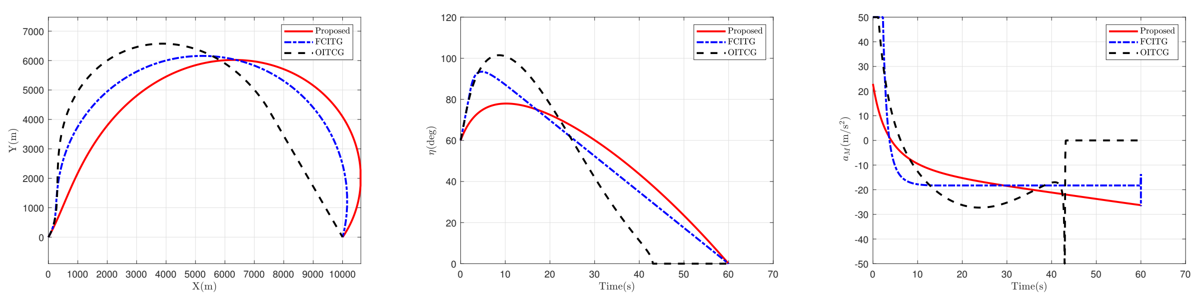
As can be observed in Fig.4, all these three methods can realize perfect strike. However, compare with other two laws, the proposed algorithm requires less energy cost and causes lower final impact-time error, as shown is Table II. FCITG law is derived utilizing circular guidance and fix-time tracking concept, not possessing optimality. Also, due to the rapid convergence rate of the lead angle, its biased term will cause command singularity at the instant of interception. Although OITCG law is designed based on optimal error dynamics, it will cause trajectory deviation and redundant energy cost due to its inaccurate estimation on time-to-go. Additionally, when the look angle gradually decreases to zero, ITCG term of OITCG law becomes less ineffective, thus causing a larger final error. Therefore, the comparative results represent that the proposed approach (20) in this paper can realize higher impact-time control accuracy and requires less energy expenditure than both FCITG method in [23] and OITCG law in [13].



(a) Missile trajectory (b) Look angle

(c) Acceleration command (d) Impact-time error

Fig. 3. Simulation results of the proposed guidance law with different impact-time constraints



(a) Missile trajectory (b) Look angle (c) Acceleration command

Fig. 4. Simulation results of different ITCG laws

VI. CONCLUSIONS

In this paper, an optimal circular impact time guidance law is proposed to address the guidance problem with terminal time constraint. Circular guidance is introduced for designing the baseline term due to its exact estimation of time-to-go. Then, the guidance command used for nullifying the predicted error is derived by minimizing a quadratic integral index proportional to the total control effort. Related theoretical analysis is provided to confirm the convergence properties of the predicted error and lead angle. Extensive simulations are also offered to testify the feasibility and advantages of the proposed algorithm.

APPENDIX

The Hamiltonian function of (17) can be formulated as

$$H = \frac{1}{2}\frac{u^2}{\hat{t}_{go}^n} + \lambda \frac{\hat{t}_{go}}{V} u \tag{29}$$

where the variable λ is the Lagrange multiplier.

The first-order optimality condition and costate equation can be given as

$$\frac{\partial H}{\partial u} = \frac{u}{\hat{t}_{go}^{n}} + \lambda \frac{\hat{t}_{go}}{V} = 0 \tag{30}$$

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial \hat{t}_f} = 0 \tag{31}$$

The control input u can be derived from Eq.(30) as

$$u = -\lambda \frac{\hat{t}_{go}^{n+1}}{V} \tag{32}$$

Combing Eq.(32) with the state equation of optimization problem (17) and integrating on both sides yields

$$\hat{t}_f\left(t_f\right) - \hat{t}_f(t) = -\frac{\lambda}{V^2}\frac{\hat{t}_{go}^{n+3}}{n+3} \tag{33}$$

Imposing the zero impact-time error at the terminal time, i.e., $\varepsilon_t(t_f) = t_d - \hat{t}_f(t_f) = 0$, the Lagrangian multiplier λ can be obtained as

$$\lambda = -\frac{(n+3)V^2\varepsilon_t}{\hat{t}_{go}^{n+3}} \tag{34}$$

Combing Eq.(34) with Eq.(32) yields the optimal solution of the problem (18) as

$$u = \frac{(n+3)V\varepsilon_t}{\hat{t}_{go}^{2}} \tag{35}$$

Accordingly, the optimal guidance command regulating the impact-time error is derived as

$$a_b = \frac{(n+3)V\varepsilon_t}{\hat{t}_{go}^{2}}\frac{\eta\sin\eta}{\sin\eta - \eta\cos\eta} \tag{36}$$

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