

A comparative study between flatness-based control with an exact observer and functional observer-based feedback control

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Abstract—This paper presents a comparative study between flatness-based control and functional observer-based feedback control, with a focus on how each method addresses control tasks in systems with complex dynamics. Both control strategies are widely utilized in applications such as robotics, aerospace, and automotive systems, where efficient and reliable control is essential. The paper explores the key principles, advantages, and limitations of each approach, providing a detailed comparison of their applicability, design complexity and robustness. Our paper builds on our previous work [11] which focused exclusively on flatness-based control for the Ball Balancing Table (BBT) and its simulation results. We now extend this by comparing it with a functional observer-based feedback controller for the same system.

Index Terms—Flatness; Multiple-Input Multiple-Output systems; Linear Functional Observer; Ball Balancing Table

I. INTRODUCTION

The theory of linear time-invariant [10] systems provides a diverse range of design methods and solutions for control problems, as demonstrated by previous research.

This includes strategies such as state feedback controllers, flatness-based control, observers, Kalman filters [8], and various other control techniques.

The stability problems of control systems with saturation have been a source of significant attention in the research community for many years and are still challenging both theoretically and practically. The application of flatness theory has been prevalent across various studies, utilizing techniques such as solving the Bezout equation and implementing pole placement to achieve desired trajectories [1] and [2].

This approach was developed in the stationary and non-stationary cases for SISO and MIMO system [1] in continuous and discrete time formalism and has demonstrated that it can reach good trajectory tracking performances [2]. Additionally, a flatness-based control law with an exact observer [11] was previously studied, where a controller was established without the need to solve the Bézout equation, providing satisfactory results in tracking reference trajectories.

On the other hand, the design of linear functional observers has been explored by numerous researchers. One of the most commonly used algebraic methods in this area is based on solving the generalized Sylvester equation. However, solving

this equation remains an area that requires further investigation and the development of more efficient techniques. This method is advantageous because it is simple and does not require any changes in coordinates. A minimal-order functional observer for time-invariant discrete-time linear systems was studied in [9] to control problems of the ball balancing table system. The evaluation of this approach was done by implementing a state feedback controller with a multifunctional observer in a control laboratory experiment.

Our proposal in this paper is to compare the robustness of flatness-based control with an exact observer strategy, as developed in [11], with functional observer-based feedback control, focusing on the advantages and limitations of both approaches in the presence of disturbances. The structure of this paper is as follows: Section I introduces the fundamental concepts, including the flatness property, the canonical controllable form of MIMO linear time-invariant systems, and the design methodology for the flatness controller. Section II details the direct method for designing a minimal linear functional observer in the continuous case, which includes the structure used and the required and sufficient conditions for its existence.

In Section III, a comparative study between flatness-based control and control based on a minimal functional observer will be performed to assess their relative performance and applicability. This study will evaluate the effectiveness of each approach in the context of system control by presenting a simulation results.

II. FLATNESS BASED CONTROL CONCEPTS

By defining all system variables with flat output, as introduced by [3] for linear systems in a continuous-time, solving the differential equations is not necessary to deduce the dynamics. Therefore, the flat output can be used to express the state, the input and the output of the system as differential functions.

The following equation describes the nonlinear system in accordance with [5], [4].

$$\dot{x}(t) = f(x(t), u(t)) \quad (1)$$

where $u(t) \in \mathfrak{R}^m$ is the control vector, $z(t)$ is the flat output and $x(t) \in \mathfrak{R}^n$ is the state vector

$$z(t) = h(x(t), u(t), \dot{u}(t), \dots, u^{(r)}(t)) \quad (2)$$

and:

$$x(t) = \mathbf{A}(z(t), \dot{z}(t), \dots, z^{(\beta)}(t)) \quad (3)$$

$$u(t) = \mathbf{B}(z(t), \dot{z}(t), \dots, z^{(\beta+1)}(t)) \quad (4)$$

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where A and B are functions that reconstruct states and input from the flat output and its derivatives, β is a positive integer. The output of the system can be expressed as follows by:

$$y(t) = g(x(t), u(t)) \quad (5)$$

Then the output can be expressed:

$$y(t) = \mathbf{C}(z(t), \dot{z}(t), \dots, z^{(\sigma)}(t)) \quad (6)$$

where σ is a positive integer. By using the explicit expressions of output $y(t)$ and control $u(t)$ in the linear case, the flat output can be related to the partial state defined by [16].

A. Exact observer

We consider the state representation defined by equation (1), where the first component of the state vector $Z(t)$ represents the flat output of the system [16]. By successively differentiating the output $y(t)$ up to order $(n-1)$ we can express it as follow:

$$\begin{aligned} y(t) &= C_C Z(t) \\ \dot{y}(t) &= C_C A_C Z(t) + C_C B_C u_C(t) \\ y^{(2)}(t) &= C_C A_C^2 Z(t) + C_C A_C B_C u_C(t) + C_C B_C \dot{u}_C(t) \\ &\vdots \\ y^{(n-1)}(t) &= C_C A_C^{n-1} Z(t) + C_C A_C^{n-2} B_C u_C(t) + \dots \\ &\quad + C_C A_C B_C u_C^{(n-3)}(t) + C_C B_C u_C^{(n-2)}(t) \end{aligned} \quad (7)$$

Then $Y(t)$ can be written in matrix form as follows

$$Y(t) = OZ(t) + MU(t) \quad (8)$$

where

$$Y^T(t) = [y^T(t) \quad \dot{y}^T(t) \quad \dots \quad y^{(n-1)T}(t)] \quad (9)$$

$$U^T(t) = [u_C^T(t) \quad \dot{u}_C^T(t) \quad \dots \quad u_C^{(n-2)T}(t)] \quad (10)$$

$$O = [C_C^T \quad (C_C A_C)^T \quad \dots \quad (C_C A_C^{n-1})^T]^T \quad (11)$$

O is the observability matrix of the pair (A_C, C_C) with dimensions $(mn \times n)$ and M is the transmission matrix such that M_0 represents the matrix of zero dimensions specifically of size $(m \times m)$."

$$M = \begin{bmatrix} M_0 & M_0 & \dots & M_0 & M_0 \\ M_1 & M_0 & & & M_0 \\ \vdots & \ddots & \ddots & & \vdots \\ M_{n-2} & & & M_1 & M_0 \\ M_{n-1} & M_{n-2} & M_{n-3} & \dots & M_1 \end{bmatrix} \quad (12)$$

The observation of the system requires that the observability matrix has rank :

$$\text{rank}(O) = n \quad (13)$$

where

$$\text{rank}(O) = \text{rank}([O \quad Y(t) - MU(t)])$$

According to [6], we consider that the system is compatible, which implies that it is soluble and admits exact solutions in $Z(t)$. Given that O 's rank corresponds to n , the solution in $Z(t)$ is unique and given by:

$$Z(t) = (O^T O)^{-1} O^T (Y(t) - MU(t)) \quad (14)$$

where :

$$s^{-1}h(t) = \int_{-\infty}^t h(\tau) d\tau \quad (15)$$

The state vector is expressed through integration by parts:

$$\begin{aligned} Z(t) &= A_C^{n-1} (O^T O)^{-1} O^T \Pi y(t) - \\ &\quad \left(A_C^{n-1} (O^T O)^{-1} O^T M \right. \\ &\quad \left. - [A_C^{n-2} \bar{B}_C \quad \dots \quad \bar{B}_C] \right) \Pi^* u_C(t) \end{aligned} \quad (16)$$

such as:

$$\Pi = [\pi_{n-1} \quad \dots \quad \pi_1 \quad \pi_0]^T \quad (17)$$

$$\Pi^* = [\pi_{n-1} \quad \dots \quad \pi_2 \quad \pi_1]^T \quad (18)$$

with $\pi_i = \text{diag}(s^{-i} \quad \dots \quad s^{-i})$ are an $(m \times m)$ matrices.

B. Flatness-based Control design of the BBT System

The modeling of the BBT system is presented in this work [9]. The following is its continuous-state space representation.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (19)$$

$$\begin{aligned} A &= \begin{pmatrix} 0.6703 & 0 & 0 & 0 & 0 & 0 \\ 0.0033 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.004 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6703 & 0 & 0 \\ 0 & 0 & 0 & 0.0033 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.004 & 1 \end{pmatrix} \\ B &= \begin{pmatrix} 0.422 & 0 \\ 0.0009 & 0 \\ 0 & 0 \\ 0 & 0.422 \\ 0 & 0.0009 \\ 0 & 0 \end{pmatrix} \\ C &= \begin{pmatrix} 0 & 0 & 78.1250 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 98.0469 \end{pmatrix} \end{aligned}$$

where:

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t) \quad x_6(t)]^T \quad (20)$$

The control law of the LTI system is defined by:

$$u_C(t) = K(s)z_d(t) - \Lambda Z(t) \quad (21)$$

Then the control law was deduced in [11]

$$u_C(t) = K(s)z_d(t) - R(s^{-1})y(t) - Q(s^{-1})u_C(t) \quad (22)$$

with:

$$R(s^{-1}) = \Lambda A_C^{n-1} (O^T O)^{-1} O^T \Pi \quad (23)$$

$$Q(s^{-1}) = \Lambda \begin{pmatrix} A_C^{n-1} (O^T O)^{-1} O^T M - \\ [A_C^{n-2} B_C \quad \dots \quad B_C] \Pi^* \end{pmatrix} \quad (24)$$

Such as :

$$S(s^{-1}) = (I_m + Q(s^{-1})) H_C$$

Where I_m represents an identity matrix of rank m , the control law can be written in the two-degree-of-freedom controller form:

$$S(s^{-1}) u(t) = K(s) z_d(t) - R(s^{-1}) y(t) \quad (25)$$

The design is focused on choosing the $Z_d(t)$ trajectory to follow and the tracking dynamics provided by $K(s)$.

In the first step, the controllability system has been verified in the previous work [11] by constructing the matrix $K_{\{A,B\}}$ and its the rank has been obtained equal to 6, the system is then controllable.

$$A_C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.67 & -2.34 & 2.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.67 & -2.34 & 2.67 \end{pmatrix}$$

$$\bar{B}_C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C_C = \begin{pmatrix} -0.246 & -0.281 & -0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.309 & -0.353 & -0 \end{pmatrix}$$

and:

$$H_C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The controllable canonical form's state vector can be written as follows:

$$Z(t) = (z_1(t) \quad \dot{z}_1(t) \quad \ddot{z}_1(t) \quad z_2(t) \quad \dot{z}_2(t) \quad \ddot{z}_2(t))^T$$

where $z_1(t)$ and $z_2(t)$ are the two flat outputs of the system.

Let us defining planned trajectories of the two flat outputs given by the following polynomial expression:

$$z_{1d}(t) = z_{1in} + (z_{1fin} - z_{1in}) \times \left(21 \left(\frac{t}{2500} \right)^5 - 35 \left(\frac{t}{2500} \right)^6 + 15 \left(\frac{t}{2500} \right)^7 \right) \quad (26)$$

$$z_{2d}(t) = z_{2in} + (z_{2fin} - z_{2in}) \times \left(21 \left(\frac{t}{2500} \right)^5 - 35 \left(\frac{t}{2500} \right)^6 + 15 \left(\frac{t}{2500} \right)^7 \right) \quad (27)$$

Such as (z_{1in}, z_{2in}) and (z_{1fin}, z_{2fin}) represent the initial and final values of the desired flat outputs, respectively, which are calculated from the following initial and final values of the system outputs: $(y_{1in}, y_{2in}) = (1, 0)$ and $(y_{1fin}, y_{2fin}) = (1.2, 0)$

C. Simulation results of flatness-based control of BBT

The controller allows the output system to track a desired trajectory without using observer dynamics as demonstrated in Figures 1 and 3. The simulation results indicate that the flatness-based control is highly effective in tracking the trajectory, with tracking errors tending asymptotically to zeros.

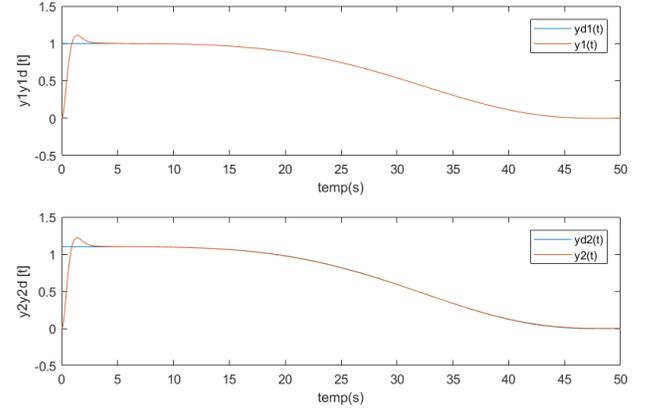


Fig. 1. Desired trajectories and system outputs

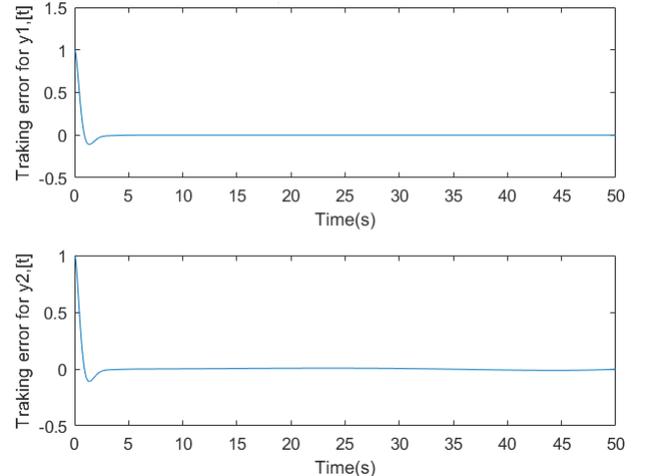


Fig. 2. Tracking errors for the system outputs

The figure 3 demonstrates the limitations of flatness-based control with the exact observer in noise rejection, even though it performs well in trajectory tracking.

III. LINEAR FUNCTIONAL OBSERVER

Consider a time invariant linear system whose state equation in this form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (28)$$

where the matrix C is a full rank in rows ($\text{rank}(C) = p$) and the system is observable. Consider the vector $V \in \mathbb{R}^n$

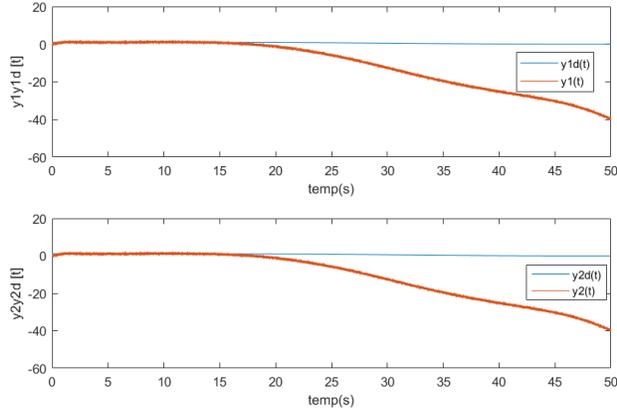


Fig. 3. Output system with noise

that we want to estimate: $v(t) = Lx(t)$ with $L \in \mathbb{R}^{l \times n}$ is a predefined matrix

Supposed that: $\text{rank}(L) = l$ and

$$\text{rank} \begin{bmatrix} L \\ C \end{bmatrix} = l + p \quad (29)$$

To reconstruct the state function $v(t)$, the structure of observer order q , $q \leq (n - p)$ is defined as follows:

$$\begin{aligned} \dot{z}(t) &= Fz(t) + Gu(t) + Hy(t) \\ w(t) &= Pz(t) + Vy(t) \end{aligned} \quad (30)$$

where $z(t) \in \mathbb{R}^q$ is the q dimensional state vector and $V(t)$ is the estimation of $V(t)$. The matrix F , G , H , and V as well as the order q of the observer are determined in such a way that the observer (25) is an asymptotic observer (30).

$$\lim_{t \rightarrow \infty} (v(t) - w(t)) = 0 \quad (31)$$

Asymptotic convergence is ensured if F is a Hurwitz matrix,

According to [14], the existence condition for the linear functionals observer ($q \times n$) matrix T such that:

$$G = TB \quad (32)$$

$$TA - FT = HC \quad (33)$$

$$L = PT + VC \quad (34)$$

For any completely observable linear system, the order $(\nu - 1) \leq (n - p)$ of an observer of linear forms is less than the order $n - p$ of a reduced order state observer.

A. Feedback controller

The pole placement method is used to assign the control loop poles to ensure that the estimation converges to zero error. The feedback control law is presented as follows

$$u_C(t) = NC * r(t) - Fx(t) \quad (35)$$

Such as $r(t)$ represents a two-dimensional reference input signal, while F and NC are the state feedback gain matrix and a constant matrix that influences the static gain, respectively [13].

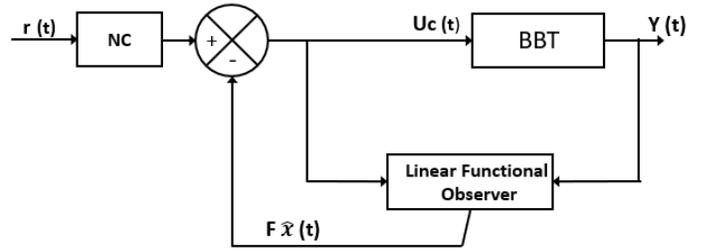


Fig. 4. A feedback control with linear functional observer

B. The design of linear functional observer using the direct approach

By defying q the smallest integer:

$$\text{rank}(\Sigma_q) = \text{rank} \left(\begin{bmatrix} \Sigma_q \\ LA^q \end{bmatrix} \right) \quad (36)$$

with :

$$\Sigma_q = \begin{bmatrix} C \\ L \\ CA \\ LA \\ \vdots \\ CA^{q-1} \\ LA^{q-1} \\ CA^q \end{bmatrix} \quad (37)$$

By derivation of $dv(t) = Lx(t)$, we obtained :

$$v^q(t) = LA^q x(t) + \sum_{i=0}^{q-1} LA^i Bu^{(q-1-i)}(t) \quad (38)$$

According to (36), we have Γ_i for $i = 0$ to q and Λ_i $i = 0$ to $q - 1$:

$$LA^q = \sum_{i=0}^q \Gamma_i CA^i + \sum_{i=0}^{q-1} \Lambda_i LA^i \quad (39)$$

The equation (38) can therefore be written as follows:

$$v^q(t) = \sum_{i=0}^q \Gamma_i CA^i x(t) + \sum_{i=0}^{q-1} \Lambda_i LA^i x(t) + \sum_{i=0}^{q-1} LA^i Bu^{q-1-i}(t) \quad (40)$$

To eliminate the state $x(t)$ let us consider the equalities:

$$\begin{aligned} Lx(t) &= v(t) \\ LAx(t) &= \dot{v}(t) - Lbu(t) \\ LA^{(q-1)}x(t) &= v^{(q-1)} - \sum_{i=0}^{q-2} LA^i Bu^{q-2-i}(t) \\ Cx(t) &= y(t) \\ CAx(t) &= \dot{y}(t) - Cbu(t) \\ CA^2x(t) &= y(t) - CABu(t) - CB\dot{u}(t) \\ CA^q x(t) &= y^q - \sum_{i=0}^{q-1} CA^i Bu^{q-1-i}(t) \end{aligned}$$

Which implies:

$$v^q(t) = \Gamma_0 y(t) + \sum_{i=1}^q \Gamma_i \left[y^i - \sum_{j=0}^{i-1} CA^j B u^{(i-1-j)}(t) \right] + \Lambda_0 v(t) + \sum_{i=1}^{q-1} \Lambda_i \left[v^i - \sum_{j=0}^{i-1} LA^j B u^{(i-1-j)}(t) \right] + \sum_{i=0}^{q-1} LA^i B u^{(q-1-i)}(t), \quad (41)$$

$$v^q(t) = \sum_{i=0}^q \Gamma_i y^{(i)} + \sum_{i=0}^{q-1} \Lambda_i v^{(i)} + \sum_{i=0}^{q-1} \Phi_i u^{(i)}(t) \quad (42)$$

For $i = 0$ to $q - 2$:

$$\Phi_i = \left[LA^{q-1-i} - \sum_{j=i+1}^q \Gamma_j CA^{j-i-1} - \sum_{j=i+1}^{q-1} \Lambda_j LA^{j-i-1} \right] B \quad (43)$$

and :

$$\Phi_{q-1} = [L - \Gamma_q C] B \quad (44)$$

The input-output differential equation (41) can be realized as the following observable state-space system of order q .

$$\dot{z}(t) = \begin{bmatrix} 0 & & & \Lambda_0 \\ 1 & \ddots & & \Lambda_1 \\ & \ddots & 0 & \vdots \\ & & \ddots & 1 \\ & & & \Lambda_{q-1} \end{bmatrix} z(t) + \begin{bmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \phi_{q-1} \end{bmatrix} u(t) + \begin{bmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{bmatrix} y(t) \quad (45)$$

$$\hat{v}(t) = [0 \quad \dots \quad 0 \quad 1] z(t) + \Gamma_q y(t)$$

By identification with the equation (30), we have :

$$F = \begin{bmatrix} 0 & & & \Lambda_0 \\ 1 & \ddots & & \Lambda_1 \\ & \ddots & 0 & \vdots \\ & & \ddots & 1 \\ & & & \Lambda_{q-1} \end{bmatrix}, G = \begin{bmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \phi_{q-1} \end{bmatrix}$$

$$H = \begin{bmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{bmatrix}, P = [0 \quad \dots \quad 0 \quad 1]$$

$$V = \Gamma_q$$

According to [15], the smallest integer q is defined such that there exist Γ_i , for $i=0$ to q and Λ_i , for $i = 0$ to $q - 1$. where:

$$LA^q = \sum_{i=0}^q \Gamma_i CA^i + \sum_{i=0}^{q-1} \Lambda_i LA^i \quad (46)$$

C. Simulation result of Functional observer-based feedback controller

The simulation results presented in Figure 6 demonstrate that the system output effectively follows the reference trajectory, confirming the satisfactory performance of the proposed control approach, and confirmed that functional observer-based control can achieve good performance with a quickly converged estimate error.

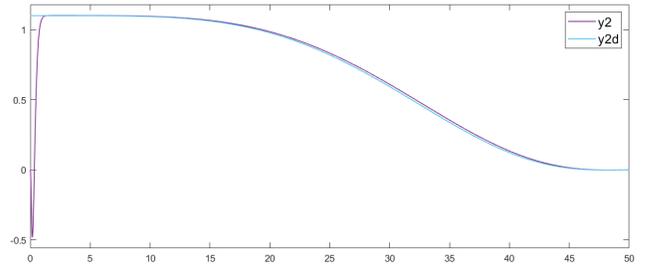
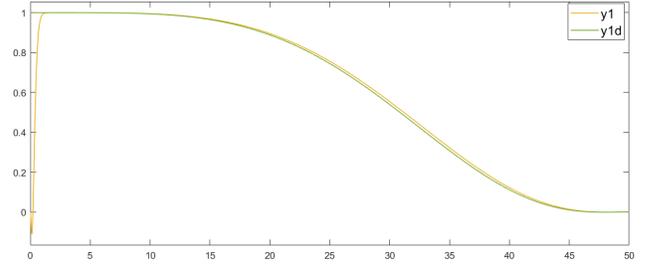


Fig. 5. Desired trajectories and system outputs

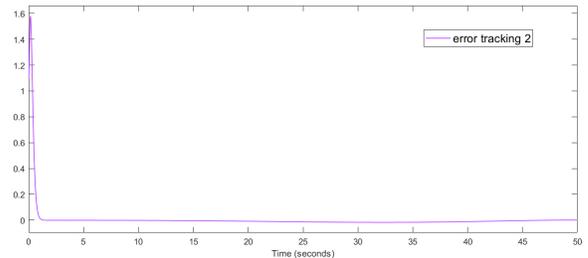
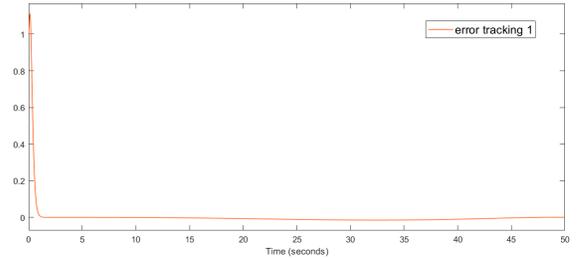


Fig. 6. Desired trajectories and system outputs

The figure 8 demonstrates the robustness of a linear functional observer, which refers to its ability to maintain accuracy in estimating the state of the system in the presence of model uncertainties and noise. The simulation results showed

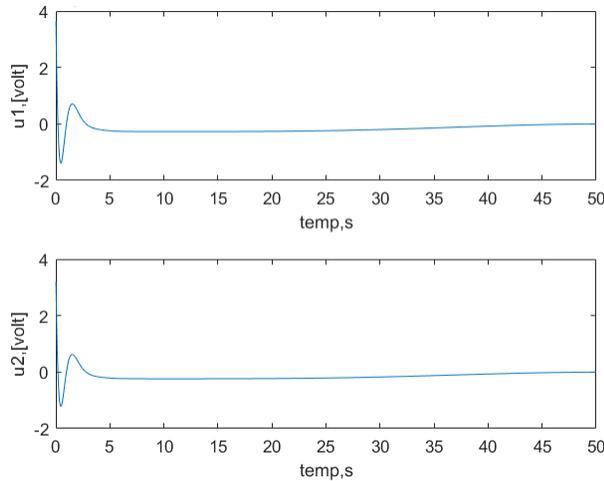


Fig. 7. Control inputs $u_1(t)$ and $u_2(t)$

that the observer can still provide reliable state estimates even with variations or errors in the system dynamics.

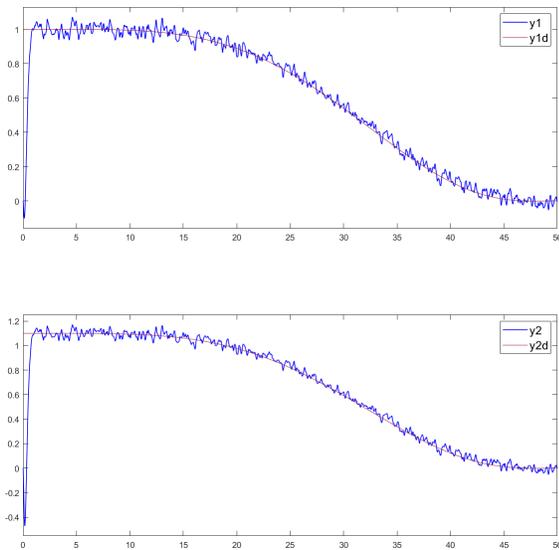


Fig. 8. Desired trajectories and system outputs

IV. CONCLUSIONS

This paper presents a comparative study of flatness-based control and a feedback controller based on a minimal functional observer, aimed at evaluating their relative performance and applicability.

The simulation results show that controlling the BBT system using the flatness-based control with exact observer approach achieves good trajectory tracking performance, but it is prone to disturbances. However, a functional observer-based feedback control makes it possible to obtain good trajectory tracking performance and robustness in the presence of disturbances. In conclusion, if the system is well modeled

and is not significantly affected by disturbances, flatness-based control offers superior performance in trajectory tracking. However, in scenarios involving uncertainties or disturbances, the robust and easier-to-implement functional linear observer serves as an effective compromise. The results obtained lead us to think about new perspectives which allow the extension of flatness-based control to incorporate minimal linear functional observers which is a second case that will be dealt with at a later time.

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