Application of the Projective Geometry Principle to Describe the Dynamics of Smart Grid Modes*

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Abstract — **When designing a managed electric distribution network (Smart Grid) with renewable energy sources and the elements of Flexible Alternative Current Transmission System (FACTS), it is necessary to take into account the level of distortion of the voltage and current curve when assessing the quality of electricity supplied to consumers. The article is devoted to the search for an approach to describing the dynamics of Smart Grid modes, which enables to study the qualitative influence of FACTS devices on the parameters of the electrical network, including non-sinusoidal voltage and current. The analysis of possible approaches and methods for describing the dynamics of the parameters of Smart Grid modes has been carried out. The approaches based on metric geometry and affine geometry have been considered. It has been established that the classical methods used to analyze static processes are not applicable in the Smart Grid. The use of the principles of projective geometry is proposed as a promising approach. The features of the application of projective geometry are shown on the example of a Smart Grid section. The proposed approach based on projective geometry allows to qualitatively describe the dynamics of Smart Grid modes.**

I. Introduction

The Smart Grid is one of the key areas of development of the electric power industry worldwide [1, 2]. These are managed electric networks with distributed generation sources, energy storage devices, and FACTS (Flexible Alternative Current Transmission System) devices [3–5]. Smart Grids are characterized by flexibility, accessibility, manageability, and energy efficiency [6, 7]. The integration of communication and electrical networks with the introduction of intelligent electronic devices and automation systems transforms traditional electrical networks into a Smart Grid. This integration implies the formation of such a property of the network as "self-healing" [8, 9].

When designing a Smart Grid, it is necessary to take into account the influence of FACTS devices on the quality of electrical energy. The introduction into the electrical network of power plants based on renewable energy sources with stochastic generation and power electronics elements with nonlinear voltage characteristics [10, 11] leads to a violation of the sinusoidal shape of the voltage and current curve [12, 13]. The appearance of higher harmonic and interharmonic components of voltage and current is critical for the Smart Grid, as it contains equipment susceptible to electromagnetic interference in the supply network (routers, computing equipment, data processing servers, etc.).

The existing methods of designing electrical networks are not suitable for the Smart Grid, as they do not take into account the features described above and do not pay due attention to the problem of non-sinusoidal. At the same time, the value of design accuracy increases: design results are the starting data for forecasting the development of the energy system and play an important role in making the final decision.

In [14], the importance of reducing harmonic components is noted. However, the primary task is to accurately predict the amplitude and phase of harmonics and interharmonics. The analysis of publications devoted to determining the harmonic order has been performed [15-17]. [18] describes a Vortex Search Algorithm for determining the order and amplitudes of harmonic and interharmonic components of current and voltage. The use of the kalman filter to determine harmonic components in real time is discussed in [19]. There is no comprehensive assessment of non-sinusoidality in all these studies. It is a problem of determining not only the order and amplitude of harmonics and interharmonics, but also the dynamics of non-sinusoidal disorder. The well-known dynamics of non-sinusoidal current and voltage provides a deeper understanding of the process and allows for proactive measures.

To evaluate the harmonic and interharmonic voltage components in a Smart Grid generated by FACTS devices, a technique based on the window Fourier transform has been developed [12, 13]. A correlation function is used to determine the order of harmonic and interharmonic components of voltage and current. It describes the process of changing the mutual connections of the input and output terminals of devices over time. The method of signal directed graphs is used for visual visualization. To determine the amplitude-frequency spectrum, a windowed Fourier transform is used using the Chebyshev window function [20].

Due to the dynamism of processes and the interaction of elements of an electrical network characteristic of the Smart Grid, in addition to the mathematical representation of non-sinusoidality and the definition of the amplitude-frequency spectrum, the task arises to describe the very "mechanism" of mode change in an electrical circuit.

The purpose of the work is to define an approach to describing the dynamics of Smart Grid modes, which enables to study the influence of FACTS devices on the parameters of the electrical network.

The object of the study is a Smart Grid section with a FACTS device.

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II. MATERIALS AND METHODS

An analysis of several approaches to the study of mode parameters in Smart Grid systems has been conducted.

If the electrical network is represented as an active impedor (Fig. 1), according to the Helmholtz–Thevenin theorem (the equivalent generator theorem) [21], it can be replaced by an equivalent voltage source whose electromotive force (EMF) is equal to the open-circuit voltage at the terminals (U_{OC}), and an input impedance relative to these terminals. If X_0=const, the variability of the operating modes is determined by a family of volt-ampere characteristics of X_L with a common origin at the coordinate origin [22-24].

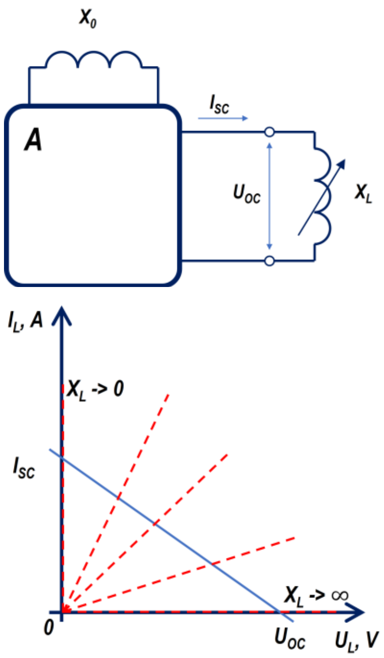

Figure 1. Representation of the Smart Grid as an active impedor with X_0=const.

In a real system X_0≠const and changes over time, including due to the operation of power electronics-based devices during voltage regulation and power flow redistribution (Fig. 2). When X_0≠const, an additional family of straight lines is formed with a common origin at the point U_{OC} (green dashed lines) [24].

In practice, when analyzing the dynamics of mode changes in a Smart Grid, it is also necessary to determine the current variation (increment) ΔI_N in N-branch resulting from a change in resistance in M-branch by a given amount ΔX_M (Fig. 3) [24].

To solve this problem, we will use the variation theorem (the theorem of mutual increments) [25]. According to the variation theorem, the current increments in an electrical circuit depend linearly on the resistance increment in one of the circuit branches. Let us determine the current increment in N-branch:

$$\Delta I_N = \frac{\Delta X_M I_{M0}}{1+\Delta X_M Y_{MM}} Y_{NM} \ , \quad (1)$$

where I_{M0} – the current in M-branch under X_M=0; Y_{MM} – input conductance; Y_{NM} – mutual conductance.

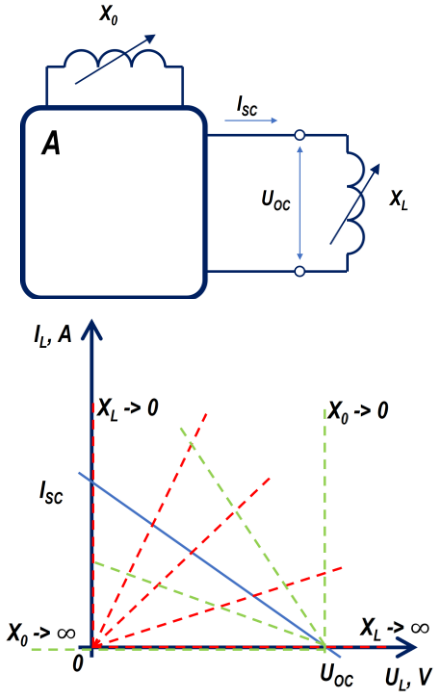

Figure 2. Representation of the Smart Grid as an active impedor with X_0≠const.

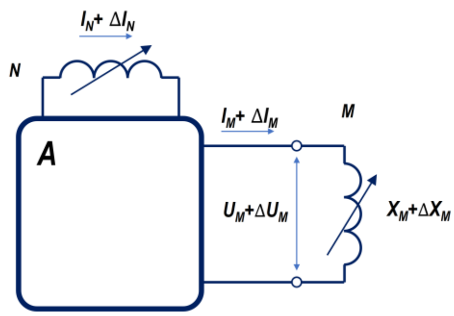

Figure 3. Representation of the Smart Grid as an active impedor with designated M- and N-branches.

To express the values in per-unit terms, the current and voltage values prior to the resistance change X_M are used. This raises the question: which values should be taken as the base values for per-unit calculations – the short-circuit test current and open-circuit voltage values (Fig. 1 and Fig. 2), or the current and voltage values prior to the resistance change (Fig. 3). The preference for per-unit calculations is due to the ability to compare operating modes of different circuits with each other. It is necessary to identify a quantity that remains constant across all possible operating modes of the circuit.

At the same time, analyzing the operating mode in terms of changes in the parameters of electrical circuit elements to solve the issue of accounting for waveform distortion in the design of Smart Grid systems. Thus, if components with frequencies different from the fundamental frequency (higher harmonics and interharmonics) arise during the operation of the network, this situation can be interpreted in circuit analysis as a change in the impedance of a circuit element for the component with frequency N, from a value $X_1^N \to \infty$ (since there was no Nth component in the network, therefore, the corresponding impedance tends to infinity) to some specific finite value [5, 10].

The formulated problem is similar to a criterion that defines the object of study for various types of geometries. Each type of geometry studies those properties of figures that remain unchanged (invariant) under a specific group of transformations [22–24]. One of the promising approaches is the use of principles from projective geometry [22, 23].

III. Results

The representation of the Smart Grid (Fig. 1) can be analyzed using metric geometry (Fig. 4). The current, voltage, and resistance lines are parallel lines. The operating mode is defined by the movement of the line OO′. It should be noted that the resistance line is shown for illustrative purposes only. From a strictly mathematical point of view, this representation is not entirely accurate: the current and voltage lines are scaled in per-unit values, while the resistance line is scaled in absolute units [22].

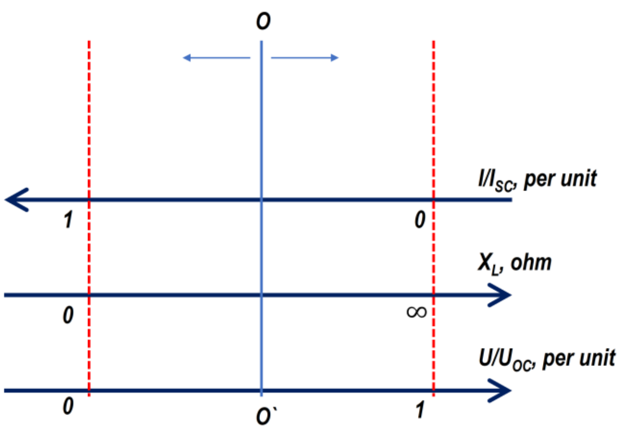

Figure 4. Representation of the operating mode with X_0=const using metric geometry.

The dependence of current on voltage in a specific operating mode can be expressed by the formula:

$$I_1 = I_{sc} - \frac{U_1}{X_0} \ . \quad (2)$$

Such a linear dependence is similar to affine transformations, which are expressed by the formulas:

$$x' = a_1x + b_1y + c_1; \quad (3)$$

$$y' = a_2x + b_2y + c_2. \quad (4)$$

Accordingly, the case with X_0=const can also be represented as an affine transformation, where S is the center of projection. The operating mode is defined by the rotation of the line SO` around point *S* within specified limits (red dashed lines in Fig. 5) [22].

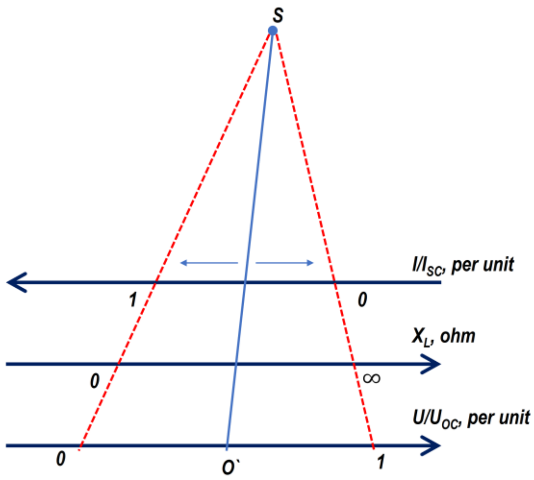

Figure 5. Representation of the operating mode with X_0=const using affine geometry.

In the case of X_0≠const, the methods of metric and affine geometry are insufficient, since a change in resistance X_0 by an arbitrary value Δ, characterizing a new steady-state operating mode, leads to changes in both the short-circuit current value I_{sc} and the open-circuit voltage U_{oc}. To describe the dynamics of operating mode changes in the Smart Grid, it is proposed to use projective transformations. Projective transformations in Cartesian coordinates are defined by the functions:

$$x' = \frac{a_1x + b_1y + c_1}{a_3x + b_3y + c_3}; \quad (5)$$

$$y' = \frac{a_2x + b_2y + c_2}{a_3x + b_3y + c_3}. \quad (6)$$

The projective coordinate of the point corresponding to the current operating mode is defined by the parameter *m*, which is determined based on various mode parameters such as R_{load}, U_{load}, I_{load}. The projective transformation originates from the center of projection *S* (Fig. 6).

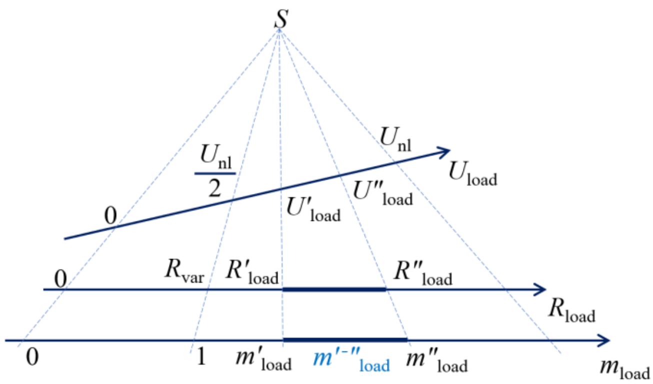

Figure 6. Projective transformations $R_{load} \rightarrow U_{load}$.

The cross-ratio m'_{load} can be defined by the following expression:

$$m'_{load} = \frac{R'_{load}}{R_{var}} = \frac{U'_{load}}{U_{nl} - U'_{load}} \ . \quad (7)$$

When the operating mode changes $R'_{\text{load}} \rightarrow R''_{\text{load}}$:

$$m_{\text{load}}^{''-'} = \frac{R''_{\text{load}}}{R'_{\text{load}}} = \frac{U''_{\text{load}}}{U_{\text{nl}} - U''_{\text{load}}} \div \frac{U'_{\text{load}}}{U_{\text{nl}} - U'_{\text{load}}} . \quad (8)$$

The essential group properties of the cross-ratio hold: $m''_{\text{load}} = m''^{-'}_{\text{load}} \cdot m'_{\text{load}}$.

Based on (9), the following can be derived:

$$\frac{U''_{\text{load}}}{U_{\text{nl}}} = \frac{\frac{U''_{\text{load}}}{U_{\text{nl}}} \cdot m_{\text{load}}^{''-'}}{\frac{U'_{\text{load}}}{U_{\text{nl}}}\left(m_{\text{load}}^{''-'} - 1\right) + 1} . \quad (9)$$

The resulting transformation with the parameter $m''^{-'}_{\text{load}}$ maps the initial operating point U'_{load} to the subsequent point U''_{load}.

IV. Conclusion

The approaches aimed at solving the problems that arise when designing a Smart Grid have been considered. It has been established that the existing methods of designing electrical networks are not suitable for the Smart Grid, as they do not take into account its features and do not pay due attention to the problem of higher harmonics and interharmonics.

The analysis of several approaches to the study of the mode parameters in the Smart Grid has been carried out. The methods based on the Helmholtz–Thevenin theorem and the variation theorem have been considered. It has been established that the use of relative units for analysis is very informative, as it allows comparing the modes of different circuits. The disadvantage of the considered approaches is that even when analyzing the same circuit, but using different theorems, different values (short-circuit current and no-load voltage or the value of current/voltage before the change) act as the basis. As a result, the same mode (physically existing in a real circuit) is described by unequal relative values.

A promising approach to describe the dynamics of Smart Grid modes is to use the principles of projective geometry. Due to invariant (unchanging) relationships, projective geometry allows for a deep and detailed study of the properties of electrical networks with variable (changeable) operating modes. The projective geometry allows to make a qualitative analysis in the electrical network. This approach enables to describe the dynamics of the mode change in the Smart Grid in a visual geometric interpretation. The study of the "mechanism" of the interaction of chain elements using projective geometry reveals additional properties of the chain. The projective geometry takes into account the mutual influence of loads and reveals additional properties of the circuit.

Future research is aimed at solving the applied problems of applying the method of projective geometry to analyze the volt-ampere and control characteristics of FACTS devices in order to study the "mechanism" of their influence on the quality of electrical energy in a Smart Grid.

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