# A Unified Approach for Optimal Cruise Airspeed with Variable Cost Index

Lucas Souza e Silva, Ali Akgunduz and Luis Rodrigues*

*Abstract*—**This paper proposes, for the first time, a unified optimal approach applicable for both fuel-powered and all-electric aircraft to solve a direct operating cost (DOC) minimization problem where the cost index (CI) is modeled as a time-varying parameter, which is either commanded by Air Traffic Control (ATC) or assigned by the airline. Furthermore, this paper demonstrates how a variable CI affects the solution of the optimization problem as it presents the equations that allow the computation of optimal constant cruise airspeed and flight time in response to step changes in the CI value. The proposed methodology is validated using a simulated flight scenario, where inputs from ATC are received during flight, requiring the aircraft to adjust its optimal airspeed, flight time, and total energy consumption accordingly. The optimal values of airspeed, flight time, and energy consumption are calculated for both a fuel-powered and an all-electric aircraft, thus allowing applications of the proposed approach to future air mobility all-electric vehicles.**

## I. INTRODUCTION

The demand for domestic and international travel is steadily increasing, indicating full recovery from the drawbacks caused by the COVID-19 pandemic [1]. Airbus projects an annual increase in passenger traffic demand by $3.6\%$ over the next 20 years [2], while Boeing forecasts, for the same period, a demand for 42000 new airplanes worldwide [3]. The vast majority of the aircraft fleet currently operating around the world and that will meet near-future demand is powered by fossil fuels [4]. This challenges modern aviation to maintain profitability while adhering to new environmental guidelines, demanding significant efforts to reduce greenhouse gas (GHG) emissions. Examples of such initiatives include the electrification of flying vehicles and the adoption of renewable or waste-derived Sustainable Aviation Fuels (SAF) as alternatives to fossil fuels [5]. In the context of fuel-based aircraft operations, although the widespread use of SAF has the greatest potential to reduce its associated GHG emissions [6], its production is still far from meeting the aviation fuel demand, as it only represented $0.3\%$ of the global production of jet fuel in 2024 [7]. For all-electric aircraft, the limited energy density in the electrical batteries prevents them from achieving energy levels comparable to those supplied by aviation fuel [8].

Unfortunately, the current production volume of SAF is insufficient to meet the fuel demands of aviation, and the energy density of modern batteries remains unsuitable for long-haul flights. An alternative for more efficient energy usage in aviation is operating the aircraft in economy mode (often called ECON), which minimizes the direct operating cost (DOC) and results in optimal airspeed values. The ECON speed can be calculated by the Flight Management System (FMS) of the aircraft or by the airline during flight planning and preparation and is based on the chosen ratio of time and energy costs, known as the cost index ($CI$).

Airlines are continuously reducing their operational costs, to increase their profit margins in a highly competitive market. Simultaneously, they are constantly challenged to exceed customers' expectations, while maintaining high levels of safety and complying with the applicable regulations and standards. In this instance, numerous factors might affect the aircraft operation after the flight plan is approved and is under execution, such as weather-related or situational conditions, ATC coordination or the airline's decision to change the aircraft's flight operational mode. To factor in the objectives of the airlines with the constraints imposed by the aircraft operation, this paper proposes a unified methodology to calculate the flight time and a constant cruise airspeed that minimize the direct operating cost of fuel-powered and all-electric aircraft in the presence of a time-varying cost index. As detailed in section II, to the best of the authors' knowledge, only references [9] - [14] available in the open literature considered a variable $CI$ once the aircraft is flying. However, none of them studied how a time-varying $CI$ affects the optimal solution to minimize DOC for fuel-powered and all-electric aircraft. The main contributions of this paper are:

1) The introduction of the cost index as a time-varying parameter in the formulation of the optimization problem to minimize DOC, which allows changes in the aircraft $CI$ imposed by the aircraft operation, with input from either ATC or the airline.
2) A unified approach to solve the minimization of DOC with variable $CI$ for fuel-powered and all-electric aircraft where the problem formulation considers the aircraft energy as a system state.
3) The validation of the proposed methodology by a simulated operational scenario.

This paper is organized as follows: Section II highlights key aspects of other work in the technical literature and how they relate to the contributions of this paper. Section III presents the methodology to perform the FMS initialization and the calculation of optimal airspeed and flight time with variable $CI$. Section IV presents a simulated scenario and discussions about the results. Section V concludes the paper.

## II. RELATED WORK

$CI$ is a trade-off parameter that balances time-related costs ($C_t > 0$) and energy-related costs ($C_e > 0$) in an aircraft

*Lucas Souza e Silva, Ali Akgunduz and Luis Rodrigues are with the Department of Electrical and Computer Engineering, Concordia University, Montreal, QC H3G 2W1, Canada.

operation and is computed by

$$CI = \frac{C_t}{C_e} \tag{1}$$

where $C_t$ is defined in units of currency per units of time and it includes maintenance costs, salaries, leasing costs and equipment depreciation among other operational expenses that are time-dependent. The term $C_e$ is associated with the cost of fuel consumed by the aircraft flying a defined route for a fuel-powered aircraft [15] or the cost of charging the electrical batteries in all-electric aircraft, and it is given in units of currency per units of energy. In this paper, $CI$ is expressed in units of energy per units of time ($J.s^{-1}$).

The minimization of DOC has been one of the primary objectives of onboard FMS since its introduction in the early 1980s [16] [17]. A range of newer studies approach the FMS economy mode problem for fuel-powered [18]–[20] and all-electric aircraft [21], [22]. Nevertheless, once $CI$ was selected in these references, it was assumed to be constant over the flight time. Other authors explore the notion of variable $CI$ in flight based on operational and environmental factors [9]–[11] or use fuzzy logic to select a new value of $CI$ based on strategic and tactical information of the flight [12]. Further contributions provide pilots with appropriate ranges for $CI$ to adjust the in-flight profile by imposing a bound on DOC [13] or show the application of a stochastic optimization methodology that assumes weather forecasts and payload variables with associated uncertainties and a probabilistic distribution to minimize total flight costs [14].

In contrast with the previous literature, this paper considers a DOC minimization problem where $CI$ is time-varying. To the best of the authors' knowledge, this is the first time a unified approach for fuel-powered and all-electric aircraft is proposed where $CI$ is a commanded time-varying parameter. Moreover, the paper shows how a variable $CI$ affects the solution of the optimization problem and validates the proposed methodology presenting in section IV a simulation of an operational scenario inspired by a case study from [12].

## III. Problem Statement and Solution

### A. Aircraft Dynamic Model and Assumptions

Consider an aircraft flying along a straight-line path and $x$ describes the longitudinal position of the aircraft along this path, $v$ is its airspeed, $D$ is the magnitude of the drag force, $L$ is the magnitude of the lift force, $T$ is the magnitude of the aircraft's thrust force, $W$ is the aircraft's weight, and $CI$ is the aircraft cost index. The following assumptions are made:

1) The aircraft cruises in steady flight without wind, at constant altitude and with constant speed. As a consequence of this assumption, we have $W = L$ and $T = D$.
2) The flight Mach number is assumed to be below the drag divergence Mach number; therefore, wave drag can be ignored.
3) The aircraft operates within its flight envelope.
4) The aircraft is assumed to be a fixed-wing aircraft, so the wing surface area $S$ is constant.
5) For fuel-powered aircraft, the specific fuel consumption $S_{fc}$ is assumed to be constant for a given altitude.
6) For all-electric aircraft, the battery is considered ideal, with negligible internal resistance and with electric charge $Q$. It operates in cruise at a constant voltage $U$ and as a consequence, $\frac{d}{dt}(QU) = \dot{Q}U$.

Based on problem assumption 1), the aircraft cruises at constant airspeed. For any $v \neq 0$, the total cruise time $t_f$ can be expressed as

$$t_f = \frac{x_f - x_0}{v} = \frac{\Delta x}{v} \tag{2}$$

where $x_0$ is the starting position and $x_f$ is the destination position. This paper proposes a unified approach that considers the energy $E$ sourced to fuel-powered aircraft by the fuel combustion in the aircraft powerplant system (noted as $E^{fuel}$) and the energy supplied to all-electric aircraft by the electrical batteries (noted as $E^{elec}$). Therefore, the dynamic model of the aircraft is

$$\dot{x} = v \tag{3}$$

$$\dot{E} = \begin{cases} \dot{E}^{fuel} = \frac{e}{g}\dot{W} & \text{if fuel} \\ \dot{E}^{elec} = \dot{Q}U & \text{if electrical} \end{cases} \tag{4}$$

where $e$ is the fuel heating value, which is constant for a certain type of fuel and it represents the energy stored per kilogram of fuel, typically ranging from $40,000$ $kJ/kg$ to $43,000$ $kJ/kg$ [23]. The gravitational acceleration $g$ is assumed to be constant and equal to $9.81$ $m.s^{-2}$, and $W_{fuel}$ is the weight of fuel available in the aircraft fuel tanks. We note that the total aircraft weight $W$ is such that the only time-varying component is $W_{fuel}$ and therefore $\dot{W}_{fuel} = \dot{W}$. In addition, $Q$ is the electrical charge available in the all-electric aircraft's battery system.

According to assumption 2), the aircraft operates below the drag divergence Mach number, and assuming that it follows a drag polar curve, one can define the magnitude of the drag force

$$D = \frac{1}{2}\rho S C_{D,0} v^2 + \frac{2 C_{D,2} W^2}{\rho S v^2} \tag{5}$$

where $\rho$ is the air density, $C_{D,0}$ is the parasitic drag coefficient at zero-lift and $C_{D,2}$ is the drag coefficient induced due to lift.

### B. Problem Formulation

The direct operating cost (DOC) of an aircraft in cruise is

$$DOC = \int_0^{t_f} (C_t - C_e\dot{E})\, dt \tag{6}$$

where $t_f$ is the total cruise time and the minus sign corresponds to energy depletion. This notion can be expanded to a flight composed of several cruise segments, where $t_f$ indicates the total flight time in each of these segments. Assuming that the cost of energy $C_e$ is constant, one can divide the cost function (6) by $C_e$, yielding

$$J = \frac{DOC}{C_e} = \int_0^{t_f} (CI - \dot{E})\, dt \tag{7}$$

As discussed in Section II, most of the related work in the literature considers $CI$ as a constant value throughout the flight or between waypoints. However, in this paper, we explore the properties of a time-varying $CI$ function modeled as a first-order filter, where $CI(0) = CI_0$ corresponds to the initial condition of the cost index, selected by the airline for the FMS initialization when preparing the aircraft flight plan. ATC determines the flight level $h$ for the aircraft cruise. Changes in the aircraft's $CI$ are expected throughout the flight from ATC to adjust the air traffic flow in a certain airspace or from the airline based on its cost management approach. The magnitude of the step change in the aircraft $CI$, noted as $CI_{in}$, is a result of multiple factors that depend on environmental, situational, or operational conditions. There are some methodologies available in the open literature, such as in [11] and [12], that could be used by ATC to compute $CI_{in}$. Therefore, we assume that $CI_{in}$ is also provided by ATC to the aircraft FMS along with the cruise flight level $h$. The result of the optimization problem will be the optimal aircraft airspeed $v^*$ and flight time $t_f^*$. In this context, the minimization of DOC for an aircraft in cruise with constant airspeed and variable cost index can be formulated as an optimal control problem as shown below

$$
\begin{aligned}
J^* = \min_{v,t_f} \quad & \int_0^{t_f} (CI - \dot{E})\, dt \\
\text{s.t.} \quad & \dot{x} = v \\
& \dot{E} = \begin{cases} \dot{E}^{fuel} = \frac{e}{g}\dot{W} & \text{if fuel} \\ \dot{E}^{elec} = \dot{Q}U & \text{if electrical} \end{cases} \\
& \tau \dot{CI} = -CI + CI_{in} \\
& D = \frac{1}{2}\rho S C_{D,0} v^2 + \frac{2C_{D,2}W^2}{\rho S v^2} \\
& CI(0) = CI_0, E(0) = E_0 \\
& x(0) = x_0, x(t_f) = x_f \\
& v > v_{min}
\end{aligned} \tag{8}
$$

where $J^*$ is the minimum DOC achieved for the minimizers of (8), which are the optimal airspeed $v^*$ and the optimal flight time $t_f^*$. We address the aircraft scheduling in this paper by introducing a lower bound $v_{min}$ on the aircraft's airspeed, which corresponds to the maximum allowed flight time that does not cause significant delays at the destination.

### C. Problem Solution

The solution of the equation in $\dot{CI}$ in (8), for the initial condition $CI(0) = CI_0$ and input $CI_{in}$, is given by

$$CI(t) = e^{-\frac{t}{\tau}}(CI_0 - CI_{in}) + CI_{in} \tag{9}$$

where $\tau$ is the time constant of the first-order filter and indicates the convergence rate of $CI$ to smoothly and continuously reach the commanded value $CI_{in}$. Considering $CI$ now as a function of time, one can rewrite the total cost function $J$ from the optimization problem (8) using the result from (9) and (2) as

$$J = \tau(CI_0 - CI_{in})(1 - e^{-\frac{\Delta x}{\tau v}}) + \frac{CI_{in}\Delta x}{v} + E_0 - E_f \tag{10}$$

where $E_f$ is the final value of the aircraft energy. Applying the necessary condition for optimality on (10) yields

$$\frac{\partial J}{\partial v} = -\frac{(CI_0 - CI_{in})\Delta x}{v^2} e^{-\frac{\Delta x}{\tau v}} - CI_{in}\frac{\Delta x}{v^2} - \frac{\partial E_f}{\partial v} = 0 \tag{11}$$

The optimal cruise airspeed $v^*$ for a variable $CI$ is the solution of (11), for any finite $\tau > 0$, $v > 0$ and $x_f > x_0$. The partial derivative of the final energy with respect to the aircraft's airspeed depends on the type of aircraft and will be discussed in section III-D.

*1) FMS Initialization:* If no ATC input is received throughout the flight, the aircraft should operate with the fixed value of $CI = CI_0$, which is the $CI$ value defined by the airline based on its strategy. In this case, a particular solution can be derived for the FMS initialization by making $\tau = \infty$ in (11), which results in

$$-CI_0\frac{\Delta x}{v_0^2} - \frac{\partial E_f}{\partial v_0} = 0 \tag{12}$$

where $v_0^*$ is the optimal airspeed computed for the FMS initialization. The optimal flight time $t_{f_0}^*$ can be computed using (2) with $v = v_0^*$.

### D. Application to fuel-powered and all-electric aircraft

As mentioned in section III-C, to compute the optimal cruise airspeed and flight time for the FMS initialization and for cruise operations with ATC input, one needs first to compute the term $\frac{\partial E_f}{\partial v}$, which depends on the aircraft's energy supply, i.e. if the aircraft is fuel-powered or all-electric. This section shows how to apply equations (2), (11) and (12) to compute the optimal airspeed and flight time for fuel-powered and all-electric aircraft.

*1) Fuel-powered aircraft:* For a fuel-powered aircraft, from (4), we can establish a relationship between the final weight $W(t_f) = W_f$ and the final fuel energy $E_f^{fuel}$ as

$$E_f^{fuel} = \frac{e}{g}W_f \tag{13}$$

The time rate of change of the aircraft weight in steady flight is expressed by

$$\dot{W} = -S_{fc}D = -S_{fc}\left(\frac{1}{2}\rho S C_{D,0}v^2 + \frac{2C_{D,2}W^2}{\rho S v^2}\right) \tag{14}$$

The solution of the separable differential equation (14) with initial condition $W_0$ is

$$W_f = k_2 v^2 tan\left[-\frac{\Delta x}{k_1 v} + arctan\left(\frac{W_0}{k_2 v^2}\right)\right] \tag{15}$$

where the constants $k_1$ and $k_2$ are defined as

$$k_1 = \frac{1}{S_{fc}\sqrt{C_{D,0}C_{D,2}}} \tag{16}$$

$$k_2 = \frac{\rho S}{2}\sqrt{\frac{C_{D,0}}{C_{D,2}}} \tag{17}$$

Replacing (15) in (13) yields

$$E_f^{fuel} = \frac{e}{g}k_2 v^2 tan\left[-\frac{\Delta x}{k_1 v} + arctan\left(\frac{W_0}{k_2 v^2}\right)\right] \quad (18)$$

Based on (18) we can compute $\frac{\partial E_f}{\partial v}$, which for a fuel-powered aircraft becomes

$$\frac{\partial E_f^{fuel}}{\partial v} = 2\frac{e}{g}k_2 v tan\left[-\frac{\Delta x}{k_1 v} + arctan\left(\frac{W_0}{k_2 v^2}\right)\right] + \frac{e}{g}\left(\frac{k_2 \Delta x}{k_1} - \frac{2v^3 W_0}{v^4 + (\frac{W_0}{k_2})^2}\right) sec^3\left[-\frac{\Delta x}{k_1 v} + arctan\left(\frac{W_0}{k_2 v^2}\right)\right] \quad (19)$$

Replacing (18) in (10), we obtain the total cost function for the FMS in cruise with ATC input for a fuel-powered aircraft. The result from (19) can be applied to (11) and (12) to compute the optimal cruise airspeed for the aircraft cruise with ATC input and for the FMS initialization, respectively, for fuel-powered aircraft. The optimal flight time can be calculated by (2) based on the optimal cruise airspeed.

*2) All-electric aircraft:* For an all-electric aircraft, we know from (4) that the total energy stored in the aircraft's battery system $E^{elec}$ depends on the electrical charge of the batteries and the battery voltage. As per assumption 6), for a constant voltage $U$, the final energy $E_f^{elec}$ available to the aircraft is

$$E_f^{elec} = Q_f U \quad (20)$$

where $Q(t_f) = Q_f$ is the final battery charge. From the definition of efficiency in the conversion of the electrical power to mechanical power, the electrical current $i$ supplied by the aircraft battery, which is equal to the time rate of change in the battery's charge $\dot{Q}$, can be expressed as

$$-i = \dot{Q} = -\frac{Tv}{U\eta} \quad (21)$$

For steady flight, $T = D$, so (21) and (5) lead to

$$\int_{Q0}^{Q_f} dQ = -\left(\frac{\rho S C_{D,0} v^3}{2U\eta} + \frac{2C_{D,2}W^2}{U\eta\rho S v}\right)\int_0^{t_f} dt \quad (22)$$

Using the result of (2), the solution of (22) is

$$Q_f = Q_0 - \frac{\Delta x}{U\eta}\left(\frac{1}{2}\rho S C_{D,0} v^2 + \frac{2C_{D,2}W^2}{\rho S v^2}\right) \quad (23)$$

Replacing (23) in (20), we obtain

$$E_f^{elec} = Q_0 U - \frac{\Delta x}{\eta}\left(\frac{1}{2}\rho S C_{D,0} v^2 + \frac{2C_{D,2}W^2}{\rho S v^2}\right) \quad (24)$$

From (24), we can now compute $\frac{\partial E_f}{\partial v}$, which for an all-electric aircraft becomes

$$\frac{\partial E_f^{elec}}{\partial v} = -\frac{\Delta x}{\eta}\left(\rho S C_{D,0} v - \frac{4C_{D,2}W^2}{\rho S v^3}\right) \quad (25)$$

With the result (24), one can obtain the total cost function $J$ for the FMS operating in cruise with an ATC input as per (10). Replacing (25) in (11) and (12), one can compute the optimal airspeed for the aircraft cruise operation under ATC input and for the FMS initialization, respectively, for an all-electric aircraft.

## IV. Results and Discussions

### A. Simulation Parameters

The simulations were performed in MATLAB (laptop equipped with 16 GB of RAM and an Intel$^R$ Core $^{TM}$ i5-1135G7 2.40GHz CPU) and use data shown in Table I for the fuel-powered Gulfstream-IV (G-IV) business jet [18], [24] and the Yuneec International E430 all-electric aircraft [25].

TABLE I
Simulation Parameters

| Parameters | Fuel-powered | All-electric |
|---|---|---|
| Wing surface area $S$ $(m^2)$ | 88.26 | 11.37 |
| Aircraft Mass $(kg)$ | 10000[1] | 472[2] |
| Zero-lift drag coefficient $C_{D,0}$ | 0.015 | 0.035 |
| Induced drag coefficient $C_{D,2}$ | 0.08 | 0.009 |
| Maximum cruise airspeed $v_{max}$ $(km/h)$ | 890 | 161 |
| Maximum Cost Index $CI_{max}$ $(J.s^{-1})$ | 3.794x10$^7$ | 4.364x10$^4$ |
| Specific Fuel Consumption $S_{fc}$ $(kg/N.s)$ | 1.92x10$^{-5}$ | N/A |
| Battery Output Voltage $U$ $(V)$ | N/A | 133.2 |
| Electrical system efficiency $\eta$ | N/A | 0.7 |

[1] Initial fuel mass.
[2] Aircraft maximum take-off mass, which is constant throughout the flight.

### B. Simulated flight scenario

A simulation of a flight scenario is shown in this section, where the optimal aircraft airspeed values were computed in MATLAB by the *fzero* function using (11) for $v$ and (12) for $v_0$ and the flight time was found by solving (2) for a fuel-powered and an all-electric aircraft. This flight scenario is inspired by the case study presented in [12], where the aircraft $CI$ is changed in flight, but with a different distance between the origin and the destination, which has been adjusted herein to create an environment where both a fuel-powered and an all-electric aircraft can operate, while keeping similar operational restrictions as presented in [12]. In contrast with the case study from [12], which only provides the new Estimated Time of Arrival (ETA) at the destination, this paper computes the optimal airspeed based on changes in $CI$ in cruise imposed by ATC and illustrates how the aircraft's airspeed reaches the optimal value and how the adjustments in $CI$ impact the aircraft ETA at the destination.

In the simulated scenario, ATC implemented two adjustments to the aircraft's $CI$ to align its operation with the prevailing air traffic conditions in the area of flight. Consequently, the aircraft's airspeed must be adjusted accordingly. Let us consider the initial position $x_0 = 0$, the destination position $x_f = 160km$ and initial cost index $CI_0 = 0.1CI_{max}$, assuming $0 \leq CI \leq CI_{max}$, where $CI_{max}$ corresponds to the maximum value of $CI$, with the aircraft operating within its envelope, as per assumption 3). The values of $CI_{max}$

are presented in Table I. The optimal cruise airspeed $v_0^*$ was computed using (12) with $CI_0 = 0.1CI_{max}$. The flight time, which in this case is the scheduled flight time $t_{f_0}^*$ is found using (2). When the aircraft is at the first intermediate position $x_{int_1} = 40km$, after time $t_1$ has elapsed, an ATC input $CI_{in_1} = 0.2CI_{max}$ is received, so the aircraft could recover from a delay in its departure. The adjusted optimal airspeed in the second flight segment $v_1^*$ and the flight time $t_{f_1}^*$ will be then computed using (11) and (2), respectively, with $x_0 = x_{int_1}$. When the aircraft is at the second intermediate position $x_{int_2}$, which is 100 $km$ distant from the origin waypoint, after time $t_2$ has passed, a new ATC input $CI_{in_2} = 0.15CI_{max}$ is received, and the aircraft is required to decrease its airspeed. The new optimal airspeed $v_2^*$ and flight time $t_3$ in the third flight segment were computed using (11) and (2), respectively, now with $x_0 = x_{int_2}$. Figure 1 summarizes the described flight scenario. The flight level $h$ is also determined by ATC and is suitable for each type of aircraft. For the fuel-powered aircraft, we assume a constant altitude of 10 $km$ above sea level, which corresponds to an air density $\rho = 0.4135\ kg/m^3$ and $S_{fc}$ as defined in Table I. For the all-electric aircraft, the altitude considered in the simulation was 1 $km$ above sea level, where the air density is considered as $\rho = 1.112\ kg/m^3$.

*1) Time constant $\tau$:* The parameter $\tau$ expresses the decay of the $CI$ function and how fast it converges to $CI_{in}$. For $CI_{in} > CI_0$, the optimal cruise airspeed for larger values of $\tau$ is smaller than the optimal cruise airspeed computed for smaller values of $\tau$. As a consequence, the total energy consumption is also smaller for aircraft operating with higher values of $\tau$. Smaller values of $\tau$ cause $CI$ to converge faster to $CI_{in}$, reducing the risks of non-compliance with the ATC requirement. Based on the observed behavior of $CI$ for different values of $\tau$, a value of $\tau = 0.01t_{f_0}^*$, was chosen as the first-order filter parameter considered in this paper.

*2) Cost index and airspeed:* Figure 2 and Figure 3 depict the cost index (left) and the aircraft's airspeed (right) as a function of the total flight time for the fuel-powered and all-electric aircraft considered in this paper, respectively. The value of the cost index converges to $CI_{in_1}$ and $CI_{in_2}$ with a time constant of $\tau$. As previously stated, the parameter $\tau$ is chosen in such a way that $CI$ converges fast to the commanded value $CI_{in}$ and the aircraft's airspeed also rapidly transitions to the optimal solutions that accommodate the ATC inputs as per (11).

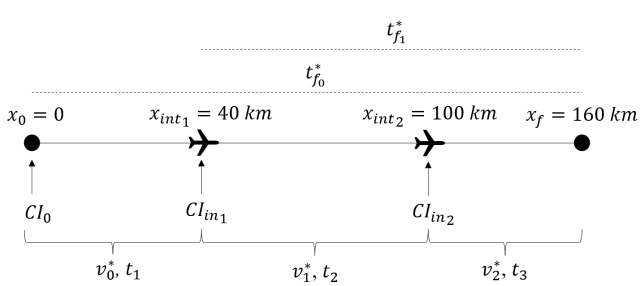


Fig. 1. Flight scenario

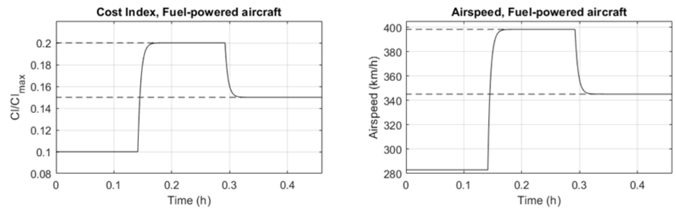


Fig. 2. Cost index (left) and airspeed (right) as a function of flight time, Fuel-powered aircraft

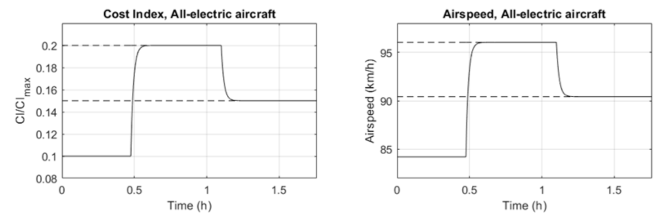


Fig. 3. Cost index (left) and airspeed (right) as a function of flight time, All-electric aircraft

*3) Energy consumption:* Figure 4 and Figure 5 show the energy available to the fuel-powered and all-electric aircraft, respectively, as a function of the distance traveled. The dashed line represents the available energy if the aircraft operated as per its original schedule, with no ATC input. However, to comply with the ATC inputs, the aircraft's airspeed was increased in the second flight segment and decreased in the third flight segment, but both revised airspeed values were higher than the airspeed that corresponds to the FMS initialization cost index. These changes resulted in a higher total energy consumption, represented by a smaller value in the final available energy.

Applying the proposed methodology for the simulated scenario, the optimal airspeed and flight time for the fuel-powered and the all-electric aircraft were determined. The changes in the aircraft airspeed resulted in a revised arrival time at the destination. Table II summarizes the simulation results. The difference between the original ETA and the actual arrival time at the destination is noted as $\Delta t_{arrival}$ and the negative sign indicates that the aircraft arrived at the destination earlier than originally scheduled.

TABLE II
SIMULATION RESULTS

| Parameter | Fuel-powered | All-electric |
|---|---|---|
| $v_0^*$ $(km/h)$ | 283.03 | 84.21 |
| $t_{f_0}^*$ | 33min55s | 1h54min |
| $t_1$ | 08min28s | 28min30s |
| $v_1^*$ $(km/h)$ | 398.24 | 96.02 |
| $t_{f_1}^*$ | 18min46s | 1h14min59s |
| $t_2$ | 09min02s | 37min29s |
| $v_2^*$ $(km/h)$ | 345.16 | 90.42 |
| $t_3$ | 10min25s | 39min49s |
| $\Delta t_{arrival}$ | -6min | -8min12s |

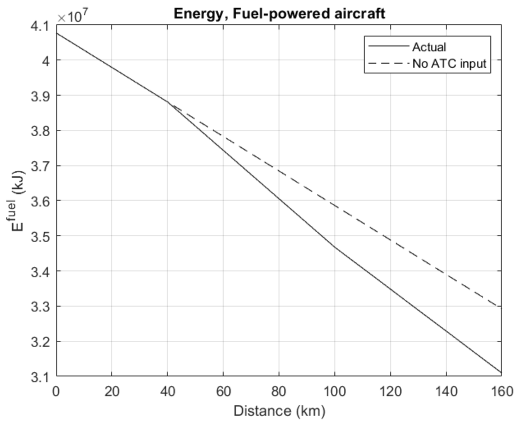


Fig. 4. Available energy as a function of distance traveled, Fuel-powered aircraft

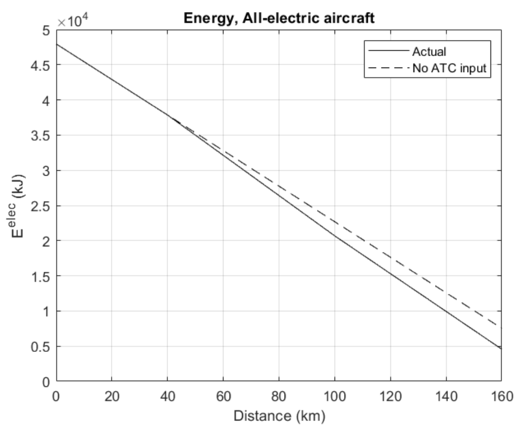


Fig. 5. Available energy as a function of distance traveled, All-electric aircraft

## V. Conclusions

This paper introduces a novel unified approach of considering a time-varying cost index in the minimization of DOC, to compute the optimal constant cruise airspeed and flight time for fuel-powered and all-electric aircraft. To the best of the authors' knowledge, this is the first time a unified approach for both fuel-powered and all-electric aircraft has been proposed where $CI$ is a time-varying signal. The proposed methodology was validated by a simulated flight scenario, where inputs from ATC were received during flight and the aircraft was required to adjust its optimal airspeed, flight time, and total energy consumption to comply with the operational restrictions imposed by ATC. The optimal values of airspeed, flight time and energy consumption were computed for both a fuel-powered and an all-electric aircraft, thus enabling applications of the proposed approach to future air mobility all-electric vehicles.

## References

[1] IATA – International Air Transport Association. "Global Air Travel Demand Continued Its Bounce Back in 2023". Available at: https://www.iata.org/en/pressroom/2024-releases/2024-01-31-02/. Accessed on 11/Jan/2024.

[2] Airbus. "Global Market Forecast 2023-2042". Available at: https://www.airbus.com/en/products-services/commercial-aircraft/market/global-market-forecast. Accessed on 10/Jan/2024.

[3] Boeing. "Commercial Market Outlook 2023-2042". Available at: https://www.boeing.com/commercial/market/commercial-market-outlook#interactive-forecast. Accessed on 13/Jan/2024.

[4] IEA – International Energy Agency. "Aviation Tracking". Available at: https://www.iea.org/energy-system/transport/aviation#tracking. Accessed on 10/Jan/2024.

[5] USA Department of Energy. "Sustainable Aviation Fuels". Available at: https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels. Accessed on 12/Jan/2024.

[6] ICAO – International Civil Aviation Organization. "Sustainable Aviation Fuel (SAF)". Available at: https://www.icao.int/environmental-protection/pages/SAF.aspx. Accessed on 08/Jan/2024.

[7] IATA – International Air Transport Association. "Disappointingly Slow Growth in SAF Production". Available at: https://www.iata.org/en/pressroom/2024-releases/2024-12-10-3/#: :text=In%202024%2C%20SAF%20production%20volumes, %25%20of%20global%20renewable%20fuel*. Accessed on 09/Jan/2025.

[8] A. Barzkar and M. Ghassemi. "Components of Electrical Power Systems in More and All-Electric Aircraft: A Review". IEEE Transactions on Transportation Electrification, VOL. 8, NO. 4, December, 2022.

[9] A. Cook, G. Tanner, V. Williams and G. Meise. "Dynamic cost indexing – Managing airline delay costs". Journal of Air Transport Management. Volume 15, Issue 1, Pages 26-35, 2009.

[10] L. Delgado, J.Martin, A. Blanch and S. Cristobal. "Hub Operations Delay Recovery based on Cost Optimisation". 6th SESAR Innovation Days, 2016.

[11] X. Prats, D. de la Torre and L. Delgado. "In-flight cost index optimisation upon weather forecast updates". IEEE/AIAA 41st Digital Avionics Systems Conference (DASC), 2022.

[12] M. G-M. Sanchez-Camacho, H. Wu, C. A. N. Cosenza, F. Krykhtine and F. Mora-Camino. "Fuzzy Setting of Cost Index for Improved Flight Management". 12th International Conference on Fuzzy Systems and Knowledge Discovery (FSKD), 2015.

[13] R. Mori. "In-flight Profile Updates by Appropriate Cost Index Selection - Operational Perspective". Trans. Japan Soc. Aero. Space Sci. Vol. 65, No. 2, pp. 56–65, 2022.

[14] I. R. de Oliveira, S. Altus, S. Tiourine, E. C. P. Neto, A. Leite and F. C. F. de Azevedo. "Stochastic Flight Plan Optimization". IEEE/AIAA 42nd Digital Avionics Systems Conference (DASC), 2023.

[15] B. Robertson, "Fuel conservation strategies: Cost index explained." Aero Quarterly, Qtr 2, pp. 26-28, 2007.

[16] H. Erzberger and H. Lee. "Constrained Optimum Trajectories with Specified Range". AIAA Journal on Guidance and Control, Vol. 3, No. 1, 1980, pp. 78–85.

[17] S. Liden. "Practical Considerations in Optimal Flight Management Computations". 1985 American Control Conference. IEEE, 1985.

[18] J. Villarroel and L. Rodrigues. "Optimal Control Framework for Cruise Economy Mode of Flight Management Systems". Journal of Guidance, Control and Dynamics. Vol. 39, No. 5, May 2016.

[19] Y. Fan, L. Yang, Q. Li, C. Nong, Z. Zheng and F. Xue. "Cost Index-based Cruise Flight Trajectory Optimization". 2020 IEEE 4th Information Technology, Networking, Electronic and Automation Control Conference, 2020.

[20] J. Sun. "OpenAP.top: Open Flight Trajectory Optimization for Air Transport and Sustainability Research". Aerospace — Open Access Aeronautics and Astronautics Journal, 2022.

[21] M. Kaptsov and L. Rodrigues. "Flight Management Systems for All-Electric Aircraft". IEEE Conference on Control Technology and Applications (CCTA), 2017.

[22] M. Wang and M. Mesbahi. "Energy Management for Electric Aircraft via Optimal Control: Cruise Phase". AIAA Propulsion and Energy Forum, 2020.

[23] A. F. El-Sayed. "Fundamentals of Aircraft and Rocket Propulsion". Springer, 2016.

[24] S. Li and L. Rodrigues. "Optimal Cruise Airspeed for Hybrid-Electric and Electric Aircraft: Applications to Air Mobility". IEEE Conference on Control Technology and Applications (CCTA), 2023.

[25] G. International. "E430 electric aircraft", 2022. Available at: https://greenwing.aero/?page id=2345. Accessed on 12/Jan/2024.