A HYBRID METHOD FOR SOLVING THE MULTI-TRAVELING SALESMAN PROBLEM *

Firas A. Houssein, Vladimir A. Kostyukov, *Joint-Stock Company "Robotics and Control Systems"*

***Abstract*— In this research work, the problem of task allocation in a multi-agent system is considered, where each agent is a robot, and each task is represented by a position, which should be visited by one agent. This problem is very similar to the multi-agent traveling salesman problem (mTSP), which, unlike the famous traveling salesman problem, involves several traveling salesmen who visit a given number of cities exactly once and return to the starting position with minimal travel costs. Therefore, the multi-agent traveling salesman problem is analyzed as a representative of the task allocation problem. The mTSP is important for the field of route optimization and task allocation between several agents. It includes two different, but interrelated subproblems: distribute cities among agents and determine the order in which each agent visits cities. In the literature, there are 3 concepts for solving mTSP with respect to solving its two constituent subproblems: the optimization concept, where both subproblems are solved simultaneously. The Cluster-First, Route-Second concept, where the question of which tasks to assign to which salesman is first decided, and then the question of the order in which each salesman solves his tasks is decided. The Route-First, Cluster-Second concept, where the question of the order in which tasks should be visited is first decided, and then this cycle is divided between agents without changing the order of visits. This paper proposes a hybrid approach to solving the mTSP, which combines the ideas of two well-known concepts: "Cluster-First, Route-Second" and "Route-First, Cluster-Second" in order to obtain their positive aspects and get rid of their weaknesses. To evaluate the effectiveness of the developed method, a comparative study was conducted. The results were evaluated based on three key criteria: the computational time, the total length of the routes travelled by the salesmen and the maximum route length among them. The analysis of the experimental data showed that when using the proposed method, the maximum path length among the routes travelled by the agents (load imbalance) is reduced by an average of 26%.**

I. INTRODUCTION

The basic idea of task allocation is to find, for a given set of tasks N_t and a set of agents Nu, a conflict-free distribution of tasks among agents that maximizes some global cost function. Each agent can perform no more than Lt tasks, and the allocation process is considered complete when $N_{min} \triangleq min\{N_t , N_u{*}L_t\}$ tasks are assigned. If each task is assigned to only one agent, then such an allocation is considered conflict-free. It is assumed that the global cost function is the sum of local functions, each of which depends on the set of tasks assigned to a particular agent. This allocation problem can be formalized as an integer (possibly nonlinear) model with binary variables x_{ij}, where x_{ij} indicates whether task j is assigned to agent i:

$$\max \sum_{i=1}^{N_u}\left(\sum_{j=1}^{N_t} c_{ij}(X_i, P_i)x_{ij}\right), \tag{1}$$

given that:
$$\sum_{j=1}^{N_t} x_{ij} \le L_t, \forall i \in I,$$
$$\sum_{i=1}^{N_u} x_{ij} \le 1, \forall j \in J,$$
$$\sum_{i=1}^{N_u}\sum_{j=1}^{N_t} x_{ij} = N_{min} \triangleq \min\{N_t, N_u * L_t\},$$
$$x_{ij} \in \{0,1\}, \forall (i,j) \in I \times J,$$

where: $x_{ij} = 1$ if agent i is assigned task j; $X_i \in \{0,1\}^{Nt}$ is a vector in which the j-th element is x_{ij}, I is a set of indices of agents $I \triangleq \{1,\dots,N_u\}$; J is a set of indices of tasks $J \triangleq \{1,\dots,N_t\}$; $P_i \in (J \cup \{\emptyset\})^{Lt}$ is a vector representing an ordered sequence of tasks for agent i; its k-th element is j if agent i performs the j-th task at the k-th point of the task chain.

When each agent must choose from a set of multiple tasks to perform, there is not only a problem of optimally distributing tasks among agents, but also of finding the optimal sequence of tasks for each agent. Thus, the problem of multiple task assignment is reduced to the multiple traveling salesman problem.

Depending on the objective function, the following types of mTSP are distinguished:

MinSum mTSP: in this variant, the objective function is to minimize the sum of travel costs for all agents. Formally, the MinSum variant is modeled as:

$$\min_{Tour_i \in TOURS}\left(\sum_{i=1}^{m} C(Tour_i)\right), \tag{2}$$

given that: $Tour_i \cap Tour_j = \emptyset, \forall\, i \neq j, i \le 1, j \le m$,

where i, j is the number of the salesman, $Tour_i$ is the route of the i-th salesman, m is the number of salesmen in the problem, *TOURS* is the set of all possible tours. $C(.)$ – cost functional (route length).

*The study was carried out with the support of the Russian Science Foundation grant No. 24-29-00492 "Development of methods of optimal task allocation in a group of mobile robotic systems", https://rscf.ru/project/24-29-00492/ based on JSC "NKB Robotics and Control Systems".

Kostyukov Vladimir Aleksandrovich – Joint-Stock Company "Robotics and Control Systems" e-mail: wkost-einheit@yandex.ru; Taganrog, Russia; phone: 88634371694; can. of eng. sc.; senior researcher.
Houssein Firas Aimanovich – Joint-Stock Company "Robotics and Control Systems" e-mail: firas94mecha@gmail.com; Taganrog, Russia; phone: 89996379357; junior researcher.

MinMax mTSP: In this variant, the objective function is the cost of the longest route (e.g. in terms of distance or time) among all the routes of traveling salesmen. This case is widely used in studies devoted to, for example, mission time reduction and load balancing among agents. Formally speaking, this variant is modeled as follows:

$$\min_{Tour_i \in TOURS} (\max_{j \in 1 \ldots m} C(Tour_j)), \quad (3)$$

given that: $Tour_i \cap Tour_j = \emptyset, \forall\, i \neq j, i \leq 1, j \leq m$,

MinSum & MinMax (Multi-Objective) mTSP: In this variant, the objective function consists of the two previous quality functionals.

In this paper, both MinSum and MinMax criteria are used, since using only MinSum may lead to a situation where several salesmen visit only one city, and the remaining cities are visited by one salesman. On the other hand, using only MinMax may lead to an excessively long overall route.

mTSP includes two interrelated subproblems:

- Distribution of tasks (cities) among salesmen.
- Determining the order of visiting tasks within the route of each salesman.

In the scientific literature, there are three main concepts for solving the mTSP, which differ in their approach to solving these subproblems:

1. The concept of simultaneous optimization: both subproblems are solved simultaneously.

2. The concept of Cluster-First, Route Second (CFRS): first, the tasks are distributed among the traveling salesmen, and then an optimal route is built for each of them (the mTSP is reduced to several traveling salesman problems).

3. The concept of Route-First, Cluster Second (RFCS): first, a common route is built for all tasks, which is then divided among the traveling salesmen (the mTSP is reduced to a single traveling salesman problem).

In the first concept of solving the mTSP problem, both of its subproblems are solved simultaneously. The algorithms or metaheuristic methods used in this concept simultaneously determine which salesman to assign a specific task to and in what order this task will be performed within the selected salesman's route.

The concept of simultaneous optimization includes many methods that can be divided into several approaches: deterministic, metaheuristic, market-based, and others. For example, in [1], the mTSP with heterogeneous vehicles was solved using an exact algorithm. The authors first proposed a formulation of the problem in the form of integer linear programming (ILP), and then developed an adaptive branching and bound algorithm that made it possible to find a suboptimal solution in 300 seconds for an example with 100 targets and 5 vehicles. In another study [2], constraint programming (CP) was used to optimally solve the mTSP, including global constraints, interval variables, and domain filtering algorithms. However, this approach turned out to be computationally expensive, as the solution time exceeded two hours for an example with 51 cities and 3 salesmen. It is worth noting that the optimization-based concept, although capable of finding optimal solutions, requires significant time costs, which limits its application to large-scale problems.

The second concept assumes a two-stage approach to solve the mTSP. In the first stage, cities are distributed among salesmen (clustering stage). In the second stage, the optimal order of visiting cities is determined for each salesman in order to optimize the specified quality function.

Thus, this concept decomposes the mTSP into several separate traveling salesman problems (TSPs) using clustering methods to reduce the search space of solutions [3-5]. Many works propose a two-stage approach to improve the efficiency of heuristic algorithms in solving large-scale mTSP. One of the first studies to propose the use of clustering methods for solving mTSP was the work of the authors [6]. They applied the neighborhood attractor scheme in combination with various heuristic algorithms, including the compression algorithm and evolutionary computation methods. The resulting combinations were tested on three different problems, and the results marked the beginning of a new direction in mTSP research. Since then, a number of works have developed approaches combining clustering algorithms with one or more heuristic optimization methods. It was found that the CFRS concept significantly reduces the computation time of the solution by dividing the problem into several simpler problems, however, this concept does not have a specific mechanism for optimizing the solution according to the MinMax criterion.

The third concept for solving the mTSP involves the following approach: first, the order in which all cities are visited is determined, which forms a single route covering all cities in the problem (the so-called "giant-tour"). This giant-tour is then divided between the salesmen without changing the order in which the cities are visited. In this way, the mTSP problem is reduced to the single TSP. This concept is known in the literature as Route First-Cluster Second (RFCS) [7, 8] and was originally used to solve vehicle routing problems, which, unlike the mTSP, take into account the capacity and carrying capacity of agents (salesmen). The author of [9] used the Bellman algorithm for directed acyclic graphs to divide the giant-tour. In simplified form, the algorithm works as follows: two nested loops, indexed as *i* and *j*, check each subsequence of cities (T_i, T_{i+1}, ..., T_j) and calculate its total load and trip cost. If a subsequence exceeds the vehicle capacity, it is excluded from consideration.

It was revealed that the RFCS concept has a more flexible mechanism for optimizing the solution according to the MinMax criterion, but at the expense of large time costs since the mTSP problem is reduced to the CF problem, therefore, the size of the solution space is (n-1)!

As a result of the review, it was revealed that the Route-First, Cluster-Second concept with the proposed modification in solving the multi-agent traveling salesman problem gives the best result along the longest route among traveling salesmen, which contributes to a more balanced load between agents. However, it is too time-consuming compared to the Cluster-First, Route-Second concept, this is due to the fact that in the first stage of the Route-First, Cluster-Second concept the size of solution's space is equal to (n-1)! solutions, where *n* is the number of cities in the mTSP, which is much larger than the size of the solution space when using the Cluster-First, Route-Second concept, which is (n/m)! (With uniform distribution of n tasks among m traveling salesmen).

II. THE CLUSTER, ROUTE, CONNECT, SPLIT CONCEPT

The larger the size of the solution space, the less likely a metaheuristic method will find an optimal solution, and the less likely it is to hit a local minimum, as a result, the Route-First, Cluster-Second concept method takes longer to work. In this regard, a hybrid method is proposed that combines both the Route-First, Cluster-Second and Cluster-First, Route-Second concepts into one to get their positive aspects and get rid of their weaknesses.

The proposed method consists of four main steps:

- Cluster: tasks (cities) are divided into clusters. The goal is to distribute the points in such a way as to minimize the size of the solution space.
- Route: For each cluster, an optimal route is built using methods such as the Ant Colony Optimization (ACO). This ensures preliminary optimization of the routes within each cluster. A detailed description of solving the TSP using ACO is described in [10].
- Connect: All clusters are connected into a single large route (giant-tour) including all cities.
- Split: The resulting giant-tour is split into subroutes, each of which is connected to a depot city and assigned to a specific salesman. The process takes into account constraints such as maximum route length, load balancing, and execution time. A detailed description of splitting algorithm is described in [10].

The essence of the proposed method is to combine the advantage of the Cluster-First, Route-Second concept in dividing the problem into several subproblems, ultimately reducing the size of the solution space and, consequently, the calculation time, and the advantage of the Route-First, Cluster-Second concept in minimizing the maximum route length among salespeople.

III. SIMULATION

To study the proposed method and conduct a comparative analysis of its performance with the CFRS and RFCS methods, 3 problems from the corresponding library of benchmark traveling salesman problems [16] (eil51, kroA100, and kroA150) with 51, 100, and 150 cities, respectively, were taken as a basis. For each of these problems, 3 scenarios were set with 3, 5, and 10 traveling salesmen. Each scenario was run 100 times to obtain an appropriate representative sample.

The performance criteria for these algorithms were as follows:

- The time to calculate the solution to the multi-travelling salesman problem.
- The sum of the paths traveled by the traveling salesmen.
- The longest path among the paths traveled by the traveling salesmen, to compare the balance and uniformity of the distribution of cities between the traveling salesmen.

The evaporation coefficient of the pheromone of ACO in all scenarios was 0.5. For the ACO-BMTSP method, the number of ants *k* was *n* * 2, the number of iterations was *n* / 2. For the KM-CACO method and the proposed method, the number of ants was n, and the number of iterations was not specified, the stopping criteria of the ant algorithm was to repeat 20 iterations without confirming the found solution. The ACO-BMTSP, KM-CACO and proposed algorithms were programmed in Python 3.8. The Intel Core i7-9700KF CPU @ 3.60GHz RAM = 32GB, Windows 10 Pro 64-bit OS were used for simulation.

IV. RESULTS AND COMPARATIVE ANALYSIS

Tables 1, 2 and 3 sequentially present the results of solving the problems eil51 with 3, 5 and 10 salesmen. Tables 4, 5 and 6 sequentially present the results of solving the problems kroA100 with 3, 5 and 10 salesmen. Tables 7, 8 and 9 sequentially present the results of solving the problems kroA150 with 3, 5 and 10 salesmen. The tables express: the average among 100 runs by the solution calculation time in seconds (μ_t), the sum of the path lengths in meters (μ_{sum}) and the maximum path length among salesmen in meters (μ_{max}). The corresponding optimal values are highlighted in bold.

TABLE I. RESULTS OF THE TASK STUDY EIL51 WITH 3 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	0.519	4.5	**0.18**
μ_{sum}	514.18	**513.43**	544.07
μ_{max}	190.87	**177.43**	187.1

TABLE II. RESULTS OF THE TASK STUDY EIL51 WITH 5 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	0.76	4.27	**0.24**
μ_{sum}	**560.6**	595.4	590.15
μ_{max}	133.33	127.08	**126.61**

TABLE III. RESULTS OF THE TASK STUDY EIL51 WITH 10 SALESMEN

Method	**CFRS**	**RFCS**	**CRCS**
μ_t	**1.31**	8.24	4.37
μ_{sum}	**769.5**	827.5	826.98
μ_{max}	99.78	93.16	**93.8**

TABLE IV. RESULTS OF THE TASK STUDY KROA100 WITH 3 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	2.11	37.52	**1.43**
μ_{sum}	28054	**26855**	27491.53
μ_{max}	11775	**9233**	9436.69

TABLE V. RESULTS OF THE TASK STUDY KROA100 WITH 5 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	3.41	38.47	**1.58**
μ_{sum}	**30658**	32360	32433
μ_{max}	7916	7111	**6903**

TABLE VI. RESULTS OF THE TASK STUDY KROA100 WITH 10 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	**6.1**	49.38	12.33
μ_{sum}	**40412**	44391	45412
μ_{max}	5550	5417	**5374**

TABLE VII. RESULTS OF THE TASK STUDY KROA150 WITH 3 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	5.87	152.61	**5.15**
μ_{sum}	34671	**32006**	32728
μ_{max}	15355	**11058**	11309

TABLE VIII. RESULTS OF THE TASK STUDY KROA150 WITH 5 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	8.41	151.4	**5.19**
μ_{sum}	**36192**	37146	37081
μ_{max}	8998	8002	**7920**

TABLE IX. RESULTS OF THE TASK STUDY KROA150 WITH 10 SALESMEN

Method	CFRS	RFCS	CRCS
μ_t	**15.21**	162.8	24.45
μ_{sum}	**45507**	49344	48983
μ_{max}	6169	5749	**5701**

Note that the proposed method has an advantage in terms of calculation time for 3 and 5 agents, and in terms of the MinMax criterion for 5 and 10 agents.

Analysis of the calculation results presented in these tables shows that the proposed method, compared to RFCS, reduces the calculation time by 88–2860%, and the maximum path length among traveling salesmen decreases by an average of 0.1–10% for problems with 5 and 10 agents and increases by an average of 2–5% for problems with 3 agents.

Compared to CFRS, the proposed method reduces the calculation time by 12–68% for problems with 3 and 5 agents and increases by an average of 60–233% for problems with 10 agents, and the maximum path length among traveling salesmen decreases by an average of 1–26%.

V. CONCLUSION

This paper proposes a hybrid approach to solving the multi-agent traveling salesman problem based on reducing the size of the solution space. The proposed method was studied on three benchmark traveling salesman problems eil51, kroA100 and kroA150 with 3, 5 and 10 agents. The quality of the solution was measured by three criteria: the time of solution calculation, the sum of the route distances of all traveling salesmen, and the maximum path length among the routes traversed by all agents. The results were compared with traditional approaches to solving the mTSP based on the ant colony optimization. Statistical analysis of the results showed that, on average, the proposed method offers a good compromise between the calculation time and the quality of the solution. In subsequent publications, we intend to show how the propose method could behave in real life problems where the tasks could have non-normal distribution. Also, the possibility of solving the dynamic mTSP are going to be studied, where the number of agents and tasks can change during the mission. minimum criterion on the set of all distances traveled by traveling salesmen.

REFERENCES

[1] 1. K. Sundar, S. Rathinam, Algorithms for heterogeneous, multiple depot, multiple unmanned vehicle path planning problems, J. Intell. Robot. Syst., Theory Appl. 88 (2–4) (2017) 513–526, http://dx.doi.org/10.1007/s10846-016-0458-5.

[2] M. Vali, K. Salimifard, A constraint programming approach for solving multiple traveling salesman problem, in: The Sixteenth International Workshop on Constraint Modelling and Reformulation, 2017.

[3] Shabanpour M., Yadollahi M., Hasani M.M. A New Method to Solve the Multi Traveling Salesman Problem with the Combination of Genetic Algorithm and Clustering Technique, IJCSNS International Journal of Computer Science and Network Security, VOL.17 No.5, May 2017.

[4] Basma H., Bashir H., Cheaitou A. A Novel Clustering Method for Breaking Down the Symmetric Multiple Traveling Salesman Problem, Journal of Industrial Engineering and Management JIEM, 2021, vol.14, no. 2.

[5] Arthur D., Vassilvitskii S. K-Means++: The Advantages of careful seeding. Proceedings of the 8th annual ACM-SIAM symposium on Discrete algorithms, 2007.

[6] Sofge, D., Schultz, A., & De Jong, K. (2002). Evolutionary computational approaches to solving the multiple traveling salesman problem using a neighborhood attractor schema. Proceedings of the Applications of Evolutionary Computing on EvoWorkshops (153-162).

[7] Beasley J.E. Route First - Cluster Second Methods for Vehicle Routing // Omega. 1983. Vol. 11, Issue 4, - P. 403-408

[8] Bozdemir M. K., Bozdemir M., Burcu Ö. Route First- Cluster Second Method for Personal Service Routing Problem // Journal of Engineering Studies and Research. 2019. Vol. 25. No. 2. - P. 18-24.

[9] Prins, C., 2004. A simple and effective evolutionary algorithm for the vehicle routing problem. Comput. Oper. Res. 31, 1985–2002.

[10] F. A. Houssein, V. F. Kostyukov and I. D. Evdokimov, "A method for solving the multi-traveling salesman problem based on reducing the size of the solution space," 2024 10th International Conference on Control, Decision and Information Technologies (CoDIT), Vallette, Malta, 2024, pp. 1729-1733, doi: 10.1109/CoDIT62066.2024.10708116.