# Optimizing Profile Block Bids in Short-Term Hydropower Scheduling: A Two-Phase Model for the Day-Ahead Market

Mohammad Jafari Aminabadi[1,2], Sara Séguin[2,3], Stein-Erik Fleten[4], Ellen Krohn Aasgård[5]

***Abstract*— This paper proposes a two-phase optimization framework for short-term hydropower scheduling in the day-ahead electricity market using profile block bids grouped in exclusive sets. The first phase solves a nonlinear deterministic model that generates a diverse and operationally feasible set of production blocks by accounting for startup costs, opportunity costs, and hydrological constraints. In the second phase, a two-stage stochastic program is used to select a subset of blocks for market submission under price uncertainty. The proposed approach captures a wide range of production scenarios while ensuring compliance with market design rules. By decomposing the problem and relaxing the binary variables, the proposed framework reduces computational time while still reaching the optimal solution. Profile block bids can be effective in complex systems with multiple reservoirs and interconnections, systems with production constraints over consecutive hours, limited operational flexibility, or high startup and shutdown costs. Numerical experiments show that the proposed model achieves comparable or better profits than hourly bidding strategies, while requiring significantly less computational time.**

## NOTATION

| | |
|---|---|
| $t \in \{1, 2, \ldots, T\}$ | Index of periods |
| $c \in \{1, 2, 3, \ldots, C\}$ | Index of hydropower plants |
| $b \in \{1, 2, 3, \ldots, B\}$ | Index of block |
| $s \in \{1, 2, \ldots, S\}$ | Index of scenarios |
| $r \in \{1, 2, \ldots, R^c\}$ | Index of plants upstream of plant c |
| $j \in \{1, 2, \ldots, J_t^c\}$ | Index of power production surface j for plant c at period t |

**Parameters**

| | |
|---|---|
| $\zeta$ | Conversion factor from $m^3/s$ to $Mm^3/h$ |
| $q_{min}^c$ | Minimum water discharge at plant $c$ ($m^3/s$) |
| $q_{max}^c$ | Maximum water discharge at plant $c$($m^3/s$) |
| $v_{min}^c$ | Minimal volume of plant $c$ ($Mm^3$) |
| $v_{max}^c$ | Maximum volume of plant $c$ ($Mm^3$) |
| $v_{Initial}^c$ | Initial volume of reservoir $c$ ($Mm^3$) |
| $m_t$ | Duration of period $t$ ($h$) |
| $P_{t,b}$ | Price of profile $b$ at hour $t$ (Phase 1) |
| $\epsilon^c$ | Startup penalty for plant c |
| $\delta_t^c$ | Inflow in period $t$ ($Mm^3$) |
| $\rho_{s,t}$ | Market price for scenario $s$ at hours $t$ |
| $\gamma$ | Penalty for each additional selected profile beyond the first one |
| $\pi^s$ | Probability of scenario $s$ |
| $\tau^{r \to c}$ | Time delay ($h$) for water transfer from upstream reservoir $r$ |
| $R_{bs}$ | Revenue from using profile $b$ in scenario $s$ |
| $C_s$ | Cost of not selecting any profile in scenario $s$ |

**Variables**

| | |
|---|---|
| $q_t^c$ | Water discharge at period $t$ ($m^3/s$) |
| $v_t^c$ | Reservoir volume at period $t$ ($Mm^3/h$) |
| $g_t^c$ | Water spillage at plant $c$ and period $t$($m^3/s$) |
| $\alpha_t^c(v_t^c)$ | Opportunity cost of water usage at plant $c$ for period $t$ |
| $\chi_{j,t}^c$ | Power production for surface $j$ (MW) |
| $oc^c$ | Lost opportunity costs of using water from the plant $c$ at period $t$ |

$$z_j \quad \begin{cases} 1, & \text{if surface j is chosen,} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{bs} \quad \begin{cases} 1, & \text{profile } b \text{ is optimal for scenario } s, \\ 0, & \text{otherwise.} \end{cases}$$

$$w_b \quad \begin{cases} 1, & 1 \text{ if profile } b \text{ is in the exclusive group,} \\ 0, & \text{otherwise.} \end{cases}$$

$$u_s \quad \begin{cases} 1, & 1 \text{ if no profile is selected in scenario } s, \\ 0, & \text{otherwise.} \end{cases}$$

## I. Introduction

European day-ahead electricity markets support various bidding formats, including hourly, flexible hourly, and block bids. Block bids, which group multiple hours into a single bid, are particularly useful in hydropower systems with intertemporal dependencies—such as cascaded reservoirs—where delays in water flow can cause mismatches between production and market prices. These bids promote stable operations and are especially suited for managing upstream-downstream conflicts [1]. By considering operational limits and cost factors, block bidding offers a structured way for producers to navigate complex market conditions [14].

Block bids are defined by an all-or-nothing acceptance rule, supporting conditional and intertemporal power delivery. Variants include regular block bids (constant power over a time window), profile block bids (variable output), and linked block bids (parent-child dependencies) [14], [18], [2]. Profile bids allow producers to tailor their energy delivery across hours, improving alignment with price variations. These bids are cleared by comparing the offered price to the weighted average market price over the selected delivery window [15].

Numerous studies have explored block bidding in hydropower scheduling. For example, Fleten and Kristoffersen [10] and Faria et al. [9] study regular block bids under price uncertainty and operational constraints. Alnæs et al. [3] analyze real bidding data from Norwegian producers and highlight the benefits of combining hourly and

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[1]M. Jafari Aminabadi is with the Department of Applied Sciences, Université du Québec à Chicoutimi (UQAC), Saguenay (QC), G7H 2B1, Canada mjaminabad@etu.uqac.ca

[2]M. Jafari Aminabadi and S. Séguin are with the Group for Research in Decision Analysis (GERAD), Montréal (QC), H3T 2A7, Canada sara.seguin@uqac.ca

[3] S. Séguin is with the department of Computer Science and Mathematics, UQAC, Saguenay (QC), G7H 2B1, Canada

[4]S.-E. Fleten is with the Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, N–7034 Trondheim, Norway

[5] E. K. Asgård is with ANEO, N–7031 Trondheim, Norway

block bids. More recent works focus on flexible bidding structures: Karasavvidis et al. [14] develop an optimization model for hydrothermal systems that includes both profile and linked bids, while Vlachos et al. [18] and Adams et al. [2] investigate adjustable and dependent bid structures. In terms of methodology, Biel et al. [6] integrate stochastic programming with neural networks for bidding in Nordic hydropower, and Schledorn et al. [16] use stochastic optimization in combined heat and power systems. Hübner et al. [11] examine exclusive bid selection and package bidding mechanisms in European markets, showing how bid limitations affect welfare and efficiency. Recent electricity market designs emphasize exclusive block bid groups, or mutually exclusive bids, which allow producers to submit alternative power delivery profiles while ensuring that only one is accepted. This reduces overcommitment and better reflects operational feasibility. In this context, [11] examine European package bidding mechanisms and the selection challenges that arise from bid limitations. Their work shows how restrictions can lead to welfare losses and proposes algorithms for better bid selection—ideas that underpin the second phase of our model, where total unimodularity plays a key role.

While most prior research has focused on regular block bids, fewer contributions address profiled blocks. This paper proposes a two-phase model for profiled block bidding in the Norwegian day-ahead market. In Phase 1, we solve a short-term hydropower scheduling problem that generates diverse and operationally feasible bidding blocks, accounting for startup costs, water use, and constraints. Phase 2 is a two-stage linear stochastic program that selects the most profitable blocks from the set, based on market price scenarios. This structure helps manage uncertainty while significantly reducing computational effort.

The remainder of the paper is structured as follows. Section II presents the methodology and mathematical formulation of the two-phase model. Section III introduces the case study. Section IV discusses the results. Section V compares model performance with hourly bidding strategies. Section VI concludes and outlines future research directions.

## II. Methodology

Short-term hydropower scheduling focuses on determining the optimal hourly production plan for one or more hydro units over a short planning horizon, while satisfying constraints such as reservoir bounds, turbine capacities, and startup costs. Power generation depends on technical factors like reservoir volume, water discharge, and net head [17], [4]. Electricity trading is commonly organized into three main markets: the day-ahead, intraday, and balancing markets. Most transactions occur in the day-ahead market, where producers and consumers submit bids for the following day, usually before noon. The market operator clears the market and publishes hourly prices and commitments around 1 p.m. [1], [8]. The intraday market allows updates closer to delivery, and the balancing market, operated by the Transmission System Operator (TSO), handles real-time imbalances [1].

This paper proposes a two-phase optimization framework to identify an optimal profile block bidding strategy for short-term hydropower scheduling in the day-ahead electricity market under price uncertainty. The methodology is designed to ensure operational feasibility while maximizing market-based profitability. In the first phase, a deterministic optimization model is solved using forecasted electricity prices and inflow data. This model incorporates key operational features such as opportunity costs, startup costs, and hydrological constraints including reservoir balance and turbine limits. The goal is to generate a diverse and feasible set of production profiles (blocks) that are compliant with the rules of block bidding in electricity markets. The minimum order duration for block bids may vary depending on market regulations. In this study, each block represents a production period ranging from a minimum of three consecutive hours up to 24 hours. This diversity allows the model to accommodate various operational conditions and prepares it to respond flexibly to a wide range of market scenarios.

The feasible blocks generated in this phase, along with their associated production costs, opportunity costs, and startup costs, are passed to the second phase. In this phase, a two-stage stochastic programming model is used to select the most profitable subset of blocks for market participation. Price uncertainty is modeled through a set of price scenarios that become available close to the bidding deadline. Based on these scenarios, the model evaluates the expected economic performance of each block and selects a fixed number (e.g., 15 blocks) that maximize the overall expected profit. This selection process reflects actual market design, where producers are typically allowed to submit a limited number of block bids grouped into exclusive sets. In such exclusive groups, at most one block can be accepted. The objective accounts for all relevant costs, including production, opportunity, and startup costs. The proposed formulation ensures computational efficiency, even for large-scale instances, and provides a structured and scenario-driven approach to support informed bidding decisions under uncertainty.

### *A. Phase 1: Profile Generation*

Phase 1 of the proposed methodology focuses on generating a set of profile block bids that define potential operational schedules for the hydropower plant over a given time horizon. To achieve this, a nonlinear deterministic optimization model is formulated, in which market prices and inflows are considered as parameters. The objective is to maximize revenue while accounting for key operational costs, including water usage, opportunity costs, and turbine startup expenses. The model generates candidate blocks by solving the hydropower scheduling problem under all operational constraints, ensuring that all blocks are physically feasible and consistent with realistic operating conditions.

Hydropower optimization is inherently nonlinear, as power production depends on water discharge, reservoir volume, and turbine efficiency. The net water head, which directly influences power generation, is determined by the forebay and tailrace elevations, as well as penstock losses. Additionally, turbine efficiency varies across units, meaning that even under similar water discharge and head conditions, different turbines may yield different power outputs.

Instead of explicitly modeling each turbine or considering all possible combinations of turbine operations, the model uses the maximum power output surface, which approximates the nonlinear relationship between water discharge, reservoir volume, and power production using polynomial regression. These surfaces, derived from combinations of feasible turbine configurations, offer a simplified yet accurate representation of turbine behavior. In addition, they help reduce the number of binary variables in the model, which lowers the overall computational complexity. This modeling approach is based on the method presented in [17]. The use of power output surfaces introduces binary variables, leading to a Mixed-Integer Nonlinear Programming (MINLP) formulation. While MINLP models can provide precise solutions, they are often computationally demanding in large-scale hydropower systems. To improve tractability, the model is formulated in a way that ensures total unimodularity in the constraint matrix. As a result, even when binary variables are relaxed, the problem still yields integer solutions. In this case, the matrix of the coefficients of the constraints is totally unimodular and therefore meets these three conditions: 1) All submatrices have elements in the set {-1, 0, 1}. 2) Each column has at most two nonzero elements. 3) There exists a partition of rows such that every column with two nonzero elements satisfies this partition. If these conditions are met, the binary selection problem can be solved as a continuous nonlinear problem while still yielding integer solutions. The optimization model aims to maximize total revenue by selecting the most efficient production profiles while accounting for key operational costs. The objective function Eq. (1) maximizes the total profit, where $P_t$ denotes the market price at time $t$, and $\chi^c_{j,t}(q^c_t, v^c_t)$ represents the power output as a function of water discharge and reservoir volume. An essential component of the model is the inclusion of water usage costs, modeled as opportunity costs that depend on reservoir volume. For simplicity, in the context of the analysis in this paper, these costs are represented by a linear function: when the reservoir is nearly full, the opportunity cost is low due to the low risk of scarcity; as the volume decreases, the opportunity cost increases, reflecting the higher value of conserving water for future use. For example, if the reservoir level drops from full capacity to 50%, the opportunity cost may increase proportionally from 1 to 5 €/MWh. This dynamic encourages more strategic water allocation, especially during periods of low storage. In addition, startup costs are included for each production unit. These costs are determined by solving a unit commitment problem, following the methodology described in [17]. The mathematical formulation of Phase 1 is presented as follows.

$$\max_{q^c_t, v^c_t, z^c_{j,t}} \quad \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} P_t \cdot \chi^c_{j,t}(q^c_t, v^c_t)\cdot z^c_{j,t} - \sum_{c\in C}\sum_{t\in T} \alpha^c_t(v^c_t) \tag{1}$$

$$\text{s.t.} \quad v^c_{t+1} = v^c_t - \zeta m_t(q^c_t + g^c_t) + \zeta\delta^c_t + \sum_{r\in R} \zeta m_{t-\tau^{r\to c}}(q^r_{t-\tau^{r\to c}} + g^r_{t-\tau^{r\to c}}), \quad \forall t, c \tag{2}$$

$$\sum_{j\in J} z^c_{j,t} \leq 1, \quad \forall t, c \tag{3}$$

$$v^c_1 = v^c_{\text{Initial}}, \quad \forall c \tag{4}$$

$$q^c_{\min} \leq q^c_t \leq q^c_{\max}, \quad \forall t, c \tag{5}$$

$$v^c_{\min} \leq v^c_t \leq v^c_{\max}, \quad \forall t, c \tag{6}$$

$$v^c_t \geq 0, \quad q^c_t \geq 0, \quad \forall t, c \tag{7}$$

$$z^c_{j,t} \in \{0, 1\}, \quad \forall t, j, c \tag{8}$$

Eq. (2) defines the water balance for each reservoir in the system. It ensures that the volume of water stored in reservoir $c$ at time $t+1$, denoted by $v^c_{t+1}$, is equal to the volume at time $t$, $v^c_t$, minus the water released for power production and spillage, $w_t(q^c_t + g^c_t)$, plus the natural inflow $\delta^c_t$, all scaled by the conversion factor $\zeta$, which converts discharge from $\mathrm{m}^3/\mathrm{s}$ to $\mathrm{Mm}^3/\mathrm{h}$. Additionally, the equation accounts for water inflow from upstream reservoirs that are hydraulically connected to reservoir $c$. These contributions are modeled with a delay $\tau^{r\to c}$, representing the travel time of water from an upstream reservoir $r$ to reservoir $c$. This formulation provides a realistic representation of reservoir interactions, especially in systems where water released from upstream plants does not immediately reach downstream reservoirs. Eq. (3) guarantees that, for each unit $c$ and every time period $t$, exactly one production surface is selected from the available set. Eq.(4) sets the initial reservoir volume to a predefined value $v^c_1 = v^c_{Initial}$, ensuring a known starting condition. Eq.(5) and (6) impose operational constraints on water discharge and reservoir volume, restricting them within their respective minimum and maximum limits to maintain system feasibility. Eq.(7) enforces non-negativity constraints on reservoir volume and water discharge to ensure physically meaningful solutions. Finally, Eq. (8) defines the binary nature of $z^c_{j,t}$. Although the model includes binary variables $z^c_{j,t}$ for production surface selection, the constraint matrix is designed to be totally unimodular. This property allows the relaxation of binary constraints without losing the integrality of the solution. In practice, this means that the mixed-integer nonlinear model can be efficiently solved as a continuous nonlinear program, reducing computational complexity while still yielding integer-accurate results.

### *B. Phase 2: Two stage stochastic profile selection optimization*

The objective of Phase 2, a two-stage stochastic binary programming model, is to select the optimal blocks based on price scenarios from among the block set generated in Phase 1. Thus, the optimal power production values and associated block costs calculated in Phase 1 are considered as inputs for Phase 2. Additionally, the scenarios incorporate the uncertainties of day-ahead market prices. The objective function in Phase 2 includes revenue from each block under different price scenarios, deducts associated costs—such as opportunity and startup costs—and incorporates a penalty term to prevent the selection of blocks that do not contribute to improving the objective function value. The mathematical formulation of the second phase is as follows:

$$\max \sum_{b \in B} \sum_{s \in S} \pi^s R_{bs} x_{bs} - \sum_{s \in S} \pi^s C_s u_s - \gamma \left( \sum_{b \in B} w_b - 1 \right) \quad (9)$$

$$\sum_{b \in B} w_b \leq N_{blocks}, \quad (10)$$

$$\sum_{b \in B} x_{bs} + u_s = 1 \qquad , \forall s \in S, \quad (11)$$

$$x_{bs} \leq w_b \quad , \forall b \in B, \forall s \in S, \quad (12)$$

$$w_b \in \{0, 1\} \qquad , \forall b \in B, \quad (13)$$

$$x_{bs} \in \{0, 1\} \quad , \forall b \in B, \forall s \in S. \quad (14)$$

Eq.(10) ensures that the total number of selected profiles does not exceed the predefined limit $N_{blocks}$, controlling the maximum number of bids submitted to the market. Eq.(11) enforces that for each scenario, exactly one decision is made——either one of the available profiles is selected, or no profile is chosen, which is indicated by $u_s$. This guarantees that the sum of selections per scenario equals one. Eq.(12) ensures that a profile $b$ can only be selected in scenario $s$ if it has already been included in the bidding set. This maintains logical consistency between the profile selection variable $w_b$ and its scenario-dependent selection $x_{bs}$. Eq.(13) specifies that each profile is either included in the bidding set or not, ensuring that no partial profile selections occur. Eq.(14) enforces a binary decision on whether a profile is selected in a specific scenario, maintaining the discrete nature of the problem. The model selects a limited number of blocks that offer the highest expected profit, while ensuring that only one block is active per scenario. This structure reflects actual market rules for exclusive bids and supports reliable bidding decisions. Based on the given price scenarios, the model selects the blocks that are expected to generate the highest profit. This profit is calculated by subtracting relevant costs, including startup costs and opportunity costs, from the expected revenues. This approach allows the bidding strategy to account for price uncertainty and ensures that the selected offers are both economically efficient and compliant with market rules.

## III. Case Study

The two-phase model has been evaluated in a case study of a hydropower system in Norway, which includes multiple reservoirs and power plants. This system consists of six interconnected reservoirs (Sverjesjøen, Falningsjøen, Innerdalsvannet, Storfossdammen, Granasjøen, and Bjørsetdammen) that supply water to five hydroelectric power plants: Ulset, Litjfossen, Brattset, Grana, and Svorkmo. Each plant has specific generation capacities and water discharge constraints. The installed capacity varies across plants, with Brattset (88 MW), Grana (82.5 MW), and Litjfossen (84 MW) having the highest output potential, while Ulset (40 MW) and Svorkmo (57.7 MW) provide additional flexibility. The reservoirs also differ significantly in volume: Innerdalsvannet (153.4 $Mm^3$), Falningsjøen (125.2 $Mm^3$), and Granasjøen (138.8 $Mm^3$) offer substantial storage capacity, whereas smaller reservoirs such as Bjørsetdammen (0.02 $Mm^3$) and Storfossdammen (1.69 $Mm^3$) are primarily used for short-term regulation and discharge routing. The system's topology incorporates a network of bypass channels and spillways, which regulate water flow between reservoirs, enhancing operational flexibility and stability.

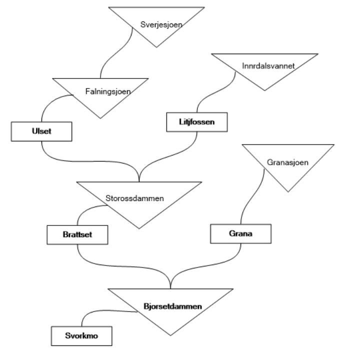


Fig. 1. System topology

The two-phase model involves solving two optimization problems with different structures. Phase 1 solves a short-term hydropower scheduling problem formulated as a mixed-integer nonlinear program (MINLP), aiming to generate feasible production profiles that account for water use, opportunity costs, and startup costs. This problem is solved using the Ipopt solver [19].

Phase 2 uses a two-stage stochastic linear program to select the most profitable subset of blocks based on multiple day-ahead price scenarios. This model is solved with the CLP solver [13]. To validate the Phase 1 results, the BONMIN solver [7], which handles nonlinear problems with binary variables, is also used. The entire implementation is done in Julia [5], and the experiments are conducted on a system with an Intel Core i5 processor and 8 GB of RAM.

## IV. Results

Since Phase 1 of the problem is solved deterministically, this section investigates the impact of the number of candidate blocks generated in Phase 1 on the performance of Phase 2. Specifically, we analyze how increasing the number of blocks in Phase 1 affects the objective value and the quality of the solution obtained in Phase 2. To evaluate this, 5 representative days were randomly selected from the dataset. Each case is characterized by different input parameters, including electricity prices and initial reservoir volumes. For each case, the second phase was solved using 30 day-ahead price scenarios. The corresponding results are summarized in Table I, which reports both the objective function values and the computation times for different numbers of blocks.

Phase 1 was executed for various numbers of blocks: 25, 50, 100, 250, 500, 750, 1000, and 1500. Given space limitations, a few of them are selected and presented in

Table I. For each case, the input data used in Phase 1 remains the same regardless of the number of blocks. Each block corresponds to a continuous production period with variable length, starting from a minimum of three hours and located within the 24-hour planning horizon. Any block that does not satisfy these criteria is excluded from the candidate set. The final set is then constructed to meet the specified numbers of blocks for each case. The feasible blocks, along with their associated opportunity and startup costs, are then passed to Phase 2. In this stage, a two-stage stochastic programming model is used to select the most profitable combination of blocks to be offered in the day-ahead electricity market. The price scenarios reflect real market conditions close to the bidding time. As shown in Table I, increasing the number of candidate blocks in Phase 1 generally leads to better objective function values in Phase 2. This indicates that having access to a richer set of block options improves bidding decisions under uncertainty.

TABLE I
OBJECTIVE FUNCTION VALUE AND COMPUTATION TIME (IN SECONDS) FOR DIFFERENT NUMBERS OF CANDIDATE BLOCKS ACROSS FIVE CASES WITH VARYING INPUT DATA

| | | **50** | **250** | **500** | **1500** |
|---|---|---|---|---|---|
| **Case 1** | Time (s) | 0.01 | 0.04 | 0.06 | 0.22 |
| | Obj. Value | 34082 | 36475 | 38521 | 38548 |
| **Case 2** | Time (s) | 0.00 | 0.02 | 0.03 | 0.15 |
| | Obj. Value | 60458.5 | 63676.3 | 65304 | 65871.3 |
| **Case 3** | Time (s) | 0.01 | 0.02 | 0.06 | 0.22 |
| | Obj. Value | 97592 | 103629 | 103629 | 104720 |
| **Case 4** | Time (s) | 0.01 | 0.02 | 0.04 | 0.11 |
| | Obj. Value | 13237.7 | 14308.8 | 15003.7 | 15079.7 |
| **Case 5** | Time (s) | 0.01 | 0.02 | 0.02 | 0.09 |
| | Obj. Value | 18329.1 | 19525.3 | 20468 | 20629 |

Fig. 2 visualizes the normalized performance of Phase 2 based on the objective values presented in Table I. The x-axis represents the number of candidate blocks used in Phase 1, while the y-axis shows the corresponding objective value expressed as a percentage of the maximum value (i.e., the value obtained with 1500 blocks, set to 100%).

These percentages were calculated by dividing the objective function value for each block size by the maximum value observed across all tested sizes, and then multiplying by 100. Formally, for a given number of blocks $n$, the normalized value is computed as:

$$\text{Percentage}_n = \left(\frac{Obj_n}{Obj_{\max}}\right) \times 100 \tag{15}$$

where $Obj_n$ is the objective function value for $n$ blocks, and $Obj_{\max}$ is the maximum value obtained (in this case, with 1500 blocks). As shown in the Fig. 2, increasing the number of candidate blocks leads to improvements in the objective function value. However, beyond 500 blocks, the improvements occur at a slower rate, indicating that the marginal benefit of adding more blocks diminishes.

Although the solution time increases slightly with the number of candidate blocks, Phase 2 remains computationally efficient. Even with up to 1500 blocks and 30 price scenarios, the problem can be solved in less than a second, which demonstrates the scalability of the proposed method.

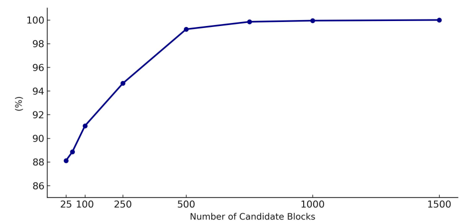


Fig. 2. Normalized objective function value for different numbers of candidate blocks.

Fig. 3 illustrates the average solution time across five representative cases for different numbers of candidate blocks. As shown in the figure, the computational time grows gradually as more candidate blocks are introduced, but remains consistently low—well below one second—even for the largest problem sizes considered. This further confirms that the relaxation of binary variables, supported by the total unimodularity of the constraint matrices, enables fast and scalable optimization in the second phase.

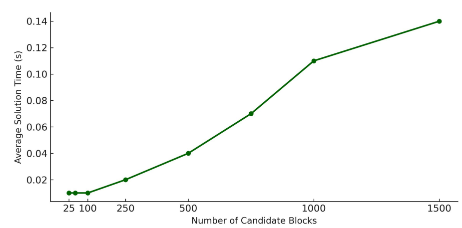


Fig. 3. Average solution time (s) for different numbers of candidate blocks.

## V. Model Evaluation

To evaluate the proposed two-phase model, we compare it with a model introduced in [12], which formulates the day-ahead hourly bidding problem as a two-stage stochastic mixed-integer nonlinear programming model. In this reference framework, first-stage decisions determine the bid volumes, while second-stage decisions reflect the actual hourly dispatch under different price scenarios. In the hourly bidding model, the imbalance between committed and realized volumes is explicitly considered, with corresponding rewards and penalties determined based on participation in the balancing market. For comparison, all input parameters such as initial reservoir volumes, inflows, and operational constraints are considered identical in both models. Given that the hourly bidding model requires separate offers for each hour of the day, each block generated in our proposed approach is also designed to span a full 24-hour period. Moreover, the water usage cost has been added to the objective function of the hourly bidding model. The hourly bidding model also follows market rules, requiring offer curves to be non-decreasing with respect to price levels. Likewise, committed volumes are determined based on market-clearing prices using linear interpolation. Additionally, the same set of price scenarios used in the second phase of the block bidding model is applied to the hourly model to ensure that both methods are evaluated under identical market uncertainty.

After the market is cleared, the profit from each submitted block and the profit from the hourly bidding model are calculated for comparison purposes. In the profiled block bidding model, since at most one block can be accepted, the selected block will be the one with the highest offered price that does not exceed the market price. This ensures compatibility with market rules. Given that the hourly bidding model is a two-stage stochastic mixed-integer nonlinear program and that the case study involves five hydropower plants and six reservoirs, solving the model under a large number of scenarios presents computational challenges. Therefore, to enable a meaningful and tractable comparison between the two models, five representative price scenarios are considered. Due to the complexity of the hourly bidding formulation, startup costs could not be incorporated in that model; hence, for consistency, startup costs were also excluded from the block bidding model in this part of the evaluation.

The comparison results are summarized in Table II, which includes multiple test cases evaluated with varying inputs and price scenarios on different days. For the five cases, the numbers of blocks is 750 and the number of scenarios is 5.

TABLE II
COMPARISON OF HOURLY BIDDING AND SELECTED BLOCK PROFIT

| Case | Hourly Bidding Profit | Selected Block Profit |
|---|---|---|
| Case 1 | 70,369 | 69,327 |
| Case 2 | 144,060 | 147,883 |
| Case 3 | 89,783 | 89,594 |
| Case 4 | 54 ,069 | 55,380 |
| Case 5 | 71,586 | 71,890 |

As shown in Table II, the proposed method provides better results or results that are very close compared to the hourly bidding model. Block bids are particularly valuable for hydropower producers because they allow for offering energy over multiple consecutive hours with operationally feasible patterns. This is beneficial in systems with reservoir dependencies, startup costs, or limited flexibility. Moreover, block bids contribute to more stable production schedules and better alignment with market prices under uncertainty. The model presented in this paper benefits from relaxing the binary variables in both phases, resulting in very short solution times. Therefore, it can be highly efficient and can be applied to complex and large-scale problems.

## VI. CONCLUSION

This paper proposes a two-phase optimization framework to help hydropower producers bid in day-ahead electricity markets using profile block bids in exclusive groups. The first phase uses a deterministic MINLP model to generate feasible production blocks, accounting for operational limits, startup, and opportunity costs. The second phase applies a two-stage stochastic program to select the most profitable blocks based on market price scenarios, respecting bidding rules. Results show the model delivers strong bidding strategies quickly, with similar or better profits than hourly bidding and lower complexity. Future work includes refining opportunity costs with seasonal inflow patterns, testing more price scenarios, and exploring a hybrid model combining hourly and block bids.

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