

Maglev System Control Using a New Adaptive Super Twisting Theory

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Abstract— A new finite time second order terminal sliding mode controller is proposed for a magnetic levitation train in this paper. Sliding mode which is nonlinear and robust controller is used because of existent the nonlinearity and uncertain terms in the dynamics of the maglev train system. To overcome on the chattering and produce a smooth control signal, the famous super twisting algorithm is used. Also the proposed control law is consists of two adaptive terms to estimate uncertain function. Moreover, the proposed controller insures the finite time stability of sliding surface and the finite time stability of the errors is achieved too, due to use terminal sliding mode theory. Simulations are performed to evaluate the controller and its high accuracy. Robustness of the proposed control scheme against uncertainty is also investigated.

I. INTRODUCTION

High speed magnetic levitation (Maglev) trains technology has been researched in recent years. The following dynamics are usually nonlinear and unstable [1, 2]. Recently, many studies have reported different control systems for Maglev trains. Traditionally, the controllers have been proposed using linearized models based on their nominal operating points. Since, some controllers are presented based on nonlinear techniques such as feedback linearization [3, 4], back-stepping [5] and the nonlinear H_∞ [6].

The control performance is affected due to the uncertainties. For the uncertain nonlinear Maglev system with unknown parameters, sliding mode (SM) controllers are proposed in [7] and [8]. However, the main challenge in SM theory is the chattering which is due to discontinuity in the controller signal [9, 10]. The discontinuous conventional SM controller commonly is replaced by a continuous to avoid chattering, that decrease accuracy [11].

Another method for eliminating chattering is higher order sliding mode (HOSM) theory. In the super-twisting (ST) second order sliding mode (SOSM) control, the only requirement is to measure the sliding variable compared to other SOSMs [12-14]. The ST SM controller is designed for maglev system in [15] but he finite time convergence analysis for uncertain nonlinear systems with SOSMs has not been performed with the use of the Lyapunov theory. Moreover some control algorithms are given with estimation of the uncertainties and only asymptotic stability

of this algorithm is proved by using the Lyapunov theory [16, 17].

To date, adaptive SMCs have been designed in order to reduce chattering [18, 19]. Based on adaptive conventional SM control theory, Maglev control systems are designed in [20].

Another challenge in the conventional SMC is that the system states cannot converge in a finite time. Terminal sliding mode (TSM) control was proposed, which insure the finite time convergence in the sliding phase [21, 22].

In this paper the control problem of Maglev system is considered. First a model of a Maglev system is considered. Then a coordinate transformation is applied. External disturbance and the effects of uncertainties cannot be measured directly. Since, the SMC theory is used. To achieve higher precision without chattering, this study uses adaptive super twisting SMC theory. Also in the proposed algorithm, terminal sliding variable is used in the sliding phase, which insures the finite time convergence in sliding phase. Therefore the applied algorithm decreases chattering effects.

II. MODELLING OF THE MAGLEV SYSTEM

The schematic of the Maglev system can be seen in Figure 1 and the parameters are listed in Table I.

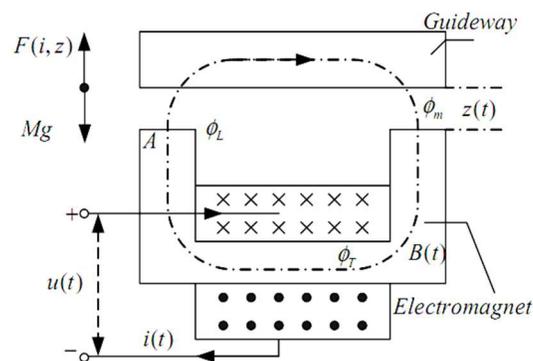


Figure 1. Schematic diagram of the magnetic levitation system [4]

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TABLE I. TABLE I. PARAMETERS

variable	Significations
ϕ_m	Gap flux
ϕ_T	Main flux
ϕ_L	Leakage flux
$u(t)$	Magnetic coil voltage
R	Magnetic coil resistance
$B(t)$	Gap flux density
μ_0	Magnetic permeability of atmosphere
F	Electromagnetic force
A	Magnetic pole area
$z(t)$	Electromagnet and track distance
N	Number of magnetic coils
M	Total mass
$i(t)$	Magnetic coil current

The assumptions taken into account in this study are listed below:

- Magnetic flux formula is given by $\phi_T = \phi_L + \phi_m$, neglecting leakage flux $\phi_L = 0$.
- Deformation and the elastic vibration of track are neglected.
- The track has infinite rigidity in regards to the electromagnetic.

The model of the Maglev system is written as [2, 4]:

$$\begin{aligned}
 B(t) &= \frac{\mu_0 N i(t)}{2z(t)}, \quad F = \frac{B(t)^2 A}{\mu_0} \\
 M\ddot{z}(t) &= Mg - F \\
 u(t) &= \frac{2Rz(t)}{\mu_0 N} B(t) + NAB\dot{(t)}
 \end{aligned} \tag{1}$$

The variables are chosen as:

$$\begin{aligned}
 x &= [x_1 \quad x_2 \quad x_3]^T = [z \quad \dot{z} \quad B]^T \\
 y &= z = x_1
 \end{aligned} \tag{2}$$

Therefore the model of the Maglev system is:

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= g - \frac{A}{\mu_0 M} x_3^2 \\
 \dot{x}_3 &= -\frac{2R}{\mu_0 AN^2} x_1 x_3 + \frac{1}{NA} u
 \end{aligned} \tag{3}$$

Based on the following change in the coordinates:

$$\begin{aligned}
 z_1 &= x_1 - x_{1d}, \\
 z_2 &= x_2, \\
 z_3 &= g - \frac{A}{\mu_0 M} x_3^2
 \end{aligned} \tag{4}$$

If z_1, z_2, z_3 are driven to zero, then x_1 converges to x_{1d} , x_2 converges to zero and x_3 converges to $\sqrt{\frac{\mu_0 M}{A} g}$.

The model of the Maglev system can be described as:

$$\begin{aligned}
 \dot{z}_1 &= z_2 \\
 \dot{z}_2 &= z_3 \\
 \dot{z}_3 &= f(x) + g(x)u + d(t)
 \end{aligned} \tag{5}$$

where:

$$\begin{aligned}
 f(x) &= \frac{4R}{\mu_0^2 MN^2} x_1 x_3^2 \\
 g(x) &= -\frac{2}{\mu_0 MN} x_3
 \end{aligned} \tag{6}$$

$d(t)$ is system uncertainty.

III. DESIGN OF THE ADAPTIVE SOTSM CONTROLLER

The conventional sliding variable for dynamical system (5) is defined as:

$$s = z_3 + \lambda_1 z_2 + \lambda_2 z_1 \tag{7}$$

where λ_1 and λ_2 are positive scalars. Note that this choice of the sliding variable for system (5) ensures that $z_1 = x_1 - x_{1d} \rightarrow 0$ when $s = 0$.

The sliding dynamics is found from

$$\dot{s} = f(x) + g(x)u + d(t) + \lambda_1 z_3 + \lambda_2 z_2 \tag{8}$$

In conventional SMC the reaching control section is designed as:

$$u = \frac{1}{g(x)} (-f(x) - \lambda_1 z_3 - \lambda_2 z_2 - k \text{sign}(s)) \tag{9}$$

where k is the reaching gain. A Lyapunov function

$$V = \frac{1}{2} s^2 \tag{10}$$

is used to ensure stabilization of S and the finite time stability condition

$$\dot{V} = s\dot{s} \leq -\eta|s| \quad (11)$$

must be satisfied, where η is a positive constant such that:

$$t_r \leq \frac{|s(0)|}{\eta} \quad (12)$$

and t_r is the reaching time.

This paper proposes a novel simple finite time adaptive terminal second order SMC to control the system of (8). First, the following terminal sliding variables are defined:

$$\begin{cases} s_0 = z_1 \\ s_1 = \dot{s}_0 + \lambda_1 s_0^{p_1/q_1} \\ s_2 = \dot{s}_1 + \lambda_2 s_1^{p_2/q_2} \end{cases} \quad (13)$$

where $\lambda_i > 0$, $p_i > q_i > 0$ and p_i and q_i are positive odd integers. The controller is designed to derive s_2 to zero in a finite time. Then from (13) yields

$$\dot{s}_1 = -\lambda_2 s_1^{p_2/q_2} \quad (14)$$

The convergence time of s_1 is t_1 as:

$$t_1 = -\frac{1}{\lambda_2} \int_{s_1(0)}^0 \frac{ds_1}{s_1^{p_2/q_2}} = \frac{|s_1(0)|^{1-p_2/q_2}}{\lambda_2(1-p_2/q_2)} \quad (15)$$

Now we have $s_1=0$ and from (13) yields:

$$\dot{s}_0 = -\lambda_1 s_0^{p_1/q_1} \quad (16)$$

The convergence time of $s_0=z_1$ is t_0 and for a solution of the system (16) is given by

$$t_0 = -\frac{1}{\lambda_1} \int_{s_0(t_1)}^0 \frac{ds_0}{s_0^{p_1/q_1}} + t_1 = \frac{|s_0(t_1)|^{1-p_1/q_1}}{\lambda_1(1-p_1/q_1)} + t_1 \quad (17)$$

Therefore the finite time convergence $s_0 = z_1$ when $s_2 = 0$ is insured. Then we have:

$$\begin{aligned} \dot{s}_2 &= \dot{z}_3 + \frac{d^2}{dt^2} \left(\lambda_1 s_0^{p_1/q_1} \right) + \frac{d}{dt} \left(\lambda_2 s_1^{p_2/q_2} \right) \\ &= f(x) + g(x)u + d(t) + \frac{d^2}{dt^2} \left(\lambda_1 s_0^{p_1/q_1} \right) \end{aligned} \quad (18)$$

$$+ \frac{d}{dt} \left(\lambda_2 s_1^{p_2/q_2} \right)$$

The controller given by:

$$\begin{aligned} u &= \frac{1}{g(x)} \left[-f(x) - \frac{d^2}{dt^2} \left(\lambda_1 s_0^{p_1/q_1} \right) \right. \\ &\quad \left. - \frac{d}{dt} \left(\lambda_2 s_1^{p_2/q_2} \right) \right. \\ &\quad \left. - k_1 |s_2|^{1/2} \text{sign}(s_2) + w - k_3 \xi \right] \end{aligned} \quad (19)$$

$$\dot{w} = -k_2 \text{sign}(s_2)$$

$$\dot{\xi} = k_2 k_3 |\xi| \text{sign}(s_2) - k_4 \text{sign}(\xi)$$

with conditions:

$$\begin{cases} k_1, k_2 > 0 \\ k_4 = L_g (L_d + |w|) + \eta \end{cases} \quad (20)$$

stabilizes system (18), and guarantees the convergence of the integral term $-w$ to the uncertainty $d(t)$ in a finite time. Parameters k_1 , k_2 , k_3 and η are positive constant and $|d(t)| \leq L_d$ and $|\dot{d}(t)| = |g(t)| \leq L_g$ are bounds of the uncertainty and its first derivative.

Proof: By controller (19) in system (18) we have:

$$\begin{cases} \dot{s}_2 = -k_1 |s_2|^{1/2} \text{sign}(s_2) - k_3 \xi + d + w \\ \dot{w} = -k_2 \text{sign}(s_2) \\ \dot{\xi} = k_2 k_3 |\xi| \text{sign}(s_2) - k_4 \text{sign}(\xi) \end{cases} \quad (21)$$

for $e = d(t) + w$, the dynamics are:

$$\begin{cases} \dot{s}_2 = -k_1 |s_2|^{1/2} \text{sign}(s_2) - k_3 \xi + e \\ \dot{e} = \dot{d}(t) - k_2 \text{sign}(s_2) \\ \dot{\xi} = k_2 k_3 |\xi| \text{sign}(s_2) - k_4 \text{sign}(\xi) \end{cases} \quad (22)$$

Now, considering:

$$V = k_2 |s_2| + \frac{1}{2} e^2 + |\xi| \quad (23)$$

with $k_2 > 0$, the time derivative of V is:

$$\begin{aligned} \dot{V} &= k_2 \frac{s_2}{|s_2|} \dot{s}_2 + e\dot{e} + \xi\dot{\xi} = \\ &k_2 \frac{s_2}{|s_2|} \left(-k_1 |s_2|^{1/2} \text{sign}(s_2) - k_3 \xi + e \right) \\ &+ e(g(t) - k_2 \text{sign}(s_2)) \\ &+ \frac{\xi}{|\xi|} (k_2 k_3 |\xi| \text{sign}(s_2) - k_4 \text{sign}(\xi)) = \end{aligned} \quad (24)$$

$$\begin{aligned} &-k_2 k_1 |s_2|^{1/2} - k_2 \frac{s_2}{|s_2|} k_3 \xi + k_2 \frac{s_2}{|s_2|} e + g(t)e \\ &-k_2 \text{sign}(s_2)e + k_2 k_3 \text{sign}(s_2) \xi - k_4 = \\ &-k_1 k_2 |s_2|^{1/2} + g(t)e - k_4 \end{aligned}$$

For $k_1, k_2 > 0$ and $k_4 = L_g (L_d + |w|) + \eta$, we have:

$$\begin{aligned} \dot{V} &= -k_1 k_2 |s_2|^{1-\gamma} + g(t)(d(t) + w) \\ &- L_g (L_d + |w|) - \eta \leq -\eta \end{aligned} \quad (25)$$

That implies

$$0 \leq V(x(t)) = V(x(0)) - \eta t \quad (26)$$

And defines

$$t \leq \frac{V(x(0))}{\eta} \quad (27)$$

Therefore (25) guarantees the finite time convergence [10, 23].

IV. SIMULATION AND RESULTS

Simulations are performed and the proposed adaptive terminal super twisting controller is compared to the approximated sliding mode controller. Parameters and controller gains are as listed in table II.

Mass position, velocity and gap flux density are plotted in figures 2 to 4. Mass position is controlled with higher precision compared to the approximated SMC. The higher speed of the proposed method is evident in figure 3. Due to the difference in flux density between the proposed method and the approximated SMC, as seen in figure 4, the proposed method demonstrates higher accuracy in position control. The reduced accuracy and lower speed in the approximated method are attributed to the use of the saturation function instead of the sign function to eliminate chattering. The control input, is plotted in figure 5. Chattering effect is removed in the proposed control system as same in the approximated SMC. As shown in figures 6 to 8 the sliding variables reached to zero using proposed controller but the precision in the approximated SMC is reduced due to approximation of control law in the boundary layer. The first adaptive term and uncertainty are plotted in figure 9. As seen in this figure, the uncertain function is estimated in a finite time and estimation error reaches to zero.

TABLE II. TABLE II. PARAMETER AND THE CONTROLLER GAINS

Parameter	Value	
Maglev System	$R(\Omega)$	0.5
	μ_0	$4\pi \times 10^{-7}$
	$A(m^2)$	0.0235
	$N(\text{turns})$	324
	$M(\text{kg})$	635
	$z_d(m)$	0.01
Adaptive Super Twisting	k_1	75
	k_2	1000
	k_3	0.1
	k_4	1000
Approximated SMC	k	400

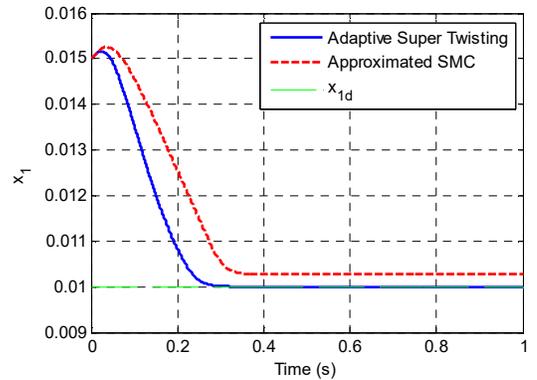


Figure 2. Position of the mass (x_1)

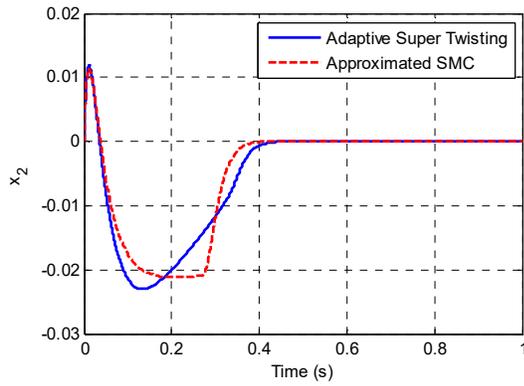


Figure 3. Velocity of the mass (x_2)

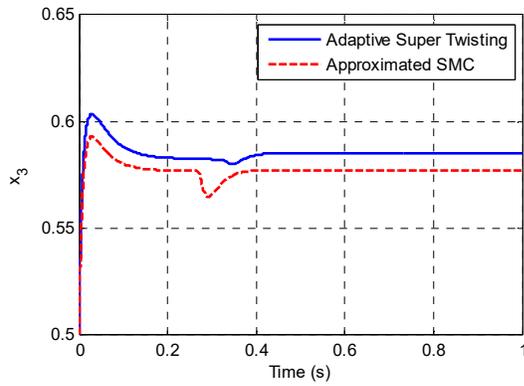


Figure 4. Gap flux density (x_3)

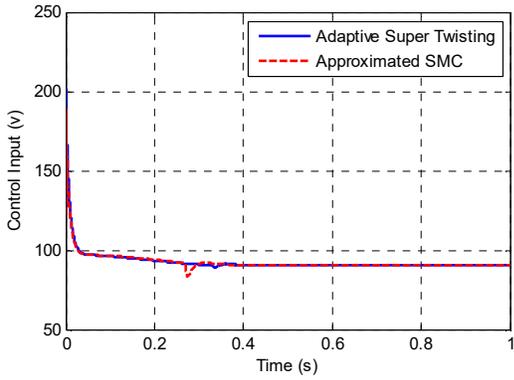


Figure 5. Control input signal generated by the controllers

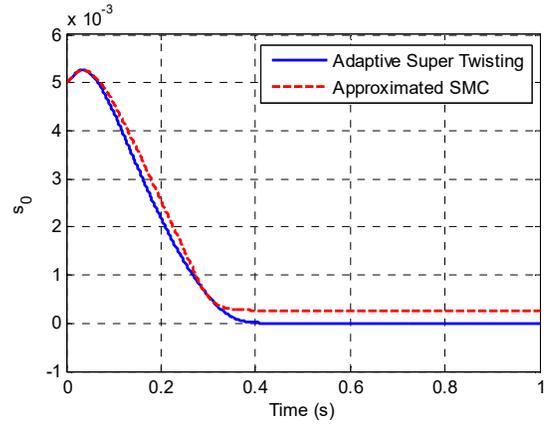


Figure 6. Sliding variable (s_0)

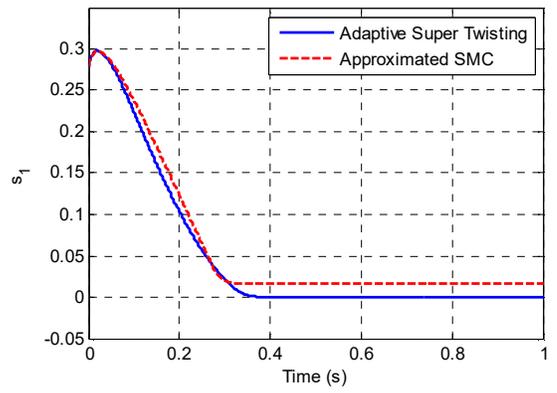


Figure 7. Sliding variable (s_1)

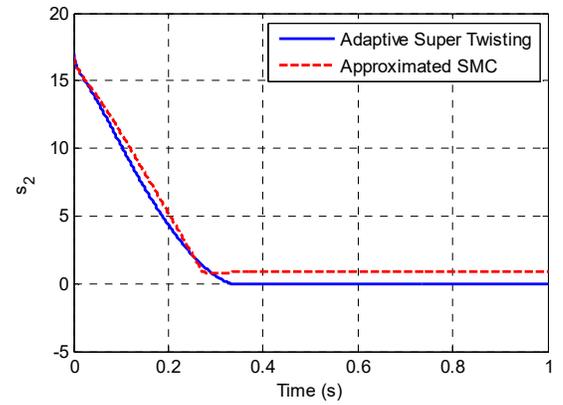


Figure 8. Sliding variable (s_2)

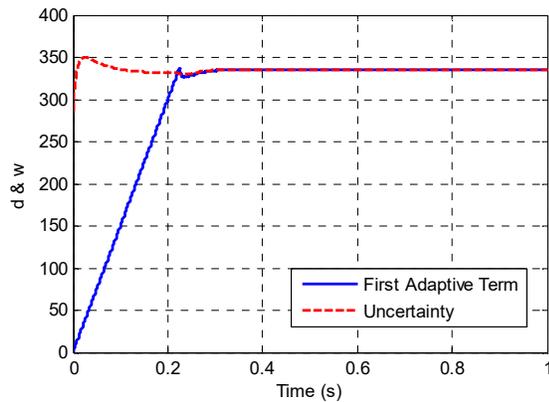


Figure 9. The first adaptive term and Uncertainty

V. CONCLUSION

This paper proposes an adaptive super twisting terminal SMC for the nonlinear magnetic levitation train system including uncertainty effects. A model of maglev systems is considered with position and velocity of the mass and magnetic flux as state variables. Due to nonlinearity and uncertainty in the model of Maglev, the SMC theory is used. To generate smooth voltage commands as control signal, the proposed controller is designed based on super twisting second order SMC theory. The proposed control law includes adaptive terms that one of them converges to the uncertainty in a finite time. Also the finite time stability is achieved in sliding phase, due to use terminal sliding mode theory. Simulation results show that the controller is able to control the position in finite time with higher accuracy than approximated SMC. Furthermore, the controller generates smooth control signals.

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