State-of-Charge Estimation of Lithium-ion Battery using Super-Twisting Algorithm with Extended State Observer

Pooja Singh, Dhanalakshmi Kaliaperumal, Rahul Kumar Sharma, and Shyam Kamal

***Abstract*—This paper considers the problem of estimating the states of a class of non-linear systems with a generalized class of uncertainties. Both matched and mismatched uncertainties have been considered in the system. An observer based on Super-Twisting Algorithm is designed to estimate the states of the system in finite time. The observer uses the estimated information of the mismatched uncertainties provided by an Extended State Observer which is designed separately. The finite-time stability of the designed observer is proved using Lyapunov technique. The designed approach ensures accuracy in estimating the states even in the presence of the generalized uncertainties which shows strong robustness. The technique is illustrated by taking the problem of State-of-Charge estimation of Lithium-ion Battery based on its simplified electrical equivalent circuit model. Simulation results are presented to highlight the effectiveness of the technique.**

***Index terms*—** Sliding Mode Control, Super-Twisting Algorithm, Extended State Observer, Parametric Uncertainty, Lithium-ion Battery.

I. INTRODUCTION

The design of observers is one of the widely considered problems in control theory [1]. Observers are used to estimate the unknown states of the system by using its model structure with the input-output information [2]. It is often used when there is a lack of information of all the states of the system. For some systems, the measurement of some of the states may not be possible due to their inaccessibility. Also, it may not always be feasible to deploy sensors for measuring each of the states as it becomes a costly method. So, the design of observers becomes very important for which model based techniques are often used [3]. As it is very hard to formulate the exact model of the system, the estimated states of the observers are often affected by model uncertainties resulting from parameter variations and external disturbances [4]. Therefore, it becomes very important to explore robust techniques for state estimation.

Sliding Mode Control (SMC) is a well-established robust control and observation scheme which has strong robustness to model uncertainties and external disturbances [1] - [3]. It is widely used in control systems as it is simple to design and implement. Its major limitation is the inherent chattering phenomenon [2]. To alleviate the problem of chattering, several approaches have been presented in the literature. Among them, the Super-Twisting Algorithm (STA) is very widely used. It is a second-order sliding mode control method. The STA is particularly effective in handling matched uncertainties and Lipschitz continuous disturbances [5]. It generates a smooth control signal, thereby minimizing chattering, and drives the states to the origin in finite time. The theoretical foundations for its stability and robustness are discussed using Lyapunov approach in [5]. This work considers a class of systems with both matched and mismatched parametric uncertainties and external disturbances for which an STA based observer has been designed to achieve the robust estimation of the states in finite time.

Observers utilizing Super-Twisting Algorithm provide accurate state estimation of non-linear systems in finite time. Super-Twisting Observer (STO) extends the robustness of traditional SMC strategies in designing observer to handle a broader class of uncertainties specifically, those that increase at a rate proportional to the square root of time [6]. This enhancement enables the STA to handle more complex disturbances which is difficult to be addressed by the schemes based on the traditional SMC techniques. However, real-world systems often exhibit uncertainties that do not conform to the general assumptions considered in the design of observers based on such techniques as in [5]. Consequently, it becomes essential to consider a more generalized class of uncertainties in the system and design an observer capable to provide the estimation of the states accurately and being robust to such class of uncertainties. For this purpose, SMC based techniques may be explored which have been used for designing control laws for various class of systems [7] - [10].

Estimation techniques must exhibit robustness to both matched and mismatched uncertainties. However, designing an observer that accurately estimates system states in the presence of generalized mismatched uncertainties is challanging. In this regard, the Extended State Obsever (ESO) has been integrated with the SMC method in [11]. This combination has been applied to systems that meet the matching constraints as described in [12]. Recently, numerous noteworthy results have been obtained by using the approach of ESO for mismatched non-linear systems [13]. It mitigates the impact of mismatched uncertainties significantly. In this work, a linear ESO is designed to obtain the information of the mismatched uncertainties present in the system. The state estimation algorithm incorporates the estimated disturbances to reduce their impact on the state estimation. This approach retains the inherent robustness of the traditional STO while broadening its applicability to systems with the generalized class of mismatched uncertainties.

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This work considers the problem of State-of-Charge (SoC) estimation of Lithium-ion (Li-ion) Battery. It is a widely used electrochemical energy storage system offering a long lifespan and fast charging capability. Estimating its internal states is difficult due to its time-varying behavior, which depends on several variables such as temperature, aging, external disturbances and sensor noises. Several schemes using Kalman filter and its variants are used to obtain SoC [14] [15]. High-gain non-linear observers are designed to estimate SoC for Lead-acid battery with useful analogies applicable to Li-ion battery [16]. A Proportional-Integral Observer is formulated in [17]. Its ease of design suits particularly where computational resources are limited. SMC is coupled with a disturbance observer for underactuated systems in [18]. This mitigates the effect of both system uncertainty and external disturbances which is crucial for battery systems exposed to dynamically changing charge-discharge cycles.

SMC based techniques have been widely used for the robust estimation of the states of various class of systems [18] - [26]. An observer is designed using adaptive STA in [23]. It is capable of suppressing the estimation error ensuring robustness to parameter variations as well as measurement noise. A dual-mode adaptive sliding mode observer which estimates the SoC is used in [24]. Its model utilizes gain adaptation to adapt to changes caused by aging as well as external interference. Other model based techniques have been used to design observers and controllers with various convergence characteristics [27] - [31]. Non-linear observers have been designed using Lyapunov technique in [27]. An estimation scheme which corrects for long-term drift and modeling inaccuracies by means of dynamic bias compensation is presented in [28]. An adaptive observer to estimate the SoC of battery using neural networks is designed in [29].

Despite significant advancements, there is a requirement for accurately estimating the SoC in the presence of a generalized class of mismatched uncertainties and disturbances. The information of accurate SoC is essential to avoid overcharging or deep discharging of Li-ion battery which may prevent its performance degradation and reduction in its lifespan. To address this, the present work designs a robust method for accurate SoC estimation while accounting for mismatched disturbances. The model parameters of the system are identified with the Recursive Least Squares (RLS) algorithm using the input-output information [4] [32]. The resulting model is then used to design a robust observer. Specifically, STO is used to ensure accurate state estimation of the battery. Further, linear ESO is integrated to handle generalized mismatched uncertainties improving the estimation accuracy.

The paper is organized as follows: Section II details the dynamic model of the Li-ion battery and estimates the model parameters using RLS scheme. Section III details the design of the Super-Twisting Observer for the nominal battery model. Section IV describes the Extended State Observer designed to estimate the uncertainties and disturbances which are to be used in the STO. Section V gives the simulation results while Section VI concludes the study.

II. MODELING OF LITHIUM-ION BATTERY

For the observer design, the dynamic model of the system is determined. In the literature, various approaches have been considered to capture the dynamic behaviour of Lithium-ion Battery. It ranges from the elctrochemical model to equivalent circuit model based schemes. Equivalent circuit models have been considered which have single RC network or multiple RC networks connected in series. Models with multiple RC networks have more accuracy but they increase the model complexity. This study models the Lithium-ion battery using single RC equivalent circuit model as illustrated in Fig. 1. This model is simple and has less complexity. In the model, R_0 indicates the ohmic resistance. R_p and C_p are the equivalent polarization resistance and capacitance respectively. V_{oc} represents the Open Circuit Voltage (OCV) of the battery. V_0 and I are the terminal voltage and current respectively. V_p is the polarization voltage which appears across the R_pC_p branch of the circuit. The only available information is input current I and the terminal voltage V_0. The objective is to obtain the SoC

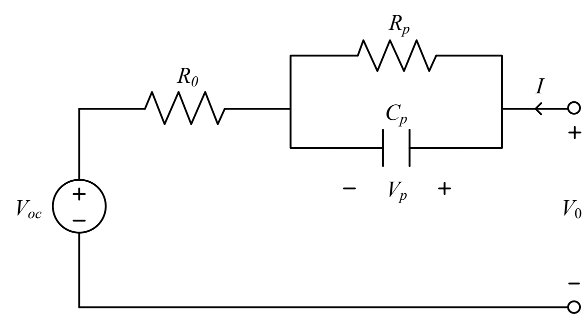

Fig. 1. Equivalent Circuit Model of Lithium-ion Battery

of the battery provided the information of the terminal voltage, V_0 and the input current, I is available by measurement. The observer utilizes the measurement information to estimate the states of the system by using the structure of its dynamic model. The dynamic behavior of the system is described by,

$$\dot{V}_p = -\frac{1}{R_pC_p}V_p + \frac{1}{C_p}I \tag{1a}$$

$$V_0 = V_{oc} + IR_0 + V_p \tag{1b}$$

The unknown model parameters of the dynamic model are estimated by using RLS technique. The corresponding values are mentioned in Table I.

TABLE I
ESTIMATED MODEL PARAMETERS

Model Parameters	Values
R_0	0.22036 Ω
R_p	0.002971 Ω
C_p	4899 F

A commonly accepted expression of SoC is,

$$z(t) = z(t_0) + \frac{1}{Q_c} \int_{t_0}^{t} \eta I(\tau) d\tau \tag{2}$$

where, η is the Coulomb efficiency and Q_c is the nominal capacity of the battery. Its derivative form is mentioned below:

$$\dot{z} = \frac{\eta}{Q_c} I \tag{3}$$

The SoC and OCV exhibit a highly non-linear relationship. Over small intervals of SoC, this relationship may be approximated by a piecewise linear curve [17]. This study uses the MATLAB Curve Fitting Toolbox to derive a linear equation which approximates the relationship between SoC and OCV over a 10 % discharge range of the battery. The linear relationship may be expressed as,

$$V_{oc} = mz + n \tag{4}$$

In the above equation, m and n are coefficients of the linearized model. From (1b),

$$V_p = V_0 - V_{oc} - IR_0 \tag{5}$$

By taking the derivative of Eq. (1b),

$$\dot{V}_0 = \dot{V}_{oc} + R_0 \dot{I} + \dot{V}_p \tag{6}$$

Using (1a),

$$\dot{V}_0 = \dot{V}_{oc} + R_0 \dot{I} - \frac{1}{R_p C_p} V_p + \frac{1}{C_p} I \tag{7}$$

Using (5),

$$\dot{V}_0 = \dot{V}_{oc} + R_0 \dot{I} - \frac{1}{R_p C_p} (V_0 - V_{oc} - IR_0) + \frac{1}{C_p} I \tag{8}$$

Rearranging the terms,

$$\dot{V}_0 = \dot{V}_{oc} + \frac{1}{R_p C_p} V_{oc} - \frac{1}{R_p C_p} V_0 + \left(\frac{R_0 + R_p}{R_p C_p} \right) I + R_0 \dot{I} \tag{9}$$

Using (4) in (9) with the approximation $\dot{V}_{oc} \approx 0$ [29],

$$\dot{V}_0 = -\frac{1}{R_p C_p} V_0 + \left(\frac{R_0 + R_p}{R_p C_p} \right) I + R_0 \dot{I} + \frac{1}{R_p C_p} (mz + n) \tag{10}$$

$$\dot{V}_0 = \frac{z}{R_p C_p} m + \frac{1}{R_p C_p} (n - V_0) + \left(\frac{R_0 + R_p}{R_p C_p} \right) I + R_0 \dot{I} \tag{11}$$

Rewriting Eqs. (3) and (11),

$$\dot{V}_0 = \frac{z}{R_p C_p} m + \frac{1}{R_p C_p} (n - V_0) + \left(\frac{R_0 + R_p}{R_p C_p} \right) I + R_0 \dot{I} \tag{12a}$$

$$\dot{z} = \frac{\eta}{Q_c} I \tag{12b}$$

The Eqs. (12) reprsent the dynamic model of Li-ion battery for which the observer is designed using Super-Twisting Algorithm.

III. Super-Twisting Observer

The observer design focuses on a class of non-linear systems defined as,

$$\dot{z}_1 = z_2 + \zeta_1(z_1, z_2, t) \tag{13a}$$

$$\dot{z}_2 = bu + \zeta_2(z_1, z_2, t) \tag{13b}$$

$$y = z_1 \tag{13c}$$

In this system, z_1 and z_2 represent the states and u is the control input. The terms ζ_1 and ζ_2 are unknown non-linear functions that capture model uncertainties, and y is the output. These uncertainties are assumed to be bounded. Here, ζ_1 represents the mismatched uncertainties and ζ_2 denotes the matched uncertainties with respect to the control input. This work aims to develop a robust observer using an STA based technique which eliminates the influence of mismatched uncertainties on state estimation.

For the nominal model, an observer based on STA is designed as,

$$\dot{\hat{z}}_1 = \hat{z}_2 + \alpha_1 |z_1 - \hat{z}_1|^{1/2} \text{sign}(z_1 - \hat{z}_1) \tag{14a}$$

$$\dot{\hat{z}}_2 = bu + \alpha_2 \text{sign}(z_1 - \hat{z}_1) \tag{14b}$$

where, $\hat{z}_1$ and $\hat{z}_2$ represent the estimated states of the system. α_1 and α_2 are the constant gain parameters of STA. Defining the error variables as,

$$e_1 := z_1 - \hat{z}_1 \tag{15a}$$

$$e_2 := z_2 - \hat{z}_2 \tag{15b}$$

The resulting error dynamics is [5],

$$\dot{e}_1 = e_2 - \alpha_1 |e_1|^{1/2} \text{sign}(e_1) + \xi_1(z_1, z_2, t) \tag{16a}$$

$$\dot{e}_2 = -\alpha_2 \text{sign}(e_1) + \xi_2(z_1, z_2, t) \tag{16b}$$

Assumption 1: The disturbance terms are globally constrained by

$$|\zeta_1| \leq \Delta_1 |e_1|^{1/2}, \quad |\zeta_2| \leq \Delta_2 \tag{17}$$

where, $\Delta_1, \Delta_2 \geq 0$

Theorem 1 ([6]): Consider the above system with the assumption (17). The origin, $e_1 = 0$ is globally asymptotically stable, provided the gain parameters α_1, α_2 satisfy the following criteria:

$$\alpha_1 > 2\Delta_1$$

$$\alpha_2 > \alpha_1 \frac{5\Delta_1 \alpha_1 + 6\Delta_2 + 4(\Delta_1 + \Delta_2/\alpha_1)^2}{2(\alpha_1 - 2\Delta_1)}$$

Proof: For the nominal model, the Lyapunov function is considered as,

$$V(e) = 2\alpha_2 |e_1| + \frac{1}{2} e_2^2 + \frac{1}{2} (\alpha_1 |e_1|^{1/2} \text{sign}(e_1) - e_2)^2 \tag{18}$$

As shown in [6], the above function is a strong Lyapunov function for the error dynamics of the system. Taking its time derivative,

$$\dot{V} = -\frac{1}{|e_1|^{1/2}} \chi^\top Q \chi + \frac{d_1}{|e_1|^{1/2}} q_1^\top \chi + d_2 q_2^\top \chi \tag{19}$$

where,

$$q_1^\top = \begin{bmatrix}(2\alpha_2 + \frac{\alpha_1^2}{2}) & -\frac{\alpha_1}{2}\end{bmatrix}$$

$$q_2^\top = \begin{bmatrix}-\alpha_1 & 2\end{bmatrix}$$

By considering the disturbance condition (17), the derivative function is,

$$\dot{V} \leq -\frac{1}{|e_1|^{1/2}}\chi^\top \tilde{Q}\chi$$

where

$$\tilde{Q} = \frac{\alpha_1}{2}\begin{bmatrix}2\alpha_2 + \alpha_1^2 - (\frac{4\alpha_2}{\alpha_1} + \alpha_1)\Delta_1 - 2\Delta_2 & \star \\ -(\alpha_1 + 2\Delta_1 + \frac{2\Delta_2}{\alpha_1}) & 1\end{bmatrix}$$

It satisfies the condition that if $\tilde{Q} > 0$ then, $\dot{V}$ is negative definite. According to the finite-time convergence properties given in [6], the finite convergence time of the error dynamics is obtained as,

$$\tilde{T} = \frac{2V^{1/2}e_0}{\tilde{\Gamma}}$$

where,

$$\tilde{\Gamma} = \frac{\gamma_{min}^{1/2} P \gamma_{min} \tilde{Q}}{\gamma_{max} P}$$

The Li-ion battery model (12) can be expressed as,

$$\dot{z}_1 = z_2 + d(z_1, z_2, t) + \zeta(z_1, z_2, t) \tag{20a}$$
$$\dot{z}_2 = u + \zeta_2(z_1, z_2, t) \tag{20b}$$

where, $z_1 := V_0$, $z_2 := \frac{m}{R_p C_p} z$ are the states of the system. $u := \frac{\eta m}{R_p C_p Q_c} I$ denotes the control input. Here, $d = \frac{R_0 + R_p}{R_p C_p} I + R_0 \dot{I} + \frac{1}{R_p C_p}(n - V_0)$. ζ and ζ_2 are bounded non-linear terms representing model uncertainties and unknown disturbances. It may be further represented as,

$$\dot{z}_1 = z_2 + \zeta_1(z_1, z_2, t) \tag{21a}$$
$$\dot{z}_2 = u + \zeta_2(z_1, z_2, t) \tag{21b}$$

where, $\zeta_1 = d + \zeta$. The battery model described above falls within the same class of systems represented by equation (13) which serves for the observer designed in this study. The observer (14) is used with the input-output measurement of the battery to compute the information of its states in the presence of the uncertain model parameters and external disturbances. Here, ζ_1 represents the mismatched uncertainty and ζ_2 represents the matched uncertainty. For the battery model presented, the uncertain terms ζ_1 and ζ_2 are supposed to satisfy the Assumption 1. However, the term d may contain a generalized expression which does not necessarily satisfy the assumption. For this generalized model, a linear ESO is designed which provides the estimated mismatched uncertainty $\hat{\zeta}_1$ to the observer. Further, the observer utilizes the estimated uncertainty effectively resulting in finite-time convergence of the error in the estimated SoC, $e = z - \hat{z}$ to zero enhancing the accuracy of the estimated SoC, $\hat{z}$.

IV. Super-Twisting Observer with ESO

A linear Extended State Observer is designed to estimate the mismatched uncertainties and disturbances, $\hat{\zeta}_1$ present in the system (21) [7].

$$\hat{\zeta}_1 = q_{11} + m_{11} z_1 \tag{22}$$

$$\dot{q}_{11} = -m_{11}(z_2 + bu + \hat{\zeta}_1) + \hat{\dot{\zeta}}_1 \tag{23}$$

$$\hat{\dot{\zeta}}_1 = q_{12} + m_{12} z_1 \tag{24}$$

$$\dot{q}_{12} = -m_{12}(z_2 + bu + \hat{\zeta}_1) \tag{25}$$

where, $\hat{\zeta}_1$ and $\hat{\dot{\zeta}}_1$ are the estimated variables of ζ and $\dot{\zeta}_1$. q_{11} and q_{12} are auxiliary variables. m_{11} and m_{12} are constant gain parameters selected by the designer. Here, these gain parameters are set at high values to achieve fast convergence. The estimation error is,

$$\tilde{e}_0 = \begin{bmatrix}\tilde{\zeta}_1 & \tilde{\dot{\zeta}}_1\end{bmatrix}^\top \tag{26}$$

$$\tilde{\zeta}_1 = \zeta_1 - \hat{\zeta}_1 \tag{27}$$

$$\tilde{\dot{\zeta}}_1 = \dot{\zeta}_1 - \hat{\dot{\zeta}}_1 \tag{28}$$

Here, $\tilde{\zeta}_1$ and $\tilde{\dot{\zeta}}_1$ are the error terms corresponding to the variables ζ_1 and $\dot{\zeta}_1$ respectively. By solving the above equation,

$$\dot{\hat{\zeta}}_1 = m_{11}\tilde{\zeta}_1 + \hat{\dot{\zeta}}_1 \tag{29}$$

Furthermore, ζ_1 is subtracted from both sides of the above equation resulting in,

$$\dot{\tilde{\zeta}}_1 = -m_{11}\tilde{\zeta}_1 + \tilde{\dot{\zeta}}_1 \tag{30}$$

Similarly,

$$\dot{\tilde{\dot{\zeta}}}_1 = -m_{12}\tilde{\zeta}_1 + \ddot{\zeta}_1 \tag{31}$$

Differentiating,

$$\ddot{\tilde{\zeta}}_1 = -m_{11}\dot{\tilde{\zeta}}_1 - m_{12}\tilde{\zeta}_1 + \ddot{\zeta}_1 \tag{32}$$

As $\ddot{\zeta}_1$ is assumed to be bounded, the stability of $\tilde{\zeta}_1$ is ensured by considering $m_{11} > 0$ and $m_{12} > 0$. The error dynamics of the observer is,

$$\dot{\tilde{e}}_0 = D_1 \tilde{e}_0 + E_1 \ddot{\zeta}_1 \tag{33}$$

where

$$D_1 = \begin{bmatrix}-m_{11} & 1 \\ -m_{12} & 0\end{bmatrix}; \quad E_1 = \begin{bmatrix}0 \\ 1\end{bmatrix} \tag{34}$$

Using the estimated uncertainty, $\hat{\xi}_1$ from the designed ESO in the Super-Twisting Observer,

$$\dot{\hat{z}}_1 = \hat{z}_2 + \alpha_1|z_1 - \hat{z}_1|^{1/2}\text{sign}(z_1 - \hat{z}_1) + \hat{\zeta}_1 \tag{35a}$$
$$\dot{\hat{z}}_2 = u + \alpha_2\text{sign}(z_1 - \hat{z}_1) \tag{35b}$$

V. Simulation Results

In this study, a cylindrical INR 18650-20R Lithium-ion Battery is considered. The battery data has been utilized for simulation purposes from the work of [32]. The selected battery features a nominal voltage of 3.6 V and a rated capacity of 2 Ah. Its maximum and minimum voltage limits are 4.2 V and 2.5 V respectively. The data is obtained for pulse-discharge test which is conducted on the battery. It is initially charged to its full capacity using the Constant Current-Constant Voltage (CC-CV) method. Further, the battery is discharged using a negative pulse current of magnitude 1 A. A relaxation period is introduced in the discharge cycle after every 10% drop in the SoC. The cycle consists of a 2-minute discharge followed by a 2-hour rest period. The input pulse current profile used is shown in Fig. 2.

For the single RC equivalent circuit model shown in Fig. 1, its unknown model parameters are estimated by using the RLS algorithm. The resulting estimated parameters are summarized in Table I. These values of the parameters are subsequently used in the battery model for observer design and simulation analysis. To validate the dynamic model of the battery, the terminal output voltage predicted by the model is compared with the experimental data obtained initially in response to the pulse current. The resulting voltage plots are shown in Fig. 3. From the obtained results, the estimated voltage achieves a Root Mean Square Error (RMSE) of 0.00375 and a Mean Absolute Error (MAE) of 0.0022 indicating high accuracy. The subsequent error plot is depicted in Fig. 4 which further validates the model accurately.

The SoC is estimated using the designed observer based on Super-Twisting Algorithm. The estimated SoC is shown in Fig. 5, while the corresponding error plot in Fig. 6 confirms finite-time convergence at 180 s. The reference SoC is obtained using the Coulomb Counting (CC) method, which is utilized to validate the estimation results. The simulation shows an RMSE of 0.00945 and an MAE of 0.00193. To estimate the mismatched uncertainties, a linear ESO is designed. The performance of the ESO is demonstrated in Fig. 7 with the associated estimation error presented in Fig. 8. As per the obtained results, the ESO accurately estimates the model uncertainties. The observer demonstrates convergence of the error in estimation of the uncertainty to a small bound about the origin within 30 s. Fig. 6 shows the error plot with RMSE 0.03537 and MAE 0.00035.

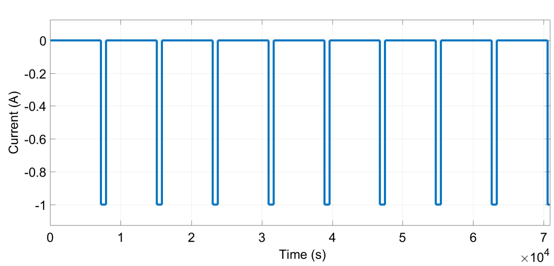

Fig. 2. Current Input, I

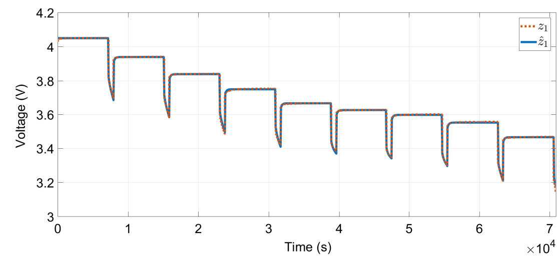

Fig. 3. Estimated Output Voltage, $\hat{z}_1$

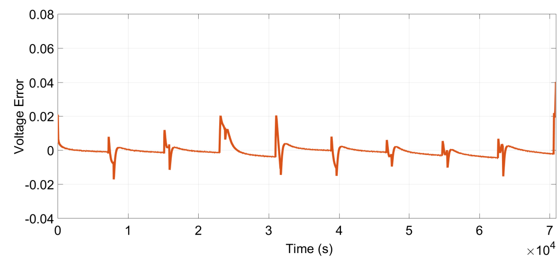

Fig. 4. Error in Estimated Output Voltage, e_1

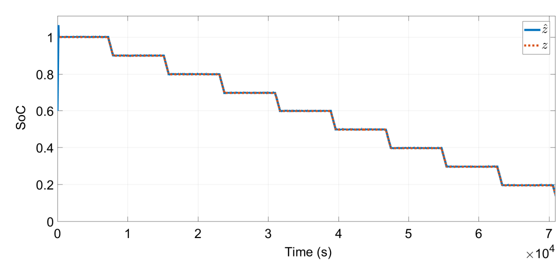

Fig. 5. Estimated State-of-Charge, $\hat{z}$

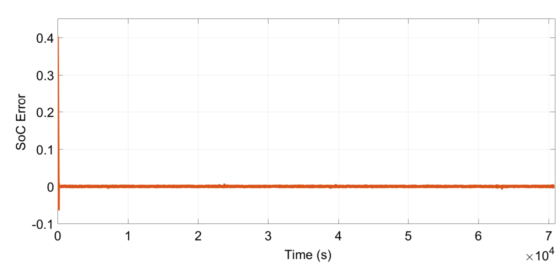

Fig. 6. Error in Estimated State-of-Charge, e

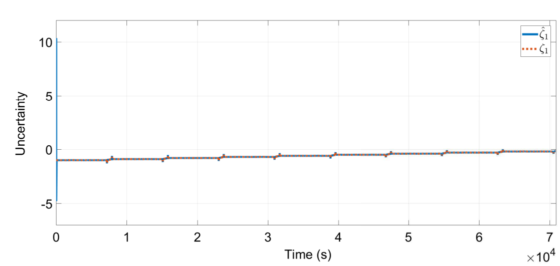

Fig. 7. Estimated Uncertainty, $\hat{\zeta}_1$

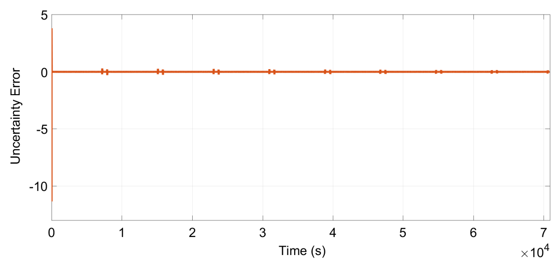

Fig. 8. Error in Estimated Uncertainty, $\tilde{\zeta}_1$

VI. Conclusion

This study focuses on obtaining the estimation of states for a class of uncertain systems using a robust observer based approach. It considers both matched and mismatched uncertainties in the design of the observer. Super-Twisting Algorithm has been used to design the observer which uses the estimated information of the mismatched uncertainties provided by the designed linear ESO. Simulation results are illustrated by designing the observer for the SoC estimation of Lithium-ion Battery. The model parameters of the battery are estimated using RLS method. The technique effectively achieves finite-time convergence of the state estimation errors resulting in accurate SoC estimation despite the effects of the generalized parametric uncertainties and external disturbances. Adaptive techniques may be used to minimize the chattering and further enhance the accuracy by avoiding overestimation of the gain parameters. Further, a time-varying model of Lithium-ion Battery may be considered by including the dependence of SoC on the OCV and the parameters of the equivalent circuit model.

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