Dynamic Mode Decomposition (DMD) for Enhanced Epilectic Seizure Prediction from EEG Signals

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***Abstract*— Epilepsy is a non-communicable neurological disorder that causes recurrent and unprovoked seizure. Ideally, predicting seizures would represent a step forward in making life easier for those who suffer from epilepsy. This work aims to predict the occurence of epileptic seizures via a novel approach that combines Dynamic Mode Decomposition (DMD), that is a data-driven modelling technique for dynamical systems with a deep learning classifier, that is based on a convolutional neural network. The resulting two-stage data-driven predictor was tailored for the analysis of encephalographic (EEG) data. The validity analysis of the approach is carried out over the CHB-MIT Scalp EEG Database, demonstrating its applicability for seizures' recognition and prediction on real clinical data.**

***Index Terms*— Dynamic Mode Decomposition, Convolutional Neural Networks, Intelligent Systems.**

I. INTRODUCTION

Epilepsy is a chronic neurological disorder of the brain that affects nearly 50 million people of all ages worldwide, characterized by unprovoked recurrent seizures [1]. Even if it is possible to live a seizure-free life if epilepsy is well-treated, the capability of accurately recognizing and predicting seizures offers significant advantages for both patients and healthcare providers [2]. With respect to patients, early detection and prediction of seizures can improve their overall quality of life, warning them about the incoming possible seizure and enabling timely interventions and harm mitigation [3].

To this end, the analysis of electroencephalography (EEG) data, which captures the electrical activity of the brain by measuring neuronal membrane potentials via electrodes placed on the scalp, provides a source of information that is crucial for identifying suitable patterns associated with seizures and has shown promising results in improving seizure detection and prediction. Leveraging the high temporal resolution of EEG data, multiple data-driven methods have been developed on the basis of the analysis of time and frequency domains features.

The present work proposes a novel solution to predict incoming seizures from the analysis of EEG data, combining two different data-driven modelling and analysis techniques. In a first phase, EEG data is processed to learn a dynamical model that captures its evolution through Dynamic Mode Decomposition (DMD) [4], that is a technique from dynamical systems and control theory capable of discovering dynamics directly from data. The advantage of DMD lies in the high interpretability of the spatio-temporal evolution of critical patterns in the data, enabling an easy representation of the main characteristics and information discovered in the learned dynamics. Starting from this DMD-based analysis, in the second phase, we employ a convolutional neural network to perform the prediction of incoming seizures by analyzing the DMD-transformed EEG data.

The main contributions of this work are highlighted as follows:

- Definition of a novel seizure detection and prediction approach leveraging on the combination of DMD and CNNs
- Classification of EEG data among three classes, ranging from normal, imminent seizure, and ongoing seizure
- Evaluation of the proposed scenario over the CHB-MIT Scalp EEG Database

The remainder of the manuscript is organized as follows: Section II discusses some related works; Section III presents the necessary background on DMD; Section IV describes the proposed DMD-based seizure detection approach; Section V demonstrates its validity over the CHB-MIT Scalp EEG Dataset; Section VI draws the conclusions and discusses future research works.

II. RELATED WORKS

Given the high sampling frequency required to collect meaningful EEG signals, several works explored how to process EEG data to extract only its most relevant features to better, and more easily, analyze it. For example, the authors of [5] developed a feature extractor model based on Hjorth's mobility parameter in combination with a Fisher Discriminant analysis classifier to detect seizure activity; the core concept is to utilize data on the rate of change in the EEG signal over time to distinguish between normal and epileptic activity. A similar objective is pursued in [6], [7], where nonlinear features from EEG data are extracted using chaotic system theory tools, such as the Lyapunov exponents. Additionally, [8] evaluates entropy-based features, such as the Kullback-Leibler relative and standard entropy.

Along with these standard approaches, machine learning methods find widespread application, due to their inherent ability to automatically learn complex patterns from data. In [9] a cross-correlation aided support vector machine classifier is proposed, whose input is related to the features extracted from crosscorrelograms of EEG signals, while [10] and [11] carry out the classification between seizure and non-seizure states by means of a Fourier transform-based neural network classifier, employing a feed-forward and convolutional neural network respectively.

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Compared to standard feed-forward networks, convolutional neural networks (CNNs) are particularly effective for time-series analysis due to their convolutional layers, which enable them to capture local patterns emerging in the time series and learn hierarchical features. The authors of [12] employ a CNN as a binary classifier while extracting features from EEG data via common spatial pattern and wavelet packet decomposition, while in [13] a CNN functions both as feature extractor and classifier, identifying important features through its one-dimensional convolutional layers and then performing the binary classification.

Other neural architectures particularly suited for time series analysis are recurrent neural networks (RNNs), which present loops that allow information to persist as an internal state. Among RNNs, Long Short-Term Memory (LSTMs) networks are among the most commonly employed, mainly due to their memory cells that are capable of maintaining information for long periods. RNNs are employed as classifiers in [14] and [15], effectively leveraging features extracted from EEG data through discrete wavelet transform and wavelet packet decomposition respectively, while the authors of [16] develop a LSTM-based classifier.

Analysis of EEG data in time-domain is also performed via data-driven dimensionality reduction approaches of the likes of Principal Component Analysis (PCA) [17] and Dynamic Mode Decomposition (DMD) [18]. PCA in particular, thanks to its capability to identify principal components, namely the directions which maximize the variance in EEG data, found application works such as [19]. In such work, the authors propose two threshold-based PCA classifiers, one for epilepsy diagnosis and the other for seizure detection, with their threshold found via normalized partial energy sequence arguments [20]. Similarly, in [21] PCA is used as a feature extractor in combination with genetic algorithms. In addition, the authors of [22] utilize PCA to reduce the dimensionality of feature maps extracted from a feed-forward neural network model, before passing them to a feed-forward classifier.

DMD, on the contrary, thanks to its capability to capture the underlying dynamics of a dynamical system in terms of leading dynamic modes, has been employed as a reduction method [23] in several settings. For instance, in [24], [25] the extracted modes are used to define the DMD spectrum; [26] focuses on vectorized and reordered modes; and [27] proposes a multi-resolution dynamic DMD, which leverages both Fourier transforms and wavelet decomposition.

Independently from the usage of either PCA or DMD to transform and reduce data, once it has been processed, it is then necessary to classify it. To do so, with a common choice being to employ deep neural networks, as illustrated in [28].

Taking inspiration from DMD-based approaches, this work proposes a novel automated seizure detection paradigm analyzing EEG data as if it were the output of a dynamical system, leveraging on DMD as a dimensionality reduction approach to idenfity its dynamic behaviour in terms of power and frequency of the principal dynamic modes, and on a bespoke CNN to classify data into three classes, namely normal, pre-ictal, and ictal.

III. Background on Dynamic Mode Decomposition (DMD)

Data-driven dimensionality reduction techniques have drawn attention from the scientific community for their ability to identify the inherent structure of high-dimensional data generated by complex systems, especially when their corresponding model is non-existent or too complicated to validate. Initially proposed in [4], DMD proved to be paricularly suitable to identify the underlying dynamics of complex systems thanks to its mode decomposition approach.

Let the dynamics of the dynamical system which originated the collected data be described by $\dot{x} = f(x)$, where $x(t) \in \mathbb{R}^n$ and $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is unknown, DMD aims to approximate such a nonlinear dynamics via a locally linear discrete-time dynamical system:

$$x(k+1) = Ax(k), \;\; k = 1, \ldots, m, \tag{1}$$

where m is the number of collected data-points, and A is an unknown linear operator which can be interpreted as the result of sampling the nonlinear dynamics every Δt time instants. As a result, the explicit representation of (1) can be expressed as:

$$x(k) = \sum_{i=1}^{n} \lambda_i^{k-1} \phi_i b_i, \tag{2}$$

where λ_i and ϕ_i are the eingenvalues and eigenvectors of A, also taking the name of *DMD modes*, while b_i are the initial condition coefficients resulting from the projection of the initial state on the eigenvectors of A. The underlying idea of DMD is to derive an optimal eigendecomposition of A in such a way that (1) fits data in a least-square sense, that is the minimization of $\sum_{k=1,\ldots,m-1} ||x(k+1) - Ax(k)||_2$.

Let $x(k) \in \mathbb{R}^n$ be a vector collecting EEG data measurements coming from n channels at time instant k, consider two $n \times m-1$ matrices of horizontally stacked measurements, X and X', for $k = 1, \ldots, m-1$ and $k = 2, \ldots, m$ respectively:

$$\begin{aligned} X &= \begin{bmatrix} | & | & & | \\ x(1) & x(2) & \ldots & x(m-1) \\ | & | & & | \end{bmatrix}, \\ X' &= \begin{bmatrix} | & | & & | \\ x(2) & x(3) & \ldots & x(m) \\ | & | & & | \end{bmatrix}. \end{aligned} \tag{3}$$

Since the columns of X' represent the evolution of the corresponding columns of X after a sampling time Δt, we may define an unknown linear operator A which allows us to recover (1), that is:

$$X' = AX. \tag{4}$$

The best fitting operator in least-square sense can be defined as follows:

$$A \approx X'X^{\dagger} = X'V\Sigma^{-1}U^*, \quad (5)$$

where $X^{\dagger}$ is the Moore-Penrose pseudoinverse, and V, Σ and U resulting from the Singular Value Decomposition (SVD) of matrix X and the * operator is complex conjugate transpose. We recall that, thanks to the properties of SVD, U and V are unitary, Σ is a real-valued diagonal matrix with its diagonal entries sorted in descending order of magnitude and that the columns of V and U are orthonormal eigenvectors of XX^T and X^TX respectively.

As the matrix resulting from (5) is usually of high dimensions ($n \times n$), it may be useful to employ a truncated/approximated SVD of the matrix X, for example by considering only the first r leading modes, which are characterized by the first r columns of U. Let the reduced matrix $\tilde{U}$ contain such r eigenvectors. By projecting the operator A onto $\tilde{U}$, it is possible to define the reduced-order operator $\tilde{A}$ as:

$$\tilde{A} = \tilde{U}^* A \tilde{U} = \tilde{U} X' \tilde{V} \tilde{\Sigma}^{-1}, \quad (6)$$

which in turn defines the reduced order dynamical system

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k), \quad (7)$$

with $\tilde{x}(t) = \tilde{U}x(t)$ being the reduced order state of the original system (1). It is then possible to compute the spectral decomposition of the learned dynamics:

$$\tilde{A}W = W\Lambda, \quad (8)$$

where W has the eigenvectors of $\tilde{A}$ as columns, and Λ the eigenvalues of the latter as diagonal elements, which are called DMD eigenvalues. Finally, the associated DMD modes, namely the eigenvectors of A, are recovered as follows:

$$\Phi = X'V\Sigma^{-1}W, \quad (9)$$

where each column of Φ is the DMD mode ϕ_i corresponding to the eigenvalue λ_i, as introduced in (2).

Finally, as a result, it is possible to represent the observed data as a composition of coupled spatio-temporal modes via a continous-time dynamic model $\hat{X}(t) = \Phi e^{\Omega t} b$, where $\Omega = log(\Lambda)/\Delta t$ and b is the collection of the initial condition coefficients in (2). Additionally, it is possible to derive power and frequency associated to each mode, namely its contribution to the overall system dynamics and its rate of oscillation over time, which constitute the DMD spectrum.

The DMD modes power can be defined as follows:

$$P_{\phi_i} = ||\phi_i||_2^2, \quad (10)$$

while DMD modes oscillation frequency can be evalutated if the correspoding eigenvalue is complex, and is defined as follows:

$$f_{\phi_i} = |\Im(\omega_{\phi_i}/2\pi)|, \quad (11)$$

Algorithm 1 DMD [4]

Input: Snaposhot matrices X, X' and approximation rank $r \in \mathbb{N}_+$

1: Collect measurement data in X, X'
2: Compute SVD of $X = U\Sigma V^*$
3: evaluate $\tilde{A}$ according to (6)
4: Compute the spectral decomposition of $\tilde{A}$ according to (8)
5: Compute the DMD modes according to (9)
6: for each DMD mode ϕ_i evaluate P_{ϕ_i} and f_{ϕ_i} as in (10), (11)

where $\omega_{\phi_i} = log(\lambda_i)/\Delta t$, with Δt time difference between sequential measurements.

The pseudocode of the DMD approach is reported in Algorithm 1.

IV. DMD-based Seizure Detection

Seizure detection in EEG records is performed using a sequential approach that includes data preprocessing, feature extraction, and classification into three categories; ”Normal” (EEG data without any seizure), ”Pre-Ictal” (EEG data up to thirty minutes before a seizure, if possible), and ”Ictal” (EEG data corresponding to a seizure). The overall pipeline is illustrated in Figure 1.

A. Data Preprocessing

EEG signals represent the electrical activity of the brain by measuring voltages fluctuations over time by means of electrodes placed on the scalp. Each electrode represents a measurement channel and is placed in a particular position in order to measure the electrical activity of specific brain regions, allowing for the analysis of brain wave patterns and identification of different brain states. Seizures correspond to abnormal neuronal activity in the brain, which can be associated to the presence of distinct patterns or spikes in the EEG signal.

It follows that, at each time instant t, a signal from each of the considered electrodes is obtained, therefore $x(t) \in \mathbb{R}^n$ is a measurement consisting of multiple components, each representing the measurements from one of the n electrodes. While the electrode position on the scalp is fairly standard according to the international 10-20 system [29], their number can vary depending on the required level of detail in the brain activity mapping.

Note that, simply stacking all the measurements together would result in a data matrix $X \in \mathbb{R}^{n \times T}$, where $n \ll T$ with T being the EEG total recorded samples, hence obtaining a so-called “short and wide”, while DMD works best on a “tall and skinny” matrix, as is the case for data sampled from a high-dimensional system. To overcome this issue, a sliding window approach is considered; EEG signals are segmented into smaller, overlapping windows of duration T_W time steps, and the data matrix is artificially made “tall and skinny” by vertically stacking time-shifted copies of the signal as in [23]:

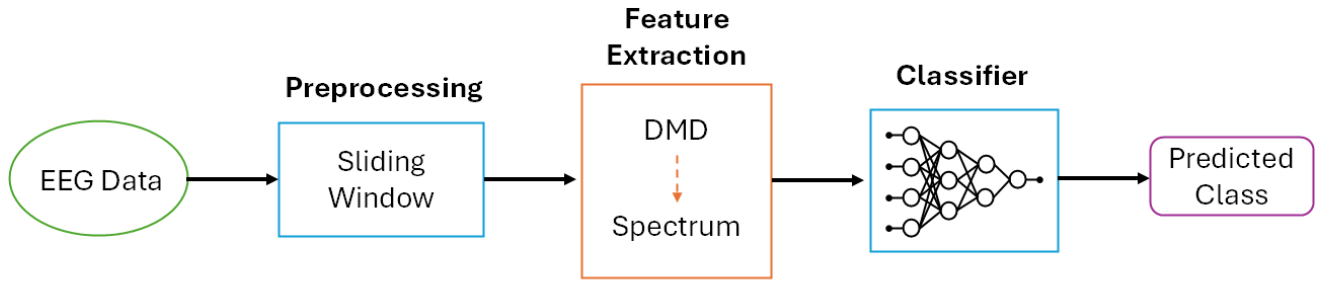

Fig. 1. Pipeline of the proposed DMD-based seizure detection approach.

$$\bar{X} = \begin{bmatrix} | & | & & | \\ x(1) & x(2) & \dots & x(T_W) \\ | & | & & | \\ | & | & & | \\ x(2) & x(3) & \dots & x(T_W+1) \\ | & | & & | \\ \vdots & \vdots & \vdots & \vdots \\ | & | & & | \\ x(T-T_W+1) & x(T-T_W+2) & \dots & x(T) \end{bmatrix}, \tag{12}$$

The data matrix resulting from the preprocessing phase $\bar{X} \in \mathbb{R}^{n\cdot(T-T_W+1)\times T_W}$ with $n \cdot (T - T_W + 1) \gg T_W$ is now "tall and skinny", making it suitable for DMD-based feature extraction purposes. Note that, as in [23], being $\bar{X}$ and similarly its corresponding $\bar{X}'$ are time-shifted stacks of the same raw data, the DMD modes are also stacks of modes of the original $n \times n$ system (1). By re-splitting the DMD modes into T_W separate columns of n elements, one obtains a DMD mode matrix relative to the corresponding T_W window, for which is now possible to extract spectral features.

B. Feature Extraction

Starting from $\bar{X}$ and its one-step time shifted counterpart $\bar{X}'$, Algorithm 1 outputs a matrix of DMD modes, $\Phi \in \mathbb{R}^{n\cdot(T-T_W+1)} \times r$, with $r \leq T_W$. After re-splitting the modes into $T - T_W + 1$ separate vectors, it is then possible to evaluate for every mode its corresponding power and frequency. Each window of T_W steps hence provides $2 \cdot r$ features, and all power and frequency values can then be stacked to form a transformed dataset of extracted features $\Psi \in \mathbb{R}^{(T-T_W+1)\times 2\times r}$, that is then fed into the CNN. This approach allows to give a straighforward interpretation of DMD modes, correlating the magnitude and frequency of a mode to the corresponding electrode which recorded the corresponding signal of length T_W.

C. Classification

Matrix Ψ is given as input to a CNN which solves the seizure detection multi-class classification problem. The convolution operation, which sits at the core of the neural architecture at hand, makes the latter well-suited for extracting classification features from the spatially organized information in the feature map. Finally, the CCN outputs a probability distribution over the considered classes, on the basis of which the most probable one is chosen. Being a standard multi-class classification task, we employed for the CNN training the categorical cross entropy loss function.

V. Results

This section evaluates the performance of the proposed approach over the CHB-MIT Scalp EEG Database, proving its effectiveness in classifying EEG data into "Normal", "Pre-Ictal", and "Ictal".

The CHB-MIT dataset [30] contains 661 EEG recordings of 23 pediatric patients (5 males, ages: 3-22; 17 females, ages:1.5-19) from the Children's Hospital in Boston. Each recording has a total of 18 channels, corresponding to electrodes, positioned according to the international 10-20 system, located at C3-P3, C4-P4, CZ-PZ, F3-C3, F4-C4, F7-T7, F8-T8, PF1-F3, FP1-F7, FP2-F4, FP2-F8, FZ-CZ, P3-O1, P4-O2, P7-O1, P8-O2, T7-P7, and T8-P8. Overall, it contains 158 seizures in around 958 hours of data, excluding the recordings of Patient 12 due to a different electrode configuration,

The recordings of each patient show a sampling frequency of 256 Hz, and are grouped into 24 cases, with each case containing annotated data related to the presence or absence of seizures.

Natively the dataset contains two types of recordings, related to periods of no epileptic activity - class "Normal", and periods when a seizure occurs - class "Ictal". In order to populate the "Pre-Ictal" class, all those records that precede, up to 30 minutes, an epileptic seizure were considered. As a result, a dataset was formed consisting of the three classes, "Normal", "Pre-Ictal", and "Ictal". The motivation behind this choice is intimately linked to the interpretability of the DMD results from the perspective of associating the spectral properties of a mode with signal recorded by an electrode.

The original data matrix X is preprocessed as discussed in section IV-A, considering a sliding window of length $T_W = 2048$, corresponding to 8 s of data. Features are extracted following the approach detailed in section IV-B, evaluating the spectral properties according to (10)-(11) of the selected $r = 200$ DMD modes. Figure 2 shows the DMD spectrum associated to the manipulated CHB-MIT dataset.

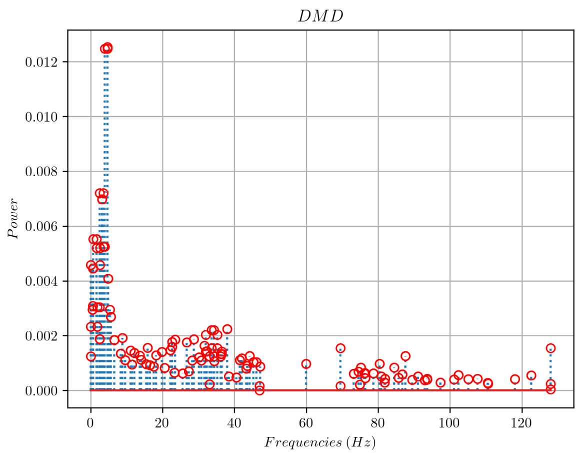

Fig. 2. Example of a DMD output obtained from approximately 8 seconds of EEG signal.

The neural architecture devoted to seizure detection is composed of two convolutional layers: the first one applies a 2D convolution to the input via 48 filters and kernel size (2,5) with 'swish' as activation function, followed by a max pooling operation of size (1,2), while the second one comprises of a 2D convolution layer of 80 filters with kernel size (2,5) and ReLU as activation function, together with a max pooling layer of size (1,2). Then, a flatten layer is followed by a fully connected one performing the multi-class classification with softmax as activation function.

Training is performed on 80% of the data using the Adam optimizer [31] and sparse categorical cross-entropy as the loss function for 22 epochs. Valuation is performed on the remaining 20% of the data using a 6-fold cross-validation. The implementation was conducted using Tensorflow and Keras on an Intel i9 9900k platform with 128 GB of RAM and a Nvidia GTX 3090.

Figure 3 shows the evolution of the value of the sparse categorical cross entropy over the training and validation sets as training epochs go by. With respect to the former, a monotonically decreasing behaviour can be considered, with a significant decrease happening during the first training rounds, meaning that the proposed architecture is suitable to solve the classification problem at hand. The behaviour in terms of the accuracy metric is shown in 4; while the evolution of the latter over the validation set appear to be quite noisy, over the training set it increases almost every other epoch.

Validation by means of the 6-fold over the corresponding set shows a promising result in terms of accuracy of prediction, with the proposed neural architecture achieving around 96% of accuracy, and a value for the loss as small as 0.38.

VI. Conclusions and Future Works

This paper has presented a novel real-time DMD-based seizure detection paradigm which revolves around the employment of DMD spectrum extracted from EEG data within

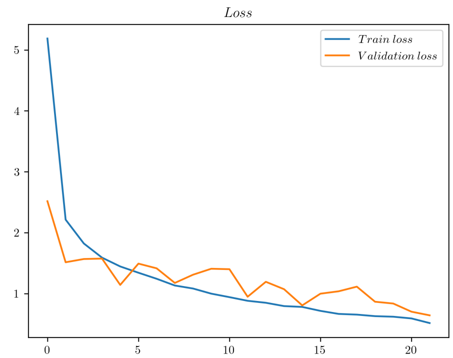

Fig. 3. Loss function evolution during training epochs.

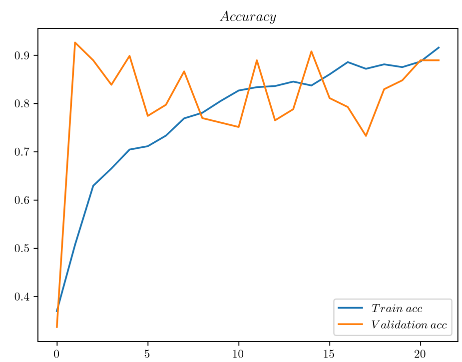

Fig. 4. Accuracy evolution during training epochs.

a CNN classifier. The proposed approach involves using sliding window methods for data preprocessing, DMD to construct a feature map based on power and frequency of the selected modes, and a custom CNN for three-class classification. Future research work aims to provide a biological interpretation of the DMD-identified modes, explicitly linking them to signals from electrodes.

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