Multi-Objective Optimization of the Taper Ratio for Conical Flywheels

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***Abstract*— This work presents a multi-objective optimization framework for the design of conical flywheels with linearly varying thickness profiles. The goal was to maximize energy storage efficiency while minimizing mass and maintaining structural integrity. Key performance metrics—energy per unit mass (e_m), shape factor (k), and mass (m)—were normalized based on their optimization direction and combined into a composite score. Correlation analysis revealed strong linear dependencies between certain variables, allowing dimensionality reduction without compromising performance sensitivity. The framework efficiently evaluated 100 taper ratio values in under one second, identifying an optimal taper ratio of $T_{opt} \approx 2.38$ that achieves a balanced trade-off among competing objectives. The methodology is computationally efficient, interpretable, and generalizable to different materials and operating conditions, offering a practical tool for early-stage flywheel design.**

I. INTRODUCTION

Flywheel Energy Storage Systems (FESSs) have gained significant attention as efficient, high-performance solutions for modern energy storage needs. Operating by converting rotational kinetic energy into electrical energy, flywheels are capable of rapid charging and discharging, offer long service lives, and require minimal maintenance, making them attractive for applications in grid stabilization, transportation, aerospace, and residential energy systems [1], [2].

The performance of a flywheel is fundamentally determined by its geometry, material properties, and rotational speed. Since the stored energy is proportional to the flywheel's moment of inertia and the square of its angular velocity, optimizing its mass distribution becomes crucial to maximize energy storage while ensuring mechanical integrity under high centrifugal stresses [1], [3]. Traditional designs often relied on thick-rimmed or constant-thickness geometries; however, advances in shape optimization have shown that significant improvements in specific energy and safety margins can be achieved by refining the rotor profile [4].

Recent works have demonstrated that parametric and topology optimization methods allow designers to balance competing objectives such as maximizing specific energy, minimizing mass, and maintaining stress constraints [3], [4]. Specifically, experimental studies have shown that conical and variable-thickness flywheels outperform conventional designs in terms of energy density and stress management [2]. Furthermore, stress-constrained optimization strategies, such as those using specific energy formulations or spline-based parametric modeling, have led to designs that enhance stored energy capacity while improving fatigue life and structural safety [4].

In this context, the present study addresses the optimization of conical flywheels with linearly varying thickness profiles. Using analytical modeling, optimization-based normalization of key performance metrics, and composite scoring functions, the work aims to determine the optimal taper ratio that simultaneously maximizes energy density and minimizes mass while ensuring that maximum stresses remain within safe limits. The findings contribute to the evolving methodologies for designing high-performance and mechanically robust flywheels for future sustainable energy storage systems.

II. METHODOLOGY

A. Parameters Definition

A conical flywheel has a varying thickness, t, given by

$$t = A + Br \tag{1}$$

where

A and B are given by

$$B = \frac{t_o - ti}{2\left(r_o - ri\right)} \tag{2}$$

and

$$A = t_i - Br_i \tag{3}$$

where t_i and t_o are the inner and outer thickness of the flywheel, respectively, and r_i and r_o are the inner and outer thickness of the flywheel, respectively. These parameters are defined by the designer in the initial stage of the design.

Additionally, the stress function, F, for conical flywheels, previously obtained in [5], is given by

$$F = c_0 r + c_1/r - (3+\nu)\left(\frac{A}{8} + \frac{Br}{15}\right)\rho\omega^2 r^3 \tag{4}$$

where

r is the radius of the flywheel,

ν is the Poisson ratio of the flywheel's material,

ρ is the mass density of the flywheel's material,

ω is the angular velocity of the flywheel,

and, considering $F\left(r_i\right) = F\left(r_o\right) = 0$, c_o and c_1 are given by

$$c_0 = \frac{\left[\left(r_i^3 + r_o^3\right) + \frac{r_i^2 r_o^2}{r_i + r_o}\right] 8B + \left(r_i^2 + r_o^2\right) 15A}{120} Z \tag{5}$$

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and

$$c_1 = -\frac{8Br_ir_o\left(1+\frac{r_i}{r_o}+\frac{r_o}{r_i}\right)+15A\left(r_i+r_o\right)}{120\left(r_i+r_o\right)}r_i^2r_o^2Z \quad (6)$$

with $Z = (3+\nu)\,\rho\omega^2$.

Equation 4 is then used to calculate the radial and hoop stress, σ_r and σ_t, respectively, in the flywheel as

$$tr\sigma_r = F \quad (7)$$

and

$$t\sigma_t = \frac{dF}{dr} + t\rho\omega^2r^2 \quad (8)$$

In this paper, the main task was to find the optimal inner-to-outer thicknesses ratio, $T = \frac{t_o}{t_i}$, that would lead to maximizing the energy density. In this regard, parameters such as energy, E, volume, V, mass, m, inertia, J, energy per unit of volume, e_v, energy per unit of mass, e_m, gyradius, r_g, and shape factor, k, were considered. E, J, V, m, e_v, e_m, r_g, and k are given by [2]

$$E = \frac{1}{2}J\omega^2 \quad (9)$$

$$J = \rho\int_{r_i}^{r_o}\int_0^{2\pi}\int_{-(A+Br)}^{A+Br} r^3dzd\theta dr \quad (10)$$

$$V = \int_{r_i}^{r_o}\int_0^{2\pi}\int_{-(A+Br)}^{A+Br} rdzd\theta dr \quad (11)$$

$$m = V\rho \quad (12)$$

$$e_v = \frac{E}{V} \quad (13)$$

$$e_m = \frac{E}{m} \quad (14)$$

$$r_g = \sqrt{\frac{J}{m}} \quad (15)$$

$$k = \frac{e_v}{\sigma_t^{\max}} \quad (16)$$

To identify the optimal T that maximizes the flywheel's energy performance while minimizing undesired characteristics such as excessive mass or stress, a multi-objective optimization (MOO) approach was employed.

MOO techniques, such as evolutionary algorithms [6], [7] and Pareto front-based methods [8], have become standard in engineering design due to their ability to explore complex trade-offs between conflicting objectives. These frameworks generate a set of non-dominated solutions, allowing designers to choose optimal configurations based on specific performance priorities. However, such approaches often involve significant computational cost and require large evaluation budgets. In contrast, the present study employs a normalized weighted scoring method for its simplicity, interpretability, and computational efficiency. This choice enables the rapid exploration of geometric parameters, which is suitable for early-stage design studies or applications with limited resources.

The methodology consisted of three main stages:

B. Normalization of Metrics Based on Optimization Direction

Each performance metric was normalized individually to allow fair comparison and combination into a composite score. A min-max normalization strategy was used:

- For metrics to be maximized (e_m, r_g, and k), the normalization was:

$$x_{norm} = \frac{x - \min(x)}{\max(x) - \min(x)} \quad (17)$$

- For metrics to be minimized (m and $\sigma_t^{\max}$), the inverted normalization was applied:

$$x_{norm} = \frac{\max(x) - x}{\max(x) - \min(x)} \quad (18)$$

This ensures that all normalized values lie within the range $[0, 1]$, with higher values always representing better performance from an optimization perspective [9], [10].

C. Correlation Analysis to Avoid Redundancy

A correlation analysis was performed to detect redundant metrics. The Pearson correlation coefficient was calculated between all pairs of normalized metrics. It was found that m and $\sigma_t^{\max}$ exhibited a correlation coefficient close to 1, as well as e_m and r_g, indicating an almost perfect linear dependence.

In MOO, the use of highly correlated objectives can introduce redundancy and bias the composite score undesirably [11]. Therefore, to avoid double counting of basically the same effect, $\sigma_t^{\max}$ and $r_g$were excluded from the final score function, and only m and e_m were retained to capture the trade-off between structural efficiency and lightweight design, to represent size, mass distribution efficiency and structural limitations.

D. Composite Score Construction

After normalization and correlation filtering, a composite score was constructed as a weighted linear combination of the remaining metrics:

$$Score = w_1e_m^{norm} + w_2r_g^{norm} + w_3k^{norm} \quad (19)$$

where w_i are weighting coefficients reflecting the relative importance of each objective.

The weighting coefficients used in the composite score were selected based on engineering relevance and correlation analysis among the performance metrics. e_m was prioritized due to its direct relevance to flywheel efficiency and was assigned the highest weight.

The optimal taper ratio T_{opt} was identified as the value maximizing the composite score.

As shown in the MATLAB code below, the optimization loop computes the relevant performance metrics and evaluates a composite score to identify T_{opt}. The total runtime for evaluating 100 taper ratios was approximately 0.50 seconds, using MATLAB R2013b on an ASUS TUF Gaming A15 laptop equipped with an AMD Ryzen 7 4800H processor and 8 GB of RAM.

```
tic;
clear; clc;

% --- Constants ---
ri = 25e-3;  % inner radius (m)
ro = 150e-3; % outer radius (m)
ti = 50e-3;  % inner thickness (m)
rho = 1;     % normalized density
omega = 1;   % normalized angular...
velocity
nu = 0.3;    % Poisson’s ratio
Z = (3 + nu) * rho * omega^2;

% --- Sweep T ---
T_values = linspace(1, 4.5, 100);

% --- Preallocate only necessary...
metrics ---
m       = zeros(size(T_values));
e_m     = zeros(size(T_values));
k       = zeros(size(T_values));

% --- Main Loop ---
for i = 1:length(T_values)
    T = T_values(i);
    to = T * ti;
    B = (to - ti) / (2 * (ro - ri));
    A = ti - B * ri;

    % Volume & mass
    V_fun = @(r) 4 * pi * r .*...
    (A + B * r);
    V = integral(V_fun, ri, ro);
    m(i) = V * rho;

    % Moment of inertia & energy
    J_fun = @(r) 4 * pi * rho *...
    r.^3 .* (A + B * r);
    J = integral(J_fun, ri, ro);
    E = 0.5 * J * omega^2;

    e_m(i) = E / m(i);

    % --- STRESS CALCULATION for shape...
    factor ---
    r_vals = linspace(ri, ro, 300);
    num_c0 = ((ri + ro)*(ri^3 + ro^3) +...
    ri^2*ro^2)*8*B + (ri + ro)*...
    (ri^2 + ro^2)*15*A;
    c0 = (num_c0 / (120*(ri + ro))) * Z;

    num_c1 = -(8*B*ri*ro*(1 + ri/ro +...
    ro/ri) + 15*A*(ri + ro));
    c1 = (num_c1 / (120*(ri + ro))) *...
    ri^2 * ro^2 * Z;

    t_fun = @(r) A + B * r;
    dF_fun = @(r) c0 - c1 ./ r.^2 -...
    Z * ((3*A/8 + 4*B * r / 15) .* r.^2);
    t_vals = t_fun(r_vals);
    dF_vals = dF_fun(r_vals);
    sigma_t = dF_vals ./ t_vals + rho *...
    omega^2 * r_vals.^2;

    sigma_t_max = max(sigma_t);
    k(i) = (E / V) / sigma_t_max;
end

%% --- Normalize for optimization ---
normalize_opt = @(x, goal) ...
    (strcmp(goal, 'max')) .* ((x - min(x))...
    ./ (max(x) - min(x))) + (strcmp(goal,...
    'min')) .* ((max(x) - x) ./...
    (max(x) - min(x)));

em_norm = normalize_opt(e_m, 'max');
k_norm  = normalize_opt(k, 'max');
m_norm  = normalize_opt(m, 'min');

%% --- Define weights for composite score ---
w1 = 0.5;  % weight for e_m
w2 = 0.2;  % weight for k
w3 = 0.3;  % weight for m

score = w1 * em_norm + w2 * k_norm + w3 * m_norm;

% --- Find optimal T ---
[score_max, idx_opt] = max(score);
T_opt = T_values(idx_opt);

%% --- Report ---
fprintf('\n--- Optimal T Based on Composite Score
fprintf('T_opt = %.3f\n', T_opt);
fprintf('Score = %.4f\n', score_max);
fprintf('e_m   = %.4f\n', e_m(idx_opt));
fprintf('k     = %.4f\n', k(idx_opt));
fprintf('m     = %.4f kg\n', m(idx_opt));

%% --- Plot optimization-normalized metrics ---
figure;
plot(T_values, em_norm, 'r', 'LineWidth', 2);...
hold on;
plot(T_values, k_norm,  'b', 'LineWidth', 2);
plot(T_values, m_norm,  'm--', 'LineWidth', 2);
plot(T_opt, score(idx_opt), 'ko',...
'MarkerSize', 8, 'MarkerFaceColor', 'y');
legend({'$e_m$', '$k$', '$m$', 'Optimal $T$'},...
'Interpreter','latex', 'Location', 'best');
xlabel('$T = t_o / t_i$', 'Interpreter','latex');
ylabel('Normalized Value');
title('Optimization-normalized Metrics with...
Optimal $T$', 'Interpreter','latex');
grid on;
toc;
```

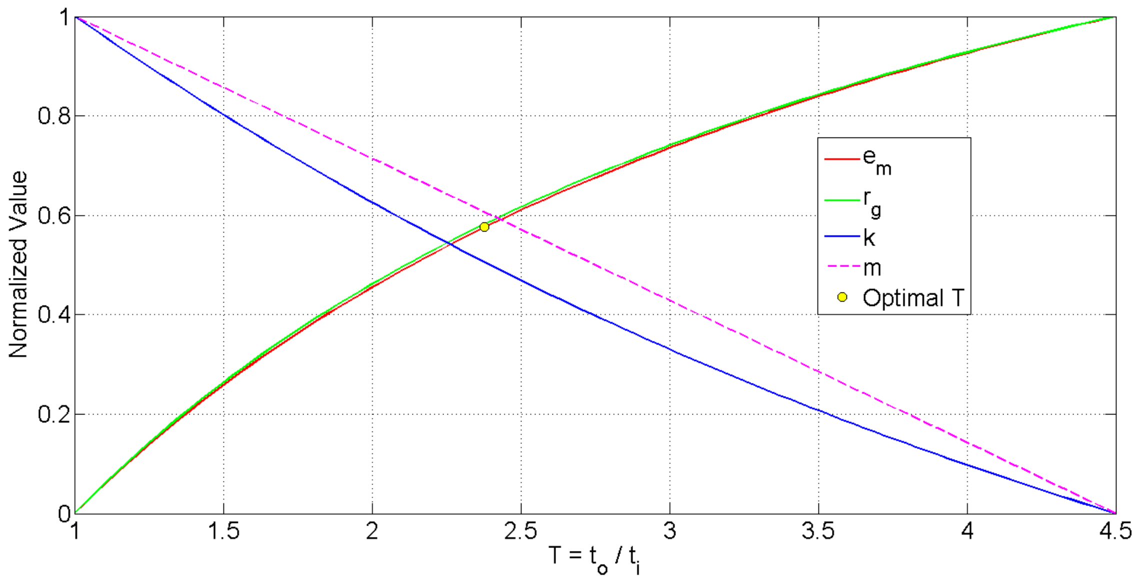

Fig. 1. Optimization-normalized performance metrics vs taper ratio (T). Higher values represent better performance.

Figure 1 presents the normalized behavior of three key performance metrics as a function of T: e_m, k, and m. Each metric has been normalized according to its optimization direction—maximization for e_m and k; and minimization for m, to enable direct comparison on a common [0,1] scale.

The trends reveal expected trade-offs: as T increases, e_m improves, indicating better energy storage efficiency and higher rotational inertia. k also increases, reflecting a more favorable balance between energy density and stress. Conversely, m decreases monotonically, which aligns with the thinning of the flywheel geometry as t_o is reduced.

The yellow marker highlights the optimal taper ratio $T_{opt} \approx 2.38$, which was identified by maximizing a composite score combining all metrics with predefined weights. This point represents a balanced trade-off between efficiency, structural safety, and mass, making it a suitable design choice for lightweight, high-performance flywheels.

III. Discussion

The results demonstrate that the proposed MOO framework effectively identifies an optimal taper ratio T that balances energy storage performance, structural safety, and material efficiency. As seen in Figure 1, the normalized trends of e_m, k, and m reveal the expected trade-offs: increasing T improves energy-related metrics but also reduces mass, up to an optimal configuration.

Correlation analysis played a central role in streamlining the optimization process. Strong linear dependence was observed between e_m and r_g, as well as between m and $\sigma_t^{\max}$. To avoid redundant objectives and potential bias in the score, only one representative metric from each pair was retained—e_m and m—while r_g and $\sigma_t^{\max}$ were excluded. Nonetheless, $\sigma_t^{\max}$ was still computed for every candidate geometry and treated as a constraint, ensuring all evaluated configurations remained within safe stress limits. This approach aligns with modern shape optimization practices that impose stress constraints rather than incorporate stress directly into the objective function.

Furthermore, the parameters A and B are not independent optimization variables, but are derived directly from inner and outer radius and thickness. For each value of T, values of A and B were recalculated to ensure geometric consistency. Their impact on the optimization is indirect: they influence mass distribution and stress levels by controlling the radial thickness profile.

The composite score was built using normalized metrics and weighting coefficients chosen based on engineering priorities and correlation filtering. e_m was assigned the highest weight due to its relevance to flywheel efficiency, while moderate weights were given to shape factor and mass. Although subjective, this weighting strategy is consistent with best practices in engineering optimization literature [9], [11], and is flexible enough to accommodate different application goals.

The optimization algorithm was implemented in MATLAB. A sweep over 100 taper ratio values was completed in approximately 0.50 seconds, confirming the method's computational efficiency. Due to the use of closed-form expressions and one-dimensional integrals, the runtime scales linearly with the number of evaluated points, making the framework suitable for rapid design iterations. Its simplicity also allows for parallelization or extension to more complex optimization spaces.

While the influence of material properties such as ρ and ω was normalized out in this study, the framework is directly extensible to real-world applications by adjusting these parameters. ν was assumed to have a minor effect on optimal geometry due to its narrow range in engineering materials (0.25–0.35). A more comprehensive sensitivity analysis of material parameters will be pursued in future work.

Finally, although the analytical expressions used in this study have been previously benchmarked [5], experimental validation and comparison with finite element models are necessary to confirm stress predictions under operational conditions. These validations will strengthen the practical applicability of the method.

IV. CONCLUSIONS

This study presented a computationally efficient and interpretable MOO framework for the design of conical flywheels with linearly varying thickness profiles. Key performance metrics—energy per unit mass, shape factor, and mass—were normalized and combined into a composite score to identify the optimal taper ratio T.

Key findings:

- A correlation analysis revealed strong dependencies between r_g and e_m, and between m and $\sigma_t^{\max}$, allowing dimensionality reduction without performance loss.
- The optimal taper ratio $T_{opt} \approx 2.38$ was identified, achieving a balanced trade-off between energy storage efficiency and structural performance.
- The method is generalizable and executes in under one second for 100 design evaluations, making it suitable for rapid parametric studies or integration into broader optimization pipelines.
- The developed methodology is generalizable to other materials and loading conditions by adjusting density and angular velocity parameters.

Future work will expand the parameter space to include additional geometric variables such as the inner-to-outer radius ratio and explore nonlinear thickness profiles (e.g., exponential tapers) for improved stress and inertia distribution. Fatigue and lifecycle analysis will also be integrated to evaluate long-term reliability, especially for applications in grid storage or electric mobility. Experimental validation and finite element comparisons are planned to confirm the analytical predictions and support manufacturing decisions.

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REFERENCES

[1] Zhihao Xu, Jie Xu, Zhiwei Chen, and Bing Zhou. A review of flywheel energy storage system technologies. *Applied Sciences*, 13(3):30, 2023.

[2] et al. Semsri, P. Experimental design of flywheel rotor with a flywheel energy storage system for residential uses. *Journal of Physics: Conference Series*, 2437:012061, 2023.

[3] S. M. Kale and P. Panchal. Stress constrained topology optimization of energy storage flywheels using a specific energy formulation. *International Journal of Mechanical Sciences*, 241:107058, 2023.

[4] L. Jiang, W. Zhang, G. J. Ma, and C. W. Wu. Shape optimization of energy storage flywheel rotor. *Structural and Multidisciplinary Optimization*, 54:97–111, 2016.

[5] Miguel Garcia, Onofre Orozco-López, Jesús Uribe-Chavira, and Andrés Blanco-Ortega. An analytical approximation of the stress function for conical flywheels. *Applied Mechanics*, 6(2), 2025.

[6] Kalyanmoy Deb. *Multi-objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, 2001.

[7] Carlos A. Coello Coello. Evolutionary multi-objective optimization: a historical view of the field. *IEEE Computational Intelligence Magazine*, 1(1):28–36, 2006.

[8] Eckart Zitzler and Lothar Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. *Evolutionary Computation*, 8(2):173–195, 2000.

[9] S. S. Rao. *Engineering Optimization: Theory and Practice*. Wiley, Hoboken, NJ, 4 edition, 2009.

[10] Evangelos Triantaphyllou. *Multi-Criteria Decision Making Methods: A Comparative Study*, volume 44 of *Applied Optimization*. Springer, Boston, MA, 2000.

[11] Robert T. Marler and Jasbir S. Arora. Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26(6):369–395, 2004.