

# Minimum Energy Policies for Machines in Job-Shops during their Idle Periods

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*Abstract* - The objective of this study is to contribute to energy efficiency in the manufacturing industry, more particularly in high energy-consuming integrated production plants. This study considers particularly job-shops where the energy necessary to realize production plans must be minimized while satisfying make-span constraints. When machines are operated in a standard way, for a given production plan, the component of energy consumption which is a variable corresponds to the unproductive periods of the machines between two operations. In this study, the cases of mechanical and thermal machines are examined to identify the energy optimal policies which allow coping efficiently with their unproductive periods. The question was at first to identify the conditions in which a shut down or an idle regime was a better solution for a machine during an unproductive period, however, the performed analysis has led to considering hybrid solutions where the machine is in a first stage shut down and then put into an idle mode until its startup to perform the next operation.

*Index Terms* – energy minimization, idle periods, industrial machines, job-shop, optimal control.

## INTRODUCTION

Today with the increasing prices of energy and the new environmental protection regulations, besides the minimization of make-span, universities, supported by industrial organizations in advanced countries, are increasingly focusing on the energy-saving issue in job-shops. The relation between industrial management styles, in general Lean management and variants, and energy savings has been of recent concern [1] while new formulations of the job-shop scheduling problem including the energy issue have been proposed. The studies of Zhang et al [2] and Meng et al [3] have shown that it is possible to diminish energy manufacturing consumption by rearranging the production process for a given demand through adequate machine selection and operations sequences. The study of Tang and Dai [4] has also shown that by adjusting power for each operation in a job-shop environment, relevant energy savings can be obtained. However, in a real context where disturbances normally appear, it seems difficult to effectively maintain optimal relative timing of the operating speeds of each machine in the whole workshop, leading to unaccounted additional delays and costs.

Given a production plan with its time horizon, the total energy spent for its realization in a job-shop is composed of two components:

- the energy spent to maintain adequate working conditions in the job-shop (lighting, temperature, humidity, air cleanliness, ...);
- the energy spent in the productive process, which is composed of the energy spent by the machines and the energy spent by the material handling and transport devices.

The energy spent by the machines is composed of startup energy, the idle energy when machines are activated at a low regime but not producing and the machining or processing energy during production time. Different studies have already considered the processing energy of several industrial machines [5, 6, 7] while when considering unproductive states, only startup energy has been considered [8]. Here, we consider that for each type of operation the machines adopt a nominal mode resulting from various considerations such as product quality, stress imposed on the machine, energy and processing time spent per part. In this study it assumed that handling and transport energy costs and delays are constant and depend only on the production plan, the layout of the job shop and the corresponding handling and transport equipment. Then, in this situation the only variable energy costs are those linked to the intervals when the machines are not productive, i.e. shut down, startup and idle operating energy costs, and minimizing total energy costs in such job shops through adequate scheduling of operations, reduces to the minimization of the variable energy associated with the unproductive periods of the machines.

The paper is organized as follows: first the considered problem is characterized, then local energy policies for unproductive periods are considered for mechanical and then thermal machines. This leads to both cases for consideration, where possible, superior hybrid solutions with a shut-down stage followed by a reduced regime for the machines before their new start-up for the next operation. Finally, bounds are established for each machine and then for the whole job-shop, giving allowance to assess the relative importance of this energy component for a given production plan.

## PROBLEM CHARACTERIZATION AND NOTATIONS

### I. Scope of the study

The class of considered job-shops is composed of a serial-parallel network of machines which process different products. The final products are obtained at the end of the sequences of operations. The main adopted objective is to minimize the total production energy  $E_T$  needed to perform the  $n$  jobs. Here, it can be considered that the total production energy  $E_T$  is composed of a constant part attached to the processing, handling and transport energy and of a variable part attached to the unproductive periods of the machines. When considering the unproductive periods of a machine, in general, the solution of turning off the machine allows its energy consumption to be set to zero during this period while the idle solution corresponds to reduced energy consumption, but over significant periods of time this consumption may no longer be negligible. Restarting a completely stopped machine to reach a certain power level compatible with production processing generally results in greater power to be used over a longer time than in the case of a machine which was in idle mode, leading to higher energy consumption. It therefore appears that there is no a priori better solution and that an optimal solution will depend on the different operating parameters characterizing the different quantities of energy to be developed in the two cases. The comparative analysis of the two basic options naturally leads to considering hybrid solutions which try to combine their advantages during the unproductive periods.

Here we examine schematically the case of machines developing mechanical energy and machines developing thermal energy, which will allow us in both cases to characterize the optimal solution and the parameters on which it depends.

### II. Basic assumptions

The assumptions common to machining and thermal processes are the following:

- The production plan is composed of jobs consisting of a sequence of operations which are performed using the machines of the job-shop.
- A machine can only execute one operation for a given job at a given time and pre-emption is not allowed.
- To start-up a machine for production, it is supposed that the machine has been set up: the product to be processed being available has been placed and the tools to be used are ready.
- It is supposed that the set-up operation can only be realized when the machine is idle or shut-down and when the product to be processed is available.
- It is supposed that once a machine has been set up and started, production begins immediately.

- Each machine is attached to handling and transport devices. The delays and energy associated with their operation are considered constant.

### III. Adopted notations

$i$ : index of jobs,  $i \in \{1, \dots, n\}$ ;  $k$  index of operations,  $k \in \{1, \dots, S_i\}$ ;  $l$ : index of machines,  $l \in \{1, \dots, m\}$ ;  $n$ : total number of jobs,  $m$ : total number of machines:  $m = |M|$  where  $M$  is the set of machines,  $m_{ik}$  is the index of the machine allowing to perform operation  $O_{ik}$ , the  $k^{\text{th}}$  operation of job  $i$ .

Let  $t_{ik}$  be the starting time of operation  $O_{ik}$  on machine  $m_{ik}$ ,  $\tau_{ik}^{su}$  be its set-up delay and  $d_{ik}$  be the processing duration,  $i=1$  to  $n$ ,  $k=1$  to  $S_i$ ;  $T_{ik}$  be the nominal transfer time of product  $i$  from machine  $m_{ik}$  to  $m_{i,k+1}$ ,  $i=1$  to  $n$ ,  $k=1$  to  $J_i$ ,  $m_{i0}$  being the stock of raw material processed by job  $i$ . The nominal transfer time is in general the transportation time of the product from one machine to the next. Here it is supposed that on machine  $l$  the operation  $O_{ik}$  is followed by operation  $O_{i'k'}$  with  $m_{ik} = m_{i'k'} = l$ . The unproductive period of machine  $l$  between operations  $O_{ik}$  and  $O_{i'k'}$  is given by  $t_{i'k'} - (t_{ik} + d_{ik})$ .

### ENERGY FOR UNPRODUCTIVE PERIODS IN MACHINING

These machines are able to perform operations such as turning, milling, grinding, ... once the necessary power is available and the piece to be worked on has been set-up on the machine. Here, available power is the driving parameter for production. Let  $P_{ik}$  and  $P_{i'k'}$  be the necessary power to perform the end of operation  $O_{ik}$  and the beginning of operation  $O_{i'k'}$  on machine  $l$ . Figure I considers an unproductive period for this class of machines and represents power evolution: in blue solid lines an idle solution and in red solid lines a shut-down solution.

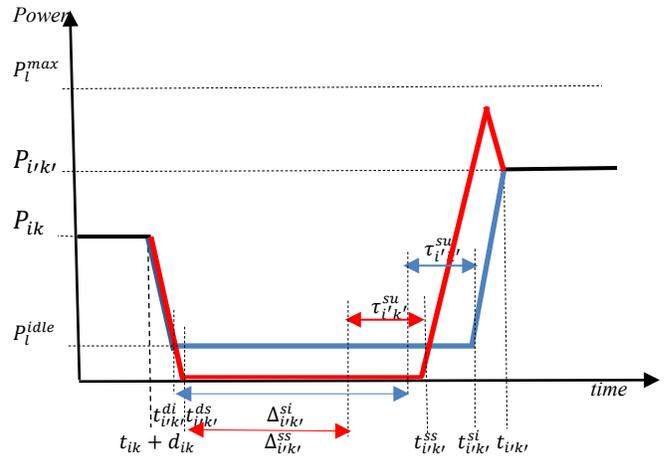


FIGURE I

POWER SETTINGS FOR MACHINING DURING UNPRODUCTIVE PERIODS

Let  $\alpha_{ik}^{idle} (= t_{i'k'}^{di} - (t_{ik} + d_{ik}))$  and  $\alpha_{ik}^{shut} (= t_{i'k'}^{ss} - (t_{ik} + d_{ik}))$  be the delays to get the reduced powers during the unproductive

period after the operation  $O_{ik}$  on machine  $l$  and  $\beta_{i'k'}^{idle}$  ( $= t_{i'k'} - t_{i'k'}^{si}$ ) and  $\beta_{i'k'}^{shut}$  ( $= t_{i'k'} - t_{i'k'}^{ss}$ ) be respectively the delays to reach at minimum energy cost and maintain power  $P_{i'k'}$  on machine  $l$ . For the shut-down and idle solutions, the respective time domains where set-up is possible are given by  $\Delta_{i'k'}^{ss}$  and  $\Delta_{i'k'}^{si}$  where  $\Delta_{i'k'}^{ss}$  is smaller than  $\Delta_{i'k'}^{si}$ . We have the following feasibility conditions for each solution:

- For the idle solution:

$$t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{idle} = \alpha_{ik}^{idle} + \tau_{i'k'}^{su} + \beta_{i'k'}^{idle} \quad (1)$$

- For the shut-down solution:

$$t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{shut} = \alpha_{ik}^{shut} + \tau_{i'k'}^{su} + \beta_{i'k'}^{shut} \quad (2)$$

Since in general the sum of transition delays for the idle solution  $\alpha_{ik}^{idle} + \beta_{i'k'}^{idle}$  is smaller than the sum of the transition delays for the shut-down solution  $\alpha_{ik}^{shut} + \beta_{i'k'}^{shut}$ , relation (1) could be satisfied and not relation (2) if the unproductive period is such as:

$$\theta_{iki'k'}^{shut} \geq t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{idle} \quad (3)$$

It appears also from Figure 1 that when the unproductive period increases, there exists a value  $\theta_{iki'k'}^{min}$  such that if:

$$t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{min} \quad (4)$$

then, the shut-down of the machine is the best solution. This value can be obtained easily from Figure 1. Beyond this value for the unproductive period, its energy cost remains constant (shut-down solution) and given on one side by:

$$E_{iki'k'}^{lim} = \frac{((P_{ik} - P_l^{idle}) \cdot \alpha_{ik}^{idle} + (P_{i'k'} - P_l^{idle}) \cdot \beta_{i'k'}^{idle})}{2} + P_l^{idle} \cdot \theta_{iki'k'}^{min} \quad (5)$$

and on the other side by:

$$E_{iki'k'}^{lim} = P_{ik} \cdot \alpha_{ik}^{shut} / 2 + \int_{t_{i'k'}^{ss}}^{t_{i'k'}} P_{i'k'}^{ss}(t) \cdot dt \quad (6)$$

Comparing expressions (5) and (6) it appears that  $\theta_{iki'k'}^{min}$  is inversely proportional to  $P_l^{idle}$ .

Figure II represents the evolution of the minimum energy cost  $E_{iki'k'}^{min}$  with respect to the unproductive period of machine  $l$  between operations  $O_{ik}$  and  $O_{i'k'}$  in the case of machining.

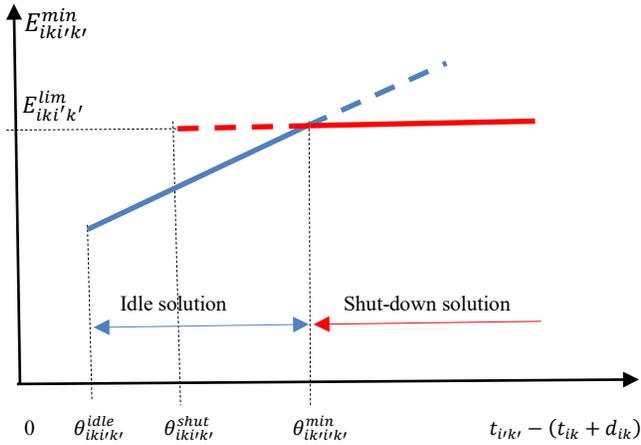


FIGURE II

IDLE AND SHUT DOWN MINIMUM ENERGIES FOR MACHINING DURING UNPRODUCTIVE PERIODS

For the intended successions of operations  $O_{ik}$  and  $O_{i'k'}$  on machine  $l$  be feasible, the unproductive period must be superior or equal to  $\theta_{iki'k'}^{idle}$  between  $\theta_{iki'k'}^{idle}$  and  $\theta_{iki'k'}^{min}$ , the idle solution is the minimum energy solution. However in this interval the energy cost increases at a rate equal to  $P_l^{idle}$ . Beyond  $\theta_{iki'k'}^{min}$ , the shut-down solution is the minimum energy solution during the unproductive period with a constant performance equal to  $E_{iki'k'}^{lim}$ .

Figure 3 considers an unproductive period for this class of machines and considers a hybrid solution

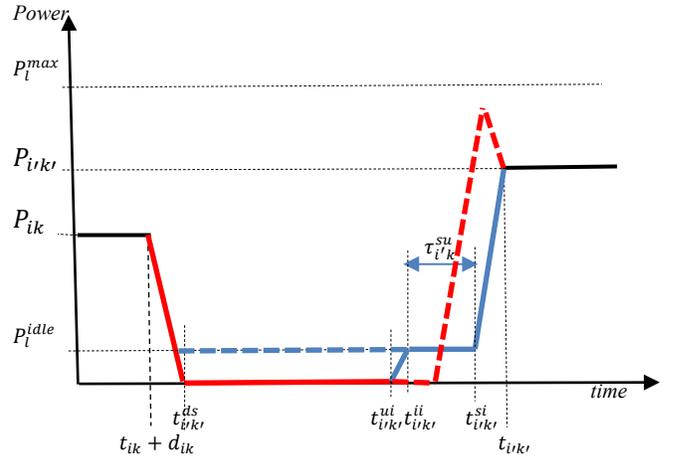


FIGURE III

HYBRID POWER SETTING FOR MACHINING DURING UNPRODUCTIVE PERIODS

From Figure I it appears that an idle regime will represent initially an additional cost with respect to the shut-down solution. This leads us to consider the case in which an initial shut-down period is followed by an idle mode before start-up, at least equal to the set-up time. From Figure III, this hybrid solution, when feasible will be always better than the idle solution. The feasibility condition is given by:

$$t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{hybr} = \alpha_{ik}^{shut} + \tau_{i'k'}^{su} + \beta_{i'k'}^{idle} + \delta \beta_l^{idle} \quad (7)$$

where  $\beta_l^{idle} = t_{i'k'}^{ii} - t_{i'k'}^{ui}$  is an operations parameter of the machine which does not depend on the performed operations. It is also clear that the maximum energy saving will be obtained in this hybrid mode when the transition to the idle regime is postponed as much as possible. The minimum energy is, when condition (7) is satisfied, a constant given by:

$$E_{i'k'}^{hybr} = \frac{(P_{ik} \cdot \alpha_{ik}^{shut} + (P_{i'k'} - P_l^{idle}) \cdot \beta_{i'k'}^{idle})}{2} + P_l^{idle} \cdot (\tau_{i'k'}^{su} + \frac{\delta \beta_l^{idle}}{2}) \quad (8)$$

While  $t_{i'k'} - (t_{ik} + d_{ik}) \leq \theta_{iki'k'}^{min}$ , this hybrid solution will be better than the idle solution, which, according to the previous results (see Figure 2), is better than the shut-down solution.

For  $t_{i'k'} - (t_{ik} + d_{ik}) \geq \theta_{iki'k'}^{min}$  we have to compare the best performance of hybrid solution  $E_{i'k'}^{hybr}$  with the performance of

the shut-down solution  $E_{ikl'k'}^{lim}$ . From Figure 3, it is clear that if  $\tau_{i'k'}$  is sufficiently small, the hybrid solution will be always better than the shut-down solution and the minimum delay of a new operation  $O_{i'k'}$  will be always equal to  $\tau_{i'k'}^{su} + \beta_{i'k'}^{idle}$ . Figure IV updates Figure II by introducing the hybrid mode for unproductive periods. The minimum value of energy  $E_{ikl'k'}^{idle}$  is given by:

$$E_{ikl'k'}^{idle} = (P_{ik} + P_l^{idle}) \cdot \alpha_{ik}^{idle} / 2 + \tau_{ikl'k'}^{su} \cdot P_l^{idle} + (P_{i'k'} + P_l^{idle}) \cdot \beta_{i'k'}^{idle} / 2 \quad (9)$$

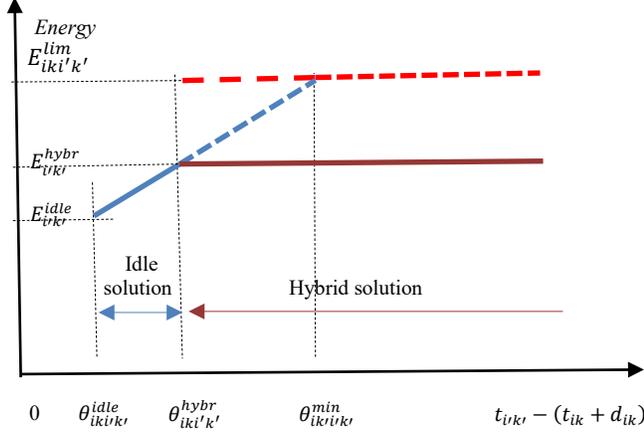


FIGURE IV

MINIMUM SPENT ENERGY FOR MACHINING DURING UNPRODUCTIVE PERIODS WITH IDLE, SHUT DOWN AND HYBRID MODES

### ENERGY FOR UNPRODUCTIVE PERIODS OF THERMAL MACHINES

These machines can perform operations such as drying, melting or molding, once the material to be processed has got the necessary temperature. Let  $T_{ik}$  and  $T_{i'k'}$  be the necessary temperature to perform operations  $O_{ik}$  and  $O_{i'k'}$  on thermal machine  $l$ . and let  $T_{su}$  be the maximum temperature allowed to perform set-up. Let  $T_{amb}$  be the ambient temperature in the job-shop and  $P_l^{max}$  be the maximum thermal power of machine  $l$ . Here we concentrate on a single industrial machine  $l$  which consumes different quantities of heat to perform thermal operations over a specific material.

#### I. Additional assumptions

To cope with thermal machines, the following additional assumptions are introduced: To process material  $i$  in thermal operation  $k$  at machine  $l$ , the material must remain at this temperature  $T_{ik}$  during a period  $d_{ik}$ . In this study, the material is supposed to be brought and maintained to temperature  $T_{ik}^{ops}$  by machine  $l$  through radiation. The case of warming by conduction could be treated in a similar way. At the very start of operations both the machine and the material are at ambient temperature  $T_{amb}$ . Four stages can be distinguished: warming of material  $i$  from an initial temperature superior or equal to  $T_{amb}$ , to temperature  $T_{ik}^{ops}$ ; maintenance of temperature

$T_{ik}^{ops}$  during period  $d_{ik}$ ; release of full heating leading the machine to idle state or shut-down; material extraction: the material can be extracted from the machine when the temperature is below  $T_{su}$ .

#### II. Modelling of thermal process

Here only the case of heating by radiation is considered. The following dynamic equations can be proposed at a macroscopic level according to the Stefan-Boltzmann law of radiation (here only the resistance is supposed to produce relevant radiation and not to be subject to relevant local conduction or convection):

$$C_{r_l} \dot{T}_{r_l} = -\sigma_{r_l} T_{r_l}^4 + P_{r_l} \quad (10)$$

$$C_{m_l} \dot{T}_{m_l} = \varepsilon_l \sigma_{r_l} T_{r_l}^4 - \frac{1}{R_{la}} (T_{m_l} - T_{amb}) - \frac{1}{R_{il}} (T_{m_l} - T_{il}) \quad (11)$$

$$C_i \dot{T}_{ik} = (1 - \varepsilon_l) \sigma_{r_l} T_{r_l}^4 + \frac{1}{R_{il}} (T_{m_l} - T_{ik}) \quad (12)$$

Here  $T_{r_l}$  is the temperature of the radiating resistance of machine  $l$ ,  $T_{m_l}$  is the temperature of the part of machine  $l$  that exchanges heat with material  $i$ ,  $P_{r_l}$  is the power supplied to the resistance,  $T_{ik}$  is the temperature of the material,  $C_{r_l}$ ,  $C_{m_l}$  and  $C_i$  are respectively the thermal capacitances of the resistance, the machine and the material,  $R_{la}$  and  $R_{il}$  are respectively the thermal resistances between the machine and the exterior and between the machine and the material,  $\sigma_{r_l}$  is a constant proportional to the Boltzmann constant,  $\varepsilon_l$  is the proportion of radiation that is received by the machine. It can be admitted that the radiating resistance submitted to a constant power  $P_{r_l}$  reach very quickly (few seconds) its equilibrium temperature given by:

$$T_{r_l} = \sqrt[4]{P_{r_l} / \sigma_{r_l}} \quad (13)$$

Then the temperature dynamics reduce to a second order linear differential system given by (11) and (12):

$$C_{m_l} \dot{T}_{m_l} = \varepsilon_l P_{r_l} - \frac{1}{R_{la}} (T_{m_l} - T_{amb}) - \frac{1}{R_{il}} (T_{m_l} - T_{ik}) \quad (14)$$

$$C_i \dot{T}_{ik} = (1 - \varepsilon_l) P_{r_l} + \frac{1}{R_{il}} (T_{m_l} - T_{ik}) \quad (15)$$

#### III. Minimum energy policy for the thermal process

The minimum energy policy during an unproductive period is given in this case by the solution of the following optimal control problem:

$$\min_{P_{r_l}(\tau)} \int_{t_{ik}}^{t_{i'k'}} P_{r_l}(\tau) \cdot d\tau \quad (16)$$

with state constraints (14) and (15), the initial conditions:

$T_{m_l}(t_{ik})$  and  $T_{il}(t_{ik})$ , and the final condition at  $t_{i'k'}$  given by:

$$T_{ik}(t_{i'k'}) = T_{i'k'}^{ops} \quad (17)$$

$$\text{and } T_{ml}(t_{i'k'}) = \frac{\varepsilon_l}{1+(1-\varepsilon_l)R_{il}/R_{ia}} T_{i'k'}^{ops} \quad (18)$$

with the power constraint:

$$0 \leq P_{r_l}(t) \leq P_{r_l}^{max} \quad t \in [t_{ik}, t_{i'k'}] \quad (19)$$

where  $P_{r_l}^{max}$  is the maximum value of the heating power for machine  $l$ . The value of  $T_{ml}(t_{i'k'})$  in relation (18) is the equilibrium temperature of machine  $l$  at the start of operation  $O_{i'k'}$ .

The Hamiltonian  $H(t)$  associated to this optimization problem [9] can be written in the affine form with respect to  $P_{r_l}(t)$ :

$$H(t) = a(\lambda_1(t), \lambda_2(t)) + b(\lambda_1(t), \lambda_2(t)) \cdot P_{r_l}(t) \quad (20)$$

where  $\lambda_1$  and  $\lambda_2$  are the adjoint variables associated respectively to equations (14) and (15). It can be shown that if  $b(t)$  is a decreasing monotonous decreasing function, then the minimization of the Hamiltonian leads to an optimal solution such that there exists  $t_{i'k'}^{S*}$  with  $t_{ik} + d_{ik} < t_{i'k'}^S < t_{i'k'}$  and  $P_{r_l}^*$  is given by:

$$P_{r_l}^*(t) = 0 \quad t \in [t_{ik} + d_{ik}, t_{i'k'}^{S*}] \quad (21)$$

$$P_{r_l}^*(t) = P_{r_l}^{max} \quad t \in [t_{i'k'}^{S*}, t_{i'k'}] \quad (22)$$

Then, Figure V considers an unproductive period for this class of machines and represents in blue the idle solution and in red the shut-down solution (solid lines for power and dotted lines for temperature).

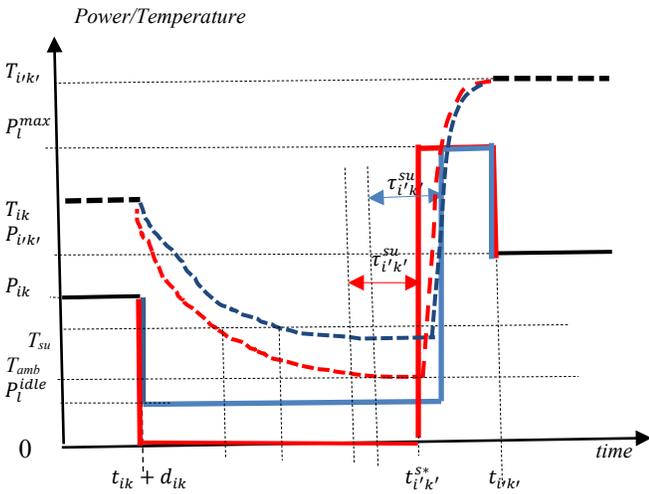


FIGURE V

POWER SETTINGS FOR THERMAL MACHINES FOR UNPRODUCTIVE PERIODS

Here  $\beta_{ik}^* = t_{i'k'} - t_{i'k'}^{S*}$  is solution of the scalar equation corresponding to the final condition:

$$[0 \ 1] \cdot e^{A(\beta_{ik}^*)} \cdot [\underline{\Delta T}_{ik}(0) - A^{-1} \cdot (I_2 - e^{A(\beta_{ik}^*)})] \cdot B \cdot P_{r_l}^{max} = T_{il}^{ops} \quad (23)$$

where  $A$  and  $B$  are respectively the state and the control matrices of the dynamic system given by (14) and (15),  $I_2$  is the identity matrix of order 2,  $e^{A(\beta_{ik}^*)}$  is the exponential matrix of  $A(\beta_{ik}^*)$ . The initial temperature conditions are such as:

$$\underline{\Delta T}_{ik}(0) = \begin{bmatrix} T_{ml}(t_{i'k'}^{S*}) - T_{amb} \\ T_{ik}(t_{i'k'}^{S*}) - T_{amb} \end{bmatrix} \quad (24)$$

The minimum necessary energy to drive the temperature of the material  $i$  in machine  $l$  from  $T_{amb}$  to  $T_{il}^{ops}$  is given by:

$$E_{ik}^{min} = P_{r_l}^{max} \cdot \beta_{ik}^* \quad (25)$$

According to this last relation,  $\beta_{ik}^*$  is also the minimum time necessary to bring the temperature of the material to  $T_{il}^{ops}$ . The conclusions which can be extracted from Figure V are like those of the machining case. When the unproductive period increases, there is a threshold beyond which it is better to shut down the machine. In Figure VI the corresponding best modes and performance are presented where thermal inertia of the machine and product is considered while in Figure 8 a hybrid solution is considered.

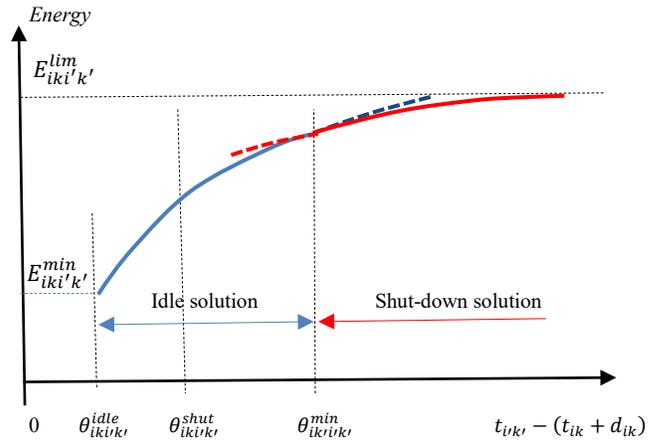


FIGURE VI

BEST PERFORMANCE FOR THERMAL MACHINES WITH IDLE OR SHUT DOWN MODES

In the case of the hybrid solution, the duration of heating at full power will be reduced during startup, but thermal losses will happen also during the idle period, making difficult to draw a general conclusion in this case.

#### ENERGY FOR UNPRODUCTIVE PERIODS IN JOB-SHOPS

The results obtained from the analysis of the case of mechanical machines lead us to consider that for each situation of unproductive period for a given machine  $l$ , an optimal policy exists and depend mainly on the unproductive period length  $t_{i'k'} - (t_{ik} + d_{ik})$  and of the following

parameters: the processing powers of the two successive operations:  $P_{ik}$  and  $P_{i'k'}$ . the idle power of the machine:  $P_l^{idle}$ . the time response parameters:  $\alpha_{ik}^{idle}$ ,  $\alpha_{ik}^{shut}$ ,  $\beta_{i'k'}^{idle}$ ,  $\beta_{i'k'}^{shut}$ ,  $\delta\beta_l^{idle}$  and the set-up time:  $\tau_{i'k'}^{su}$  for operation  $O_{i'k'}$  on machine  $l$ .

Then, the minimum energy to be spent during an unproductive period of machine  $l$  between operations  $O_{ik}$  and  $O_{i'k'}$  can be written  $\varphi_{iki'k'}(t_{ik}, t_{i'k'})$ . According to Figures II, IV and VI, this minimum energy admits an inferior and a superior bound. This can be written as:

$$E_{iki'k'}^{min} \leq \varphi_{iki'k'}(t_{ik}, t_{i'k'}) \leq E_{iki'k'}^{max} \quad (23)$$

Then, the total energy  $E_{Tot}^{up}$  spent during the unproductive periods associated to a production plan, the following bounds:

$$\sum_i \sum_k E_{iki'k'}^{min} \leq E_{Tot}^{up} \leq \sum_i \sum_k E_{iki'k'}^{max} \quad (24)$$

The total energy  $E_{Tot}^{pr}$  spent during the productive periods associated to a production plan, is given by:

$$E_{Tot}^{pr} = \sum_{i=1}^n \sum_{k=1}^{S_i} \int_{t_{ik}}^{t_{ik}+d_{ik}} P_{m_{ik}}(\tau) \cdot d\tau \quad (25)$$

The total energy associated to a production plan is the sum of the energy spent during the productive and unproductive periods over all the machines of the job-shop:

$$E_{tot} = E_{tot}^{up} + E_{tot}^{pr} \quad (26)$$

The expected global efficiency of the job-shop given by the ratio  $\rho_E = E_{tot}^{pr}/E_{tot}$  with respect to the considered production plan will be such as:

$$1 - \sum_i \sum_k E_{iki'k'}^{max}/E_{tot} \leq \rho_E \leq 1 - \sum_i \sum_k E_{iki'k'}^{min}/E_{tot} \quad (27)$$

Here, maximizing this efficiency ratio will be equivalent to minimizing the total energy spent in the job-shop to realize the production plan. This will be performed by solving a scheduling problem where  $\varphi_{iki'k'}(t_{ik}, t_{i'k'})$  will be the cost assigned to the transition from operation  $O_{ik}$  to operation  $O_{i'k'}$  at instant  $t_{ik}$  and  $t_{i'k'}$ .

## CONCLUSION

It has been considered that in the case in which the machines of a job-shop are operated in a standard way, the amount of energy necessary to perform the different processings resulting from a production plan, can be predicted and taken as a fixed energy amount attached to the production plan. The remaining component of the energy attached to this production plan is relative to the unproductive periods of the different machines. These unproductive periods are, despite their name, essential since they allow for a given machine the successive processing of the tasks of the different production process which have to use it. In this paper, it has been shown that the reduction of duration of these unproductive periods leads to a decrease in the energy spent on a machine to ensure transition from one operation to the next. So, the objective of

minimizing energy and the classical concern of limiting as much as possible the production make-span when considering the job-shop scheduling problem, are not antagonistic in this case, contrary of what happens, for example when an increase of speed of processing leads to an increase of energy consumption. This study shows that the schedule of the successive operations assigned to a machine has a direct influence on its energy performance and on the performance of the whole job-shop to achieve a production plan. The study has assumed static production plans, but the proposed approach can be adapted to more realistic situations where job-shop schedules can frequently change, since the analysis has been performed in a decentralized way and a limited look-ahead period will be sufficient to allow its implementation. This study provides a new basis to approach the scheduling of operations in job-shops.

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