NEW APPROACH FOR THE IDENTIFICATION OF A CLASS OF TIME-VARYING PARAMETERS DYNAMIC SYSTEMS

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Abstract. This paper presents a novel method for the identification of a class of dynamic systems with time-varying parameters. The targeted class includes uncertain systems with a one-to-one correspondence between state equations and unknown parameters. The approach leverages the swapping lemma and a series of filters to reformulate the parameter identification problem as a state observer design problem. Unknown parameters are determined by solving nonlinear equations using the continuous-time Newton's method. For clarity, the proposed technique is demonstrated through two illustrative examples. Simulation results indicate that the estimated parameters converge to their true values almost instantaneously, with a tracking error that can be reduced to an arbitrarily small level.

INTRODUCTION

The identification of time-varying parameter systems is still an open problem. A general method similar to that for constant parameter systems has yet to be determined. In the meantime, solving this problem for particular classes of systems is an important and desirable step. A literature review suggests several approaches for various classes of time-varying parameters systems.

The approach to the identification of linear time-varying parameter systems proposed in [1] is similar to that for linear time-invariant parameter systems. The adaptation law is designed using the gradient projection method and involves a projection operator. The main limitation of the approach is that the derivatives of the unknown parameters are considered as disturbances, unlike the method proposed in this paper. The least squares (LS) method is proposed in the literature for time-varying parameter systems as well. In [2] a non-recursive LS method is used for time-varying parameters. A polynomial approximation based on Taylor expansion with a bounded regressor vector is built and used to approximate the time-varying parameters. In [3] a new matrix forgetting factor recursive LS algorithm is proposed for time-varying parameters assuming a random walk model. Adaptive-like estimation approaches can also be found in the literature. In [4] a modified version of the LS algorithm is provided to estimate time-varying parameters by means of a polynomial approximation. Most of the works aforementioned are only able to follow slowly varying parameters. They can only ensure convergence to a neighborhood of the true value. A recursive Finite Time convergent algorithm has been presented in [5]. It is a nonlinear recursive version of the LS algorithm and is based on the generalized Super-Twisting Algorithm (STA) [6]. Other methods such as the extended Kalman filter, modulating functions and subspaces are also presented in the literature [7]. The methods presented in this literature review are based on numerous assumptions on the dynamics of the unknown time-varying parameters. These assumptions considerably limit their application to practical engineering cases.

This paper puts forward an innovative approach for the identification of a class of dynamic systems with time-varying parameters. This class is composed of uncertain systems that have as many state equations as unknown parameters. The main contribution of the paper is that the swapping lemma and a series of filters are used to convert the parameter identification problem into a state observer design problem. The unknown parameters are obtained by solving nonlinear equations using the continuous-time Newton's method. The proposed method is presented using two illustrative examples followed by simulation results.

The paper is organised as follows. Section 1 presents the proposed method using an illustrative example for time-varying parameter systems whose parameters appear linearly in their models. Section 3 presents the method for systems that are nonlinear in their parameters. Simulation results are presented in sections 2 and 4. The paper ends with a conclusion.

I. FIRST ILLUSTRATIVE EXAMPLE

This section presents the proposed identification method using an illustrative example. Consider the following state space representation of a linear time-varying parameter second order system.

$$\frac{d}{dt}x_1(t) = -x_1(t) + a(t)x_2(t) + b(t)u(\text{t}) \quad (1)$$

$$\frac{d}{dt}x_2(t) = -x_2(t) + b(t)x_1(t) + a(t)u(\text{t}) \quad (2)$$

The state variables are x_1 and x_2 and they are assumed measurable. The input u(t) is also measurable. a(t) and b(t) are two unknown time-varying parameters. The objective is to propose an algorithm to estimate a(t) and b(t). First, consider (1) and add $\sigma_1 x_1(t)$ on both sides of the equality sign. The parameter σ_1 is any known positive real number. The following equation is obtained.

$$\frac{d}{dt}x_1(t) + \sigma_1 x_1(t) = -(1-\sigma_1)x_1(t) + a(t)x_2(t) + b(t)u(\text{t}) \quad (3)$$

Next, use $\frac{1}{s+\sigma_1}$ to filter both sides of (3). This results in the following equation.

$$x_1(t) = -(1-\sigma_1)\frac{1}{s+\sigma_1}[x_1] + \frac{1}{s+\sigma_1}[ax_2] + \frac{1}{s+\sigma_1}[bu] \quad (4)$$

Where the notation $\frac{1}{s+\sigma_1}[y]$ indicates that the signal y is filtered using $\frac{1}{s+\sigma_1}$. Next, (4) is expanded using the swapping lemma [8]. This results into the following equation,

$$x_1(t) = -(1-\sigma_1)\frac{1}{s+\sigma_1}[x_1] + a\frac{1}{s+\sigma_1}[x_2] + b\frac{1}{s+\sigma_1}[u] - \frac{1}{s+\sigma_1}[\dot{a}\frac{1}{s+\sigma_1}[x_2] + \dot{b}\frac{1}{s+\sigma_1}[u]] \quad (5)$$

The followings equations are the state space representations of the filters $\frac{1}{s+\sigma_1}[*]$ that appear is (5).

$$\dot{v}_{1x1} = -\sigma_1 v_{1x1} + x_1(t) \quad (6.1)$$

$$\dot{v}_{1x2} = -\sigma_1 v_{1x2} + x_2(t) \quad (6.2)$$

$$\dot{v}_{1u} = -\sigma_1 v_{1u} + u(t) \quad (6.3)$$

$$\dot{w}_1 = -\sigma_1 w_1 - v_{1x2}\dot{a} - v_{1u}\dot{b} \quad (6.4)$$

The equation (5) can therefore be rewritten into the following form.

$$x_1(t) = -(1-\sigma_1)v_{1x1} + v_{1x2}a + v_{1u}b + w_1 \quad (7)$$

Next, the same transformation is applied to (2). The term $\sigma_2 x_1(t)$ is added on both sides of the equality sign and the resulting equation is filtered using $\frac{1}{s+\sigma_2}$. This results into the following equation

$$x_2(t) = -(1-\sigma_2)\frac{1}{s+\sigma_2}[x_2] + b\frac{1}{s+\sigma_2}[x_1] + a\frac{1}{s+\sigma_2}[u] - \frac{1}{s+\sigma_2}[\dot{b}\frac{1}{s+\sigma_2}[x_1] + \dot{a}\frac{1}{s+\sigma_2}[u]] \quad (8)$$

The parameter σ_2 is any positive real number different from σ_1 . The following state space representations of the filtered signals are used.

$$\dot{v}_{2x1} = -\sigma_2 v_{2x1} + x_1(t) \quad (9.1)$$

$$\dot{v}_{2x2} = -\sigma_2 v_{2x2} + x_2(t) \quad (9.2)$$

$$\dot{v}_{2u} = -\sigma_2 v_{2u} + u(t) \quad (9.3)$$

$$\dot{w}_2 = -\sigma_2 w_2 - v_{2x1}\dot{b} - v_{1u}\dot{a} \quad (9.4)$$

Equation (8) is then rewritten equivalently as

$$x_2(t) = -(1-\sigma_2)v_{2x2} + v_{2x1}b + v_{2u}a + w_2 \quad (10)$$

The following variables are defined to rewrite equations (7) and (10) in a compact form

$$W = (w_1 \quad w_2)^T \qquad \theta = (a \quad b)^T$$

$$A = \begin{bmatrix} -\sigma_1 & 0 \\ 0 & -\sigma_2 \end{bmatrix} \quad \Psi = \begin{bmatrix} v_{1x2} & v_{1u} \\ v_{2u} & v_{2x1} \end{bmatrix}$$

$$Y = (x_1 + (1-\sigma_1)v_{1x1} \quad x_2 + (1-\sigma_2)v_{2x2})^T$$

Consequently, an equivalent representation of the original system described by equations 1 and 2 is obtained and has the following compact state space representation.

$$\frac{d}{dt}W = AW - \Psi\frac{d}{dt}\theta \quad (10)$$

$$Y = \Psi\theta + W \quad (11)$$

The variable W is the new state vector. It is no longer measurable because of the way its components are defined at equations 6.4 and 9.4. Y is the system output. Its components are measurable. The matrix of parameters A is completely known and Hurwitz. The matrix Ψ is composed of signals that are available. It's therefore a know matrix. The state space representation is still uncertain since the vector of unknown parameters θ and its derivative appear in the state and output equations.

Our objective is to use this new representation of the system to be identified to find the unknown parameters a(t) and b(t). If we are able to find an estimate of the state vector W then equation (11) could be used to compute θ. We have therefore changed the parameter identification problem into a state observer design problem followed by a resolution of a linear equation.

The structure of equations 10 and 11 suggests obtaining a dynamic equation to estimate θ from equation 11 and then substituting these dynamics into the dynamic equation to estimate W. The implementation of this idea leads to the following algorithm to estimate the unknown time-varying vector parameter $\theta = (a \quad b)^T$.

$$\frac{d}{dt}\widehat{W} = A\widehat{W} - \rho\{Y - \widehat{W} - \Psi\hat{\theta}\} \qquad (12)$$

$$\frac{d}{dt}\hat{\theta} = \rho\Psi^{-1}(Y - \widehat{W} - \Psi\hat{\theta}) \qquad (13)$$

$\widehat{W}$ is an estimate for the state vector $W = (w_1 \quad w_2)^T$ and $\hat{\theta}$ is an estimate for θ. The parameter ρ is any positive real number. Equations 12 and 13 are respectively a state observer and a continuous-time Newton's algorithm. For the observer to be stable, the design parameter ρ, σ_1 and σ_2 should be such that the matrix ($A - \rho$I) is Hurwitz.

Note that when $\theta = (a \quad b)^T$ is constant then its derivative is zero. Consequently, equation 10 will not appear in the equivalent representation of the original system since equations 6.4 and 9.4 will not be required. Moreover, the state vector W will not appear in Equation 11. The parameter estimation algorithm reduces to the following equation.

$$\frac{d}{dt}\hat{\theta} = \rho\Psi^{-1}(Y - \Psi\hat{\theta}) \qquad (14)$$

The proposed parameter identification approach is evaluated in simulation at the next section.

II. SIMULATION RESULTS

A simulation is used to validate the performance of the proposed algorithm. The followings are used to simulate the unknown parameters. $a(t) = 2 + \sin(50t)$ and $b(t) = 4 + \sin(3t)$ The design parameters are $\rho = 1000$ and $\sigma_1 = 10\rho$ and $\sigma_2 = 12\rho$. They have been chosen to ensure that the matrix ($A - \rho$I) is Hurwitz and that the Newton's algorithm (Equation 13) converges very quickly.

First, the identification is carried out when the system is operating in open loop. The input signal is a unit step in this case. Simulation results are illustrated below. The figures show the system state variables, true parameters and estimated parameters. One can see that estimated parameters converge and perfectly track the true parameters. The identification algorithm is activated at t = 0.1 second. The initial conditions for the estimated parameters are set to zero. These results clearly demonstrate that the proposed algorithm is able to identify the class of time-varying parameter systems.

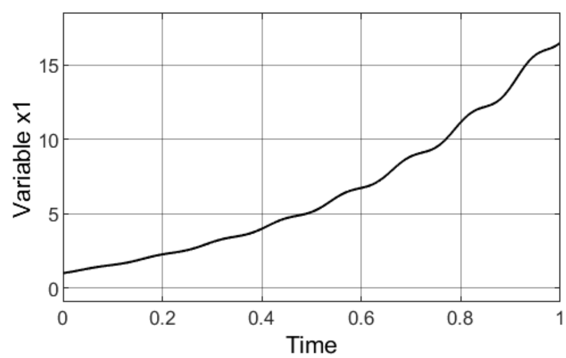

Fig. 1: Example 1: Variable x1 when the system is in open loop

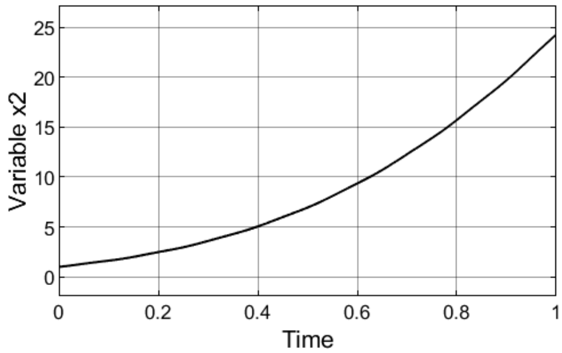

Fig. 2: Example 1: Variable x2 when the system is in open loop

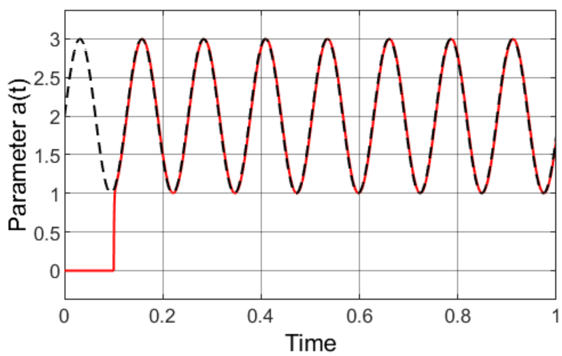

Fig. 3: Example 1: True and estimated parameters a(t) when the system is in open loop

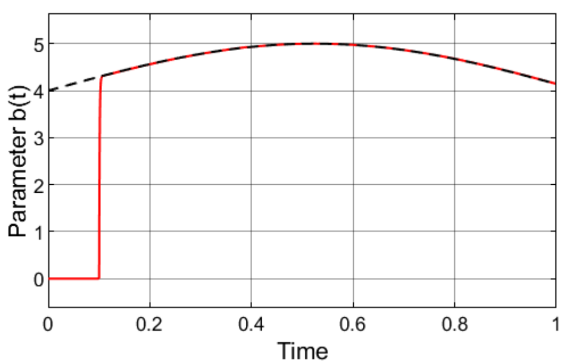

Fig. 4: True and estimated parameters b(t) when the system is in open loop

The second identification is carried out when the system is operating in a closed loop. The input equation is a state feedback control of the form.

$$u = -20x_1 - 30x_2 + 1$$

The following figures shows the simulation results. The system state variables, true parameters and estimated parameters are again illustrated. The results clearly show that estimated parameters perfectly track true parameters. Therefore the proposed algorithm works also well when the identification is performed in closed loop.

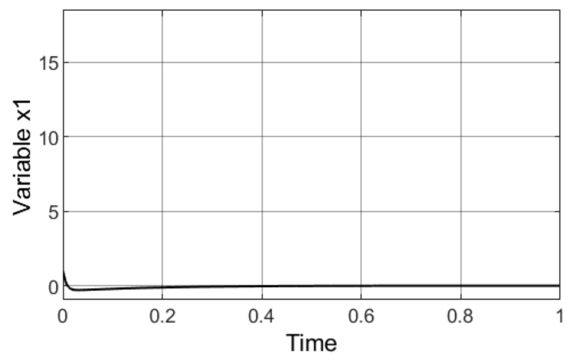

Fig. 5: Example 1: Variable x1 when the system is in closed loop

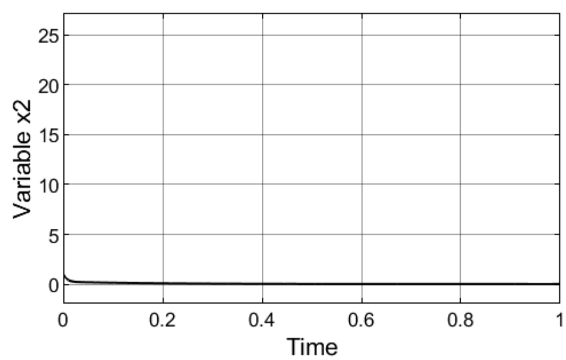

Fig. 6: Example 1: Variable x2 when the system is in closed loop

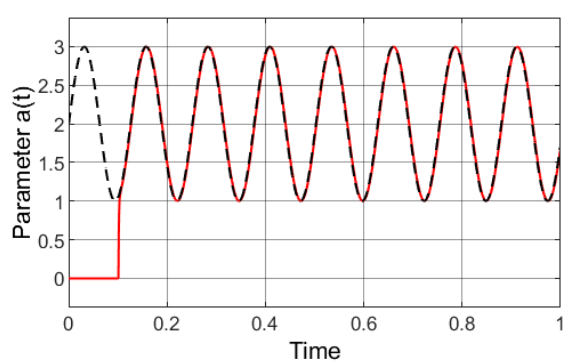

Fig. 7: Example 1: True and estimated parameters a(t) when the system is in closed loop

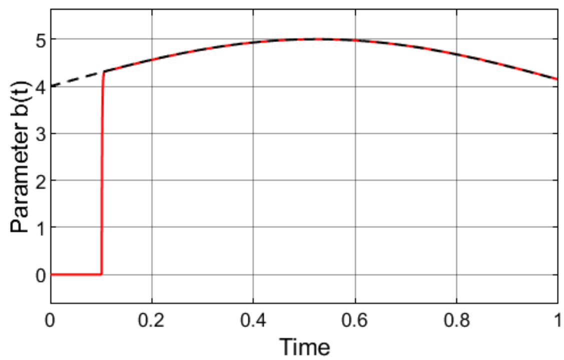

Fig. 8: Example 1: True and estimated parameters b(t) when the system is in closed loop

III. SECOND ILLUSTRATIVE EXAMPLE

The second illustrative example is presented in this section. The system in the previous example was linear in its parameters. The example in this section is nonlinear in its parameters. Our aim is to show that the proposed method can also be applied to this class of system with time-varying parameters, as long as the system has as many unknown parameters as it has state equations. Consider now the following second order system

$$\frac{d}{dt}x_1(t) = -x_1(t) + a(t)^2 x_2(t) + \ b(t)^2 u(\mathrm{t}) \quad (15)$$

$$\frac{d}{dt}x_2(t) = -x_2(t) + b(t)x_1(t) + \ a(t)b(t)u(t) \quad (16)$$

The state variables are x_1 and x_2. The input is u. a(t) and b(t) are two unknown time-varying parameters. Note that $a(t)^2$ and $b(t)^2$ appear in Equation 15, while $a(t)b(t)$ appears in Equation 16. The system is indeed nonlinear in its parameters and has two unknown time-varying parameters. The objective is to propose an algorithm to estimate a(t) and b(t). First, consider (15) and add $\sigma_1 x_1(t)$ on both sides of the equality sign. The parameter σ_1 is any known positive real number.

$$\frac{d}{dt}x_1(t) + \sigma_1 x_1(t) = -(1-\sigma_1)x_1(t) + a(t)^2 x_2(t) + \ b(t)^2 u(\mathrm{t}) \quad (17)$$

Next, $\frac{1}{s+\sigma_1}$ use to filter both sides of (2). This results in the following expression.

$$x_1(t) = -(1-\sigma_1)\frac{1}{s+\sigma_1}[x_1] + \frac{1}{s+\sigma_1}[a^2x_2] + \frac{1}{s+\sigma_1}[b^2u] \quad (18)$$

Where the notation $\frac{1}{s+\sigma_1}[y]$ indicate that the signal y is filtered using $\frac{1}{s+\sigma_1}$. Next, (18) is expanded using the swapping lemma [8]. This yields,

$$x_1(t) = -(1-\sigma_1)\frac{1}{s+\sigma_1}[x_1] + a^2\frac{1}{s+\sigma_1}[x_2] + b^2\frac{1}{s+\sigma_1}[u] - \frac{1}{s+\sigma_1}\left[2a\dot{a}\frac{1}{s+\sigma_1}[x_2] + 2\dot{b}b\frac{1}{s+\sigma_1}[u]\right] \quad (19)$$

The followings are state space representation of filters that appear is (19).

$$\dot{v}_{1x1} = -\sigma_1 v_{1x1} + x_1(t) \quad (20.1)$$

$$\dot{v}_{1x2} = -\sigma_1 v_{1x2} + x_2(t) \quad (20.2)$$

$$\dot{v}_{1u} = -\sigma_1 v_{1u} + u(t) \quad (20.3)$$

$$\dot{w}_1 = -\sigma_1 w_1 - 2av_{1x2}\dot{a} - 2bv_{1u}\dot{b} \quad (20.4)$$

The equation (19) can therefore be written as followings

$$x_1(t) = -(1-\sigma_1)v_{1x1} + v_{1x2}a^2 + v_{1u}b^2 + w_1 \quad (21)$$

Next, the same transformation is applied to (16). The term $\sigma_2 x_1(t)$ is added on both sides of the equality sign. The resulting equation is filtered using $\frac{1}{s+\sigma_2}$ and then expanded using the swapping lemma. The result is the following equation

$$x_2(t) = -(1-\sigma_2)\frac{1}{s+\sigma_2}[x_2] + b\frac{1}{s+\sigma_2}[x_1] + ab\frac{1}{s+\sigma_2}[u] - \frac{1}{s+\sigma_2}\left[\dot{b}\frac{1}{s+\sigma_2}[x_1] + \dot{a}b\frac{1}{s+\sigma_2}[u] + \dot{b}a\frac{1}{s+\sigma_2}[u]\right] \quad (22)$$

The parameter σ_2 is any known positive real number different from σ_1 . The following state representations of the filtered signals are used.

$$\dot{v}_{2x1} = -\sigma_2 v_{2x1} + x_1(t) \quad (23.1)$$

$$\dot{v}_{2x2} = -\sigma_2 v_{2x2} + x_2(t) \quad (23.2)$$

$$\dot{v}_{2u} = -\sigma_2 v_{2u} + u(t) \quad (23.3)$$

$$\dot{w}_2 = -\sigma_2 w_2 - (v_{2x1} + a v_{1u})\dot{b} - b v_{1u}\dot{a} \qquad (23.4)$$

Equation (22) is rewritten as follows

$$x_2(t) = -(1-\sigma_2)v_{2x2} + v_{2x1}b + v_{2u}ab + w_2 \qquad (24)$$

The following variables are defined to rewrite (21 and 24) in a compact form.

$$W = (w_1 \quad w_2)^T \ \theta = (a \quad b)^T$$

$$A = \begin{bmatrix} -\sigma_1 & 0 \\ 0 & -\sigma_2 \end{bmatrix} \ \mathrm{F}(\theta) = \begin{bmatrix} v_{1x2}a^2 + v_{1u}b^2 \\ v_{2x1}b + v_{2u}ab \end{bmatrix}$$

$$Y = (x_1 + (1-\sigma_1)v_{1x1} \quad x_2 + (1-\sigma_2)v_{2x2})^T$$

Consequently, an equivalent representation of (15 and 16) is

$$\frac{d}{dt}W = AW - (\frac{\partial}{\partial\theta}F(\theta))\frac{d}{dt}\theta \qquad (25)$$

$$Y = \mathrm{F}(\theta) + \mathrm{W} \qquad (26)$$

The following algorithm is proposed to estimate the unknown time-varying vector parameter $\theta = (a \quad b)^T$.

$$\frac{d}{dt}\widehat{W} = A\widehat{W} - \rho\{Y - \widehat{W} - \mathrm{F}(\hat{\theta})\} \qquad (27)$$

$$\frac{d}{dt}\hat{\theta} = \rho(\frac{\partial}{\partial\theta}F(\hat{\theta}))^{-1}(Y - \widehat{W} - \mathrm{F}(\hat{\theta}))$$

$$\hat{\theta}(0)\mathit{such\ that}\ (\frac{\partial}{\partial\theta}F(\widehat{\theta(0)}))^{-1}\text{exists.}$$

$\widehat{W}$ is an estimate for the vector $W = (w_1 \quad w_2)^T$ and $\hat{\theta}$ is an estimate for θ. Note that when $\theta = (a \quad b)^T$ is constant than $\widehat{W}$ is set to zero and the parameter estimation algorithm becomes

$$\frac{d}{dt}\hat{\theta} = \rho(\frac{\partial}{\partial\theta}F(\hat{\theta}))^{-1}(Y - \mathrm{F}(\hat{\theta})) \qquad (28)$$

$$\hat{\theta}(0)\mathit{such\ that}\ (\frac{\partial}{\partial\theta}F(\widehat{\theta(0)}))^{-1}\text{exists}$$

IV. SIMULATION RESULTS

The system describes by Equations 15 and 16 is simulated using the following data for the unknown parameters. $a(t) = 2 + \sin(50t)$ and $b(t) = 4 + \sin(3t)$. The following parameters were also used for the identification algorithm: $\rho = 1000$ and $\sigma_1 = 10\rho$ and $\sigma_2 = 12\rho$.

Two simulations were carried out. In the first simulation, the system to be identified operate in open loop and the input signal is a unit step. The system was simulated in closed loop during the second simulation. The control equation still has the following equation

$$u = -20x_1 - 30x_2 + 1 \qquad (29)$$

Simulation results are illustrated below. Figures 9 to 12 show the state variables and estimated parameters for the identification carried out in open loop. Similar data for the identification performed in closed loop are illustrated in Figures 13 to 16.

The simulation results show that estimated parameters converge and perfectly track actual parameters. The identification module is activated at t = 0.1 s and initial conditions for the estimated parameters are 0.1 and 0.1 respectively. These results clearly demonstrate that the proposed method is able to identify the class of time-varying parameter systems under study. The performance of the proposed identification approach is unchanged if the system to be identified is operating in closed or open loop.

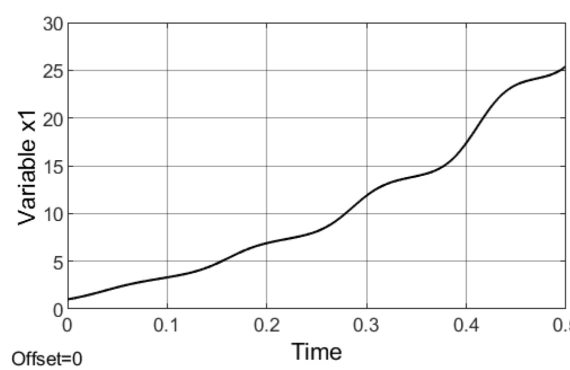

Fig. 9: Example 2: Variable x1 when the system is in open loop

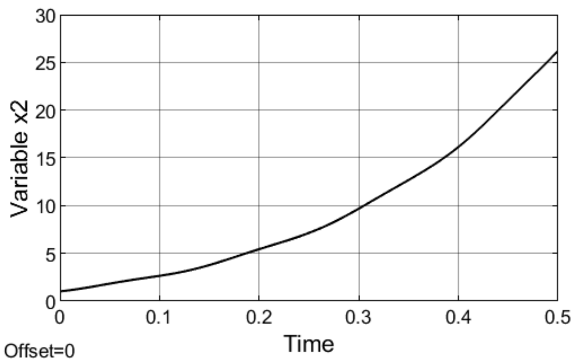

Fig. 10: Example 2: Variable x2 when the system is in open loop

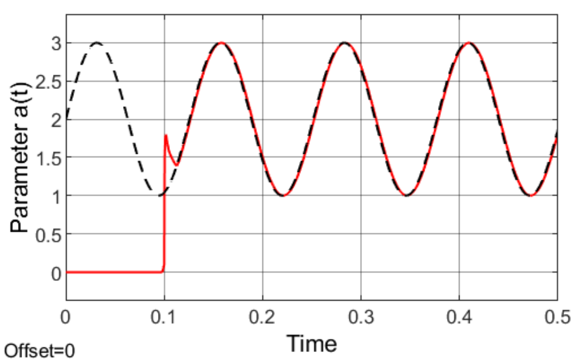

Fig. 11: Example 2: True and estimated parameters a(t) when the system is in open loop

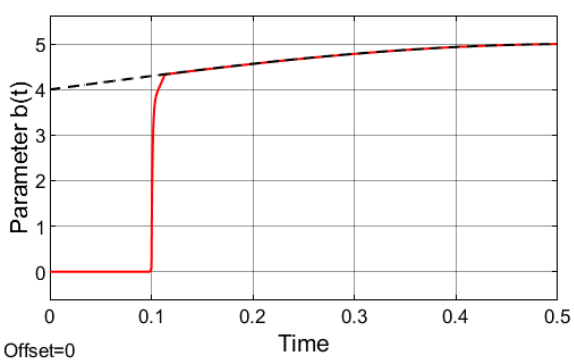

Fig. 12: Example 2: True and estimated parameters b(t) when the system is in open loop

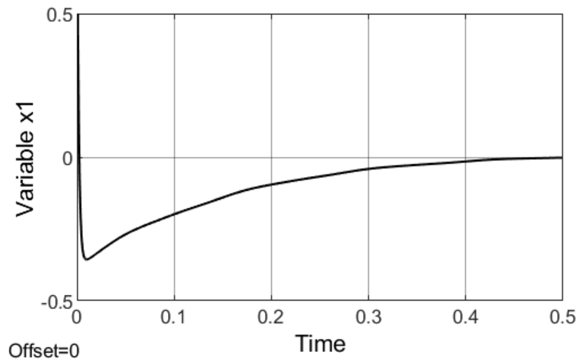

Fig. 13: Example 2: Variable x1 when the system is in closed loop

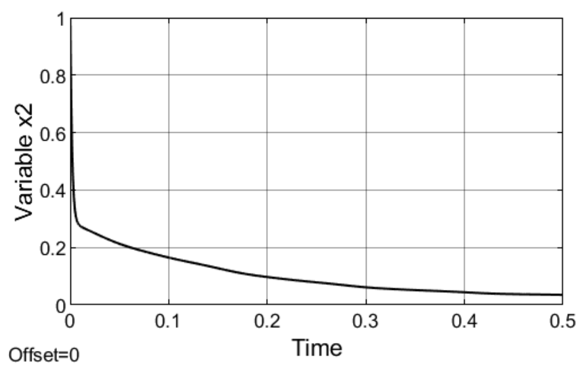

Fig. 14: Example 2: Variable x2 when the system is in closed loop

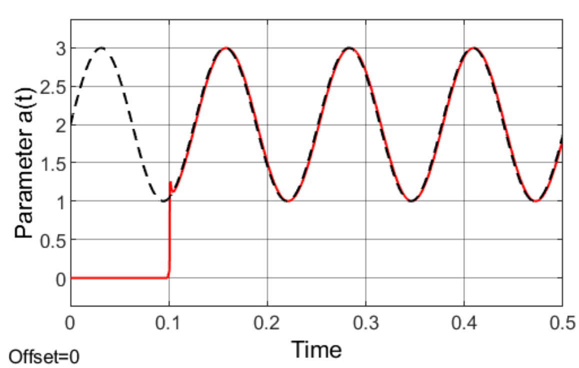

Fig. 15: Example 2: True and estimated parameters a(t) when the system is in closed loop

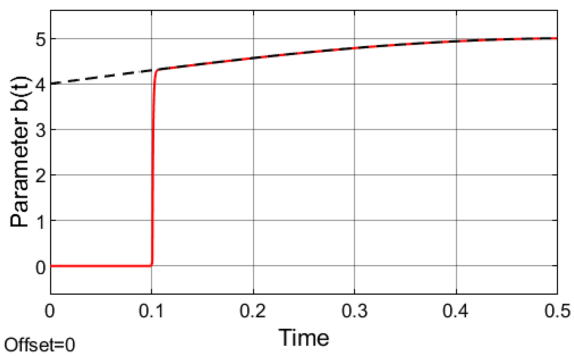

Fig. 16: Example 2: True and estimated parameters a(t) when the system is in closed loop

V. Conclusion

This paper presents a new approach for the identification of a class of time-varying parameter systems, using two illustrative examples. The parameters to be determined appear linearly in the dynamic model of the system in the first example, while they appear nonlinearly in the dynamic model in the second illustrative example. The main contribution is to transform the parameter identification problem into an observer design problem followed by the resolution of a linear or non-linear equation. The proposed algorithm has been evaluated in simulation when the system to be identified operates in both open and closed loops. The results show that estimated parameters converge almost instantaneously to their true values. The next step is to extend this approach to a wider class of time-varying parameter systems.

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