Modelling the mobile investment strategies under competition using mathematical programming

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***Abstract*— In this paper, we explore the Mobile Investment Strategy (MIS) problem within the French telecommunications market, which includes four Mobile Network Operators (MNOs): Bouygues Telecom, SFR, Free, and Orange France (OFR). We adopt the perspective of OFR, the incumbent operator, focusing on its critical network investment decisions related to the deployment of the latest mobile network technologies. Additionally, we examine user responses to the investment choices made by each MNO, a dynamic significantly shaped by competitive forces. To model and address the MIS problem, we propose a Mixed-Integer Non-Linear Program (MINLP), which we linearize and enhance by incorporating historical data from various datasets to empirically evaluate its performance.**

I. INTRODUCTION

A. Context

Global mobile data traffic is experiencing significant growth, with a forecasted compound annual growth rate of 23% from 2023 to 2028 [4]. Two primary factors contribute to this increase. First, the number of wireless devices—such as smartphones, tablets, personal computers, etc.—is expected to reach 25 billion worldwide by 2028, representing a 66% increase compared to 2022 [5]. Second, the average data usage per device is rising due to the popularity of data-intensive services like video streaming. The average usage per smartphone is projected to exceed 20 gigabytes (GB) in 2023 and reach 47 GB by the end of 2028 [4].

To accommodate this growing traffic demand, Mobile Network Operators (MNOs) must invest in the densification, expansion, and upgrading of their networks. From 2015 to 2022, French MNOs invested approximately €15.9 billion in equipment to support 4G and 5G networks [1].

In addition to network investments, MNOs are also service providers that create offers for users, granting access to new technologies like 5G while enhancing customer experience at competitive prices. In this context, understanding the mechanisms driving subscriber migration from legacy offers to new ones, as well as the interdependencies between network and user dynamics, is crucial for network planners to optimize their decisions and enhance MNO revenues.

B. Guidelines and investment dynamics

MNOs manage several network assets including existing telecommunication sites on which they can install new generation of mobile technologies to provide better network connectivity and bandwidth. They can also construct new telecommunication sites for densification (to help offload traffic from nearby sites with strained capacity) or coverage extension (that is installing sites where people do not have access to mobiles services so far) purpose. Mobile technologies are deployed on specific spectrum frequencies which give each site a certain capacity in terms of number of users and traffic that it can carry out. Now, the mobile network evolution is also subject to logistical, regulatory, and budgetary limitations. In particular, investment budgets (in terms of capital expenditures and operational costs) allocated by MNOs to network infrastructure upgrades are limited and relatively consistent from one year to another, which restricts the amount of network changes possible at each time period.

These investments enable MNOs to offer connectivity services through diverse commercial plans tailored to specific user categories (e.g., young subscribers, gamers) and to promote particular technologies. MNOs accurately forecast subscriber growth in different geographical areas, allowing them to assign the latest compatible mobile technology to enhance user experience and ensure profitability.

In addition, these investments are guided by regulatory authorities that assess and publish performance based on various criteria. In France, for example, ARCEP (Autorité de Régulation des Communications Electroniques, des Postes et de la distribution de la Presse) evaluates mobile network coverage and user quality of experience. Coverage reflects service availability across regions, while quality of experience measures service speeds, such as video downloads or streaming. These performance criteria push MNOs in formulating strategies that comply with regulatory requirements.

Finally, most mobile markets are dominated by a few vertically integrated MNOs. In Europe, for instance, each country typically has two to five domestic network operators. In this competitive landscape, operators strive to develop efficient investment strategies regarding scale and timing. Deploying a new technology at the optimal moment in a specific area can significantly enhance a MNO's market share. Consequently, determining the right timing for technology deployment, while considering competitive dynamics, user behavior, and regulatory factors, is crucial for MNOs and raises several interesting algorithmic challenges. Note that such multi-years strategies are typically devised annually.

C. State of the art

Network planning is a key aspect of optimizing telecommunication networks and is extensively covered in the literature. In particular, multi-period network planning optimization has gained significant attention over the years, with various mathematical programming variants being explored.

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In [10], the authors examine the design of minimum cost mobile investment strategies that consider load balancing across multiple technologies. They propose an exact solution approach based on integer linear programming. The authors of [7] focus on a multiyear investment planning problem, introducing a non-linear programming formulation that incorporates both network and subscriber dynamics. This formulation is linearized and enhanced with valid inequalities to address small to medium-sized instances (up to a few hundred sites). Additionally, they propose a heuristic to generate high-quality feasible solutions for larger instances.

A variant of multi-period network planning is explored in [14], where modular capacities can be added or removed throughout the planning horizon to adapt to demand fluctuations. The work in [11] tackles the multi-period capacity expansion problem specifically for tree network topologies. Lastly, [8] incorporate demand uncertainty into their analyses of multi-period network planning optimization for fixed optical and mobile networks, respectively.

The works cited above tackle different variants of the multi-period network planning from a single operator's perspective that seeks to optimize a given criterion (that generally consists in minimizing costs or maximizing revenues). However they do not take account the highly competitive reality of the telecommunications market. Actually, for a target operator, the planning and investment decisions of its competitors may have a huge impact on its revenues and positioning in the market. We should mention however that competition among telecommunication operators has been studied through the lens of game theory and for different applications related to infrastructure and service deployment or sharing, for fixed and wireless [13] (including mobile [9], [16]) networks. In particular, the authors in [16] formulate a finite discrete-timing game for the roll-out of a new mobile technology over multiple sites and time periods.

D. Contribution

In this paper, we investigate the Mobile Investment Strategy (MIS) problem. While the French telecommunications market comprises mainly four Mobile Network Operators (MNOs) —Bouygues Telecom, SFR, Free, and Orange France (OFR)—, we adopt the perspective of OFR, the incumbent operator. Our focus is on the key network investment decisions of OFR, specifically regarding the deployment of the latest mobile network technology. We also consider user reactions to the investment decisions made by each MNO, a dynamic significantly influenced by competitive forces. To address this problem, we propose a Mixed-Integer Non-Linear Programming (MINLP) model and we enhance it by leveraging historical data from multiple datasets, allowing to empirically test its performance.

The remainder of this manuscript is organized as follows: Section II formally introduces the MIS problem and presents a MINLP formulation to model it. In Section III, we outline a procedure for deriving instances from real data and share preliminary experimental results using these instances. Finally, Section IV offers concluding remarks and perspectives.

II. Mobile Investment Strategies problem under competition

A. Notations and problem definition

We consider a competitive telecommunication market composed of a set $\mathcal{I}$ of MNOs that will also be referred to as competitors. From a network perspective, each competitor operates a set of network **sites** denoted $\mathcal{S}_i$, $i \in \mathcal{I}$, with a set of legacy **mobile technologies** $\mathcal{G}$ (*e.g* $\{2G, 3G, 4G\}$) and intends to deploy the next-generation mobile technology, denoted by *NG* (*e.g* 5G or 6G). From a *business* perspective, the network operators compete over a base of potential customers distributed in a set of geographical **areas** denoted by $u_a \in \mathcal{A}$. Each potential customer either wants to subscribe to an **offer** within the set proposed by one of the operators for a legacy mobile technology denoted by $\mathcal{O}_i$ (*e.g* $\{o2G, o3G, o4G\}$ corresponding to 2G, 3G, 4G-offers) or eventually the one based on the NG-technology, denoted by NO_i (*e.g* a 5G or a 6G-offer). We note $\mathcal{O} = \cup(\mathcal{O}_i \cup \{NO_i\})_{i \in \mathcal{I}}$. The evolution over time of customers repartition across different geographic areas and operators as well as available offers per operator is the main indicator that we observe for the analysis of users dynamics. We therefore denote by $u^t_{a,i,o}$ the number of customers in the area $a \in \mathcal{A}$ subscribing to an offer $o \in \mathcal{O}_i \cup \{NO_i\}$ proposed by the network operator $i \in \mathcal{I}$ at time-interval $t \in \mathcal{T}$, where $\mathcal{T}$ denotes the discrete time horizon considered for the planning (*e.g* 6 months or 3 years). By convention, the initial subscriber distribution states (at the beginning of the time horizon) are denoted by $u^0_{a,i,o}$. We assume that the mobile market has reached a certain level of maturity when it has the following properties: (i) the set of potential customers is constant over time for each geographic area and (ii) customers are only interested in upgrading their offer to benefit from the NG-technology (through the offers $\{NO_i, i \in \mathcal{I}\}$). As explained in what follows, we consider the competitors' launch for new offerings supported by their deployment strategies for the *NG*-technology as the main driver for these evolutions. In particular, we focus on the **design of the NG-deployment** plan of a specific network operator, say $\tau \in \mathcal{I}$, modeled by binary decision variables $z^t_s, s \in \mathcal{S}_\tau, t \in \mathcal{T}$, equal to 1 if the NG-technology is installed on site s at time-interval t (0 otherwise). In this work, we assume deployment plans of its competitor's $i \in \overline{\mathcal{I}} = \mathcal{I} \setminus \{\tau\}$ to be known and modeled by parameters $Z^t_{s,i} \in \{0;1\}, s \in \mathcal{S}_i, t \in \mathcal{T}$. Based on radio propagation properties and sites' environmental features (*e.g* urban our rural), we are able to determine the set of areas $\mathcal{A}_s$ which are under the NG-coverage of site $s \in \cup_{i \in} \mathcal{S}_i$ when the NG-technology is installed; symmetrically, the set of sites of each operator $(\mathcal{S}_{a,i})_{i \in \mathcal{I}}$ which covers the area $a \in \mathcal{A}$ if the NG-technology is installed. Combining this information with network operators deployment plans, we derive the **coverage status** of the different areas by operators, represented by binary variables $r^t_a \in \{0;1\}, a \in \mathcal{A}, t \in \mathcal{T}$ for operator τ and by parameters $R^t_{a,i}, a \in \mathcal{A}, t \in \mathcal{T}, i \in \overline{\mathcal{I}}$ for its competitors: formally we have r^t_a (resp $R^t_{a,i}, i \in \overline{\mathcal{I}}$) equal to 1 if the NG-technology is installed at time-interval

$t \in \mathcal{T}$ on at least one site covering area $s \in \mathcal{S}_{a,\tau}$ (resp. $s \in \mathcal{S}_{a,i}, i \in \overline{\mathcal{I}}$), and 0 otherwise. As for the migration and/or churn mechanism, we assume that the u_a potential customers of any area $a \in \mathcal{A}$ are aware of its coverage status by all operators, denoted by the $|T|$-dimensional binary vector $C_a^t =< r_a^t, (R_{a,i}^t)_{i \in \overline{\mathcal{I}}} >$ at each time-interval t, and that they decide of their operator and offer for the next time-interval $t+1$ according to the the migration and churn mechanism: $f^a_{C,o,o'} : \{0;1\}^{|\mathcal{I}|} \times \mathcal{O} \times \mathcal{O} \longrightarrow [0,1]$, denoting the percentage of $u^t_{a,i,o}$ customers of each legacy offers $o \in \mathcal{O}_i \cup \{NO_i\}$ of network operator $i \in \mathcal{I}$ that switch to the offer $NO_{i'}$ of network operator $i' \in \mathcal{I}$ if $C_a^t = C$, condition that will be characterized by boolean variable $\delta^t_{C,a}$. We illustrate such a mechanism with a bipartite graph given on Figure 1. In our example, we consider two operators, $\mathcal{I} = \{OFR, Free\}$, within a geographical area referred to as "Paris." We present the migration and churn function applicable between any two consecutive time periods under the following conditions: the first operator, OFR, has deployed the NG-technology, while the second operator, Free, has not (i.e., $C =< 1, 0 >$). It is important to note that for any offer, the sum of outgoing percentages equals 100%. Additionally, the migration and churn effects are more pronounced towards the NO_{Orange} offer than towards the NO_{Free} offer, which is logical given that we are illustrating a scenario where OFR is the only operator that has already deployed the NG-technology.

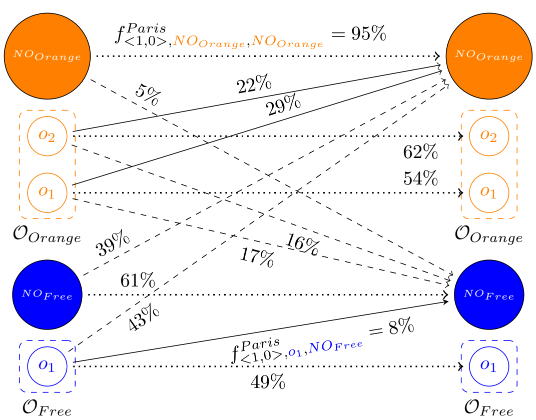

Fig. 1. Illustration of migration and churn dynamic between two periods for NG-deployment status equal to $< 1, 0 >$. We consider a French example with two operators Orange and Free and their offers for "Paris" area.

In terms of capacity planning, we focus exclusively on operator τ, assuming the competitors networks properly dimensioned (as we do not have access to their dimensioning rules or budget). We begin by considering that the sites equipped with NG-technology have a bandwidth capacity denoted by $CAPA_{NG}$. This is based on the common assumption that all modules for each sector are installed at a site simultaneously with the NG-antenna, allowing for the mutualization of field interventions and a reduction in operational costs. Moreover we consider that any NO_τ user has an average traffic demand D_{NG} that can be dispatched to any site of the operator τ, provided this site covers the area of the user and that the NG-technology is installed: we therefore introduce notations $u^t_{a,\tau,NO_\tau,s}$ standing for the number of NO_τ users of area $a \in \mathcal{A}$ of operator τ that are communicating though site $s \in \mathcal{S}_{a,\tau}$ at time interval $t \in \mathcal{T}$. In this context, our goal is to maximize the market share of our operator τ with respect to its offer NO_τ at the end of the time horizon considered. Beside this objective, the NG-technology deployment plan of $\tau \in \mathcal{I}$ will be driven by budget constraints, characterized by a maximal number of sites that can be deployed at each time interval $t \in \mathcal{T}$, denoted by Z^t; moreover, annual ranking of MNOs by regulators (such as ARCEP in France) pushes to consider strategic guidelines relying on minimum percentages of the population to be under the coverage of the NG-technology to be fulfilled over the time horizon, $\underline{QA}^t, t \in \mathcal{T}$.

As a synthesis, we aim at designing the NG-technology deployment plan that maximizes the market share of τ at the end of the time horizon, while respecting several operational constraints and budget guidelines, assuming that competitors' deployment plans are known. This problem is referred to as P^{MIS} in the following. From a game theoretical viewpoint, this optimization problem refers to the problem of finding the best response of the competitor of interest τ to its competitors' profile of strategies.

B. MILP model for the problem

The P^{MIS} problem can be formulated as the following Mixed Integer Non-Linear Program.

The objective (1) is to maximize the market share (namely the number of subscribers) of the NO-offer for the operator of interest τ. Constraints (2-3) define the coverage status of the areas using the NG-technology by the operator τ according its deployment plan z_s^t while equations (4) enable a proper identification of the coverage vector at any time-interval, with respect to deployment strategies of the competitors. Equalities (5) model the temporal dynamics of the users within the offers of the whole set of competitors, relying on the migration and churn mechanism function f. Inequalities (6-7) ensure a proper dimensioning of the NG-network of the operator of interest τ: first we virtually associate the traffic induced by NO_τ users of each area to site covering it with the NG-technology of operator τ; second we ensure that the cumulative traffic generated by NO_τ-users steered to a given site of operator τ can be handled by this site's capacity. Constraints (8) control the number of sites where the NG-technology can be installed by the operator τ for each time-interval, while constraints (9) ensure that the threshold of percentage of the population to be under NG-coverage by operator τ are met. Finally, equations (10-14) are the trivial and integrity constraints. Note that the equality (5) contains a quadratic term which can be linearized using classical method (likewise in [12]).

III. Case study for 5G deployment in France

In this section, we outline the methodology used to derive test instances from real-life data extracted from ANFR [3] (Agence Nationale des FRéquences) and INSEE [6] public databases. We construct various instance sizes in order to

$$\max_{z,u,r} \sum_{a\in\mathcal{A}} u^{t_{|\mathcal{T}|}}_{a,\tau,NO_\tau} \tag{1}$$

$st.$

$$r^t_a \le \sum_{s\in\mathcal{S}_{a,\tau}} z^t_s \qquad a\in\mathcal{A}, t\in\mathcal{T}, \tag{2}$$

$$z^t_s \le r^t_a \qquad a\in\mathcal{A}, s\in\mathcal{S}_{a,\tau}, t\in\mathcal{T}, \tag{3}$$

$$\delta^t_{a,C} = \left(c_\tau r^t_a + (1-c_\tau)(1-r^t_a)\right)\prod_{i\in\overline{\mathcal{I}}}\left(c_i R^t_{a,i} + (1-c_i)(1-R^t_{a,i})\right) \qquad \forall C=(c_i)_{i\in\mathcal{I}}\in\{0;1\}^{|\mathcal{I}|}, a\in\mathcal{A}, t\in\mathcal{T}, \tag{4}$$

$$u^t_{a,i,o} = \sum_{C\in\{0;1\}^{|I|}} \delta^t_{a,C} \sum_{i'\in\mathcal{I}} \sum_{o'\in\mathcal{O}_{i'}\cup\{NO_{i'}\}} f^a_{C,o',o}\cdot u^{t-1}_{a,i',o'} \qquad \forall a\in\mathcal{A}, i\in\mathcal{I}, o\in\mathcal{O}_i\cup\{NO_i\}, t\in\mathcal{T}, \tag{5}$$

$$u^t_{a,\tau,NO_\tau} = \sum_{s\in\mathcal{S}_{a,\tau}} u^t_{a,\tau,NO_\tau,s} \qquad \forall a\in\mathcal{A}, t\in\mathcal{T}, \tag{6}$$

$$\sum_{a\in\mathcal{A}_s} D_{NG} u^t_{a,\tau,NO_\tau,s} \le CAPA_{NG} z^t_s \qquad \forall a\in\mathcal{A}, t\in\mathcal{T}, \tag{7}$$

$$\sum_{s\in\mathcal{S}_\tau} z^t_s - z^{t-1}_s \le \overline{Z}^t \qquad \forall t\in\mathcal{T}, \tag{8}$$

$$\sum_{a\in\mathcal{A}} u_a r^t_a \ge \underline{QA}^t \cdot \sum_{a\in\mathcal{A}} u_a \qquad \forall t\in\mathcal{T}, \tag{9}$$

$$r^t_a \in \{0;1\} \qquad \forall a\in\mathcal{A}, t\in\mathcal{T}, \tag{10}$$

$$z^t_s \in \{0;1\} \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{11}$$

$$\delta^t_{a,C} \in \{0;1\} \qquad \forall C\in\{0;1\}^{|\mathcal{I}|}, a\in\mathcal{A}, t\in\mathcal{T}, \tag{12}$$

$$u^t_{a,i,o} \in \mathbb{N} \qquad \forall a\in\mathcal{A}, i\in\mathcal{I}, o\in\mathcal{O}_i, t\in\mathcal{T}, \tag{13}$$

$$u^t_{a,i,o,s} \in \mathbb{N} \qquad \forall a\in\mathcal{A}, i\in\mathcal{I}, o\in\mathcal{O}_i, s\in\mathcal{S}_i, t\in\mathcal{T}. \tag{14}$$

assess the scalability of our approach and evaluate the obtained solutions from a business perspective.

A. Framework

■ **Sites**: To analyze the *existing sites* and their deployment dynamics for a specific operator within a given geographical area, we start by filtering out ANFR data using regional tables. This database is fed by all the actors managing radioelectric installations in France (among which, MNOs) [3]. Several spatial and temporal information related to these installations, namely the list of telecommunication *sites*, *stations* installed over a site, *antennas* per station, and *transmitters* placed on top of each antenna and frequency *bands* associated with each antenna, are available on this database. For example, Figure 2 depicts the evolution of 4G site deployment by the end of 2022 for Bouygues Telecom and SFR. A comparative analysis of these data with Orange's deployment history data have shown a difference on a magnitude of 5% for 4G transmitters thus allowing to consider ANFR datasource as reliable. Due to the huge number of existing sites in France (over 56,000 sites shared among four major MNOs) and the proximity of many sites, observing the investment strategies at a site level can harden the analysis. Therefore, we propose a clustering approach allowing to group sites close to each other. The process involves $i)$ constructing a set of existing sites in a geographical zone, $ii)$ determining the number of clusters using the elbow method [15] and $iii)$ assigning each site to a cluster using the k-means clustering algorithm to minimize intra-cluster variance. We assume that a technology is considered deployed in a cluster when 20% of transmitters are present. Besides, from *Transmitter Installation Date* attribute in the *Transmitter* table, we can get the deployment date of each technology on each site. Then, we define the deployment sequence of a legacy technology $g \in \mathcal{G}$ by a competitor $i \in \mathcal{I}$ as a mathematical construction that associates to each existing site $s \in \mathcal{S}_i$, a time period t at which g has been deployed on s by operator i. The process of constructing the deployment sequence of a historical technology, in a geographical zone, from the ANFR data, includes two stages $i)$ first, we generate the set of existing sites $\mathcal{S}_i$ of competitor i within the geographical zone, $ii)$ we recover the deployment time of technology g in each site $s \in \mathcal{S}_i$. Finally, to generate *potential sites*, a proportion of the existing sites is randomly transformed into potential sites for the target operator to roll-out the new mobile technology.

■ **Areas**: To create the set of geographical areas $\mathcal{A}$, we rely on the Contours...IRIS® 2022 edition database from INSEE. This database maps IRIS (Ilots Regroupés pour l'Information Statistique) contours [2], which are uniform geographical divisions in France used for statistical data dissemination. The geographical areas can be defined as sets of IRIS, Communes, Departments, or Regions, and IRIS zones can also be clustered using polygonal spatial clustering methods. Using data from ANFR and INSEE, two methods are proposed to calculate the coverage parameters allowing to determine which sites $\mathcal{S}_a$ actually cover an area $a \in \mathcal{A}$ and, consequently, the fraction of subscribers present in this area. The first one is called *Coverage Radius Approach* and allows to define a coverage radius for each site and technology by determining the intersection area between the coverage

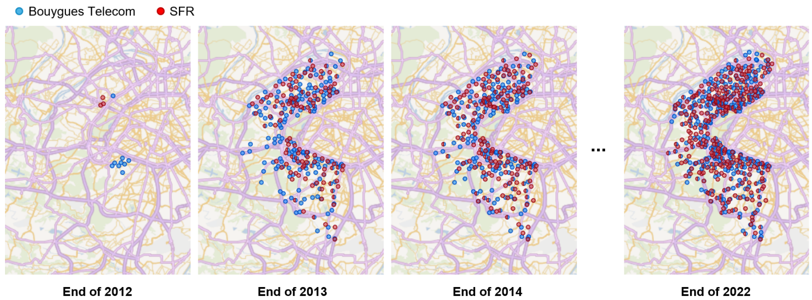

Fig. 2. Deployment of 4G technology transmitters by Bouygues Telecom and SFR in Hauts-de-Seine (Paris area)

radius (set to 2 km) and the geographical area, divided by the area's total surface. The second one is the so-called *Voronoi Diagram Approach* and uses Voronoi diagrams generated from clustered sites and corresponding Voronoi cells. The coverage is computed using the intersection of a Voronoi cell with the area, divided by the area's total surface.

■ **Users**: Exploiting another INSEE database related to French population census [6], we can determine the initial number of users in each IRIS and therefore also in each area. In particular, the number $u^0_{a,i,o}$ of initial subscribers to an offer o_i of operator $i \in \mathcal{I}$ in area $a \in \mathcal{A}$ is computed by multiplying the number of users in a by the market share of the offer of each operator.

■ **Deployment strategies**: We define the deployment strategy of a technology $g \in \mathcal{G}$ by a competitor $i \in \mathcal{I}$ within a geographical area as a mathematical framework that associates each area $a \in \mathcal{A}$ with a time period $t \in \mathcal{T}$ during which operator i has deployed technology g in that area. While a deployment strategy focuses on the areas covered, a deployment sequence pertains to the sites installed. To construct a deployment strategy for a technology by competitor i, we first generate the site deployment sequence, then compute the area coverage for each time period. An area is considered covered if the sites where the technology is deployed reach at least a specified threshold, say 70%, of the users in that area.

■ **Model parameters** We assume that two mobile technologies, namely $\mathcal{G} = \{3G, 4G\}$, are already available in the French mobile market and a next mobile generation, say $NG = 5G$, is currently under deployment by the target operator τ = OFR and its competitors. The market includes 3 additional operators, that is to say $\mathcal{I} \backslash \{\tau\} = \{$Bouygues Telecom, SFR, Free$\}$. We suppose that the three mobile technologies are proposed via three commercial offers $\mathcal{O} = \{o3G, o4G, o5G\}$ that we suppose comparable across operators. Our framework is based on a number of other hypotheses that are supposed to capture the expected network and users dynamics through a set of parameters that are detailed in what follows. First, we suppose that the unitary demands at each time period and for the NG technology, ranges as follows $D_{NG} \in [0, 006, 0, 010]$. We assume an operator may decide to deploy this technology on a given site and at each time period by providing it with a capacity denoted $CAPA_{NG} \in [800, 1000]$ unit of capacity. We assume that parameters $\underline{QA^t}$ for $t \in \mathcal{T}$ express strategic guidelines and take their values in $[0.1, 0.7]$ and increase over time. Users on an area can upgrade their old offers to the newest one $NO = o5G$. When considering all four MNOs on the market, the percentage of users within an area willing to shift is modeled by function f, and depends on the coverage state of the area. We consider that users subscribed to an old offer are equally divided between the operators covering the area in which they belong. Finally, the time horizon is taken as 5 years with annual time periods ($t \in \{1, 2, 3, 4, 5\}$).

■ **Instance size**: First, we study the whole Metropolitan France. Both the areas $\mathcal{A}$ and the existing sites $\mathcal{S}$ are set as the 96 departments composing France. Each area is covered by one and only one site, and conversely, each site covers one and only one area. Then, we study the deployment on two french departments, namely: Hauts-de-Seine and Mayenne. For each department, we will construct instances with all the existing sites and instances with 50 clustered sites. To assess the impact of densification and expansion, we generate instances with a proportion of 10% and 50% of potential sites. To compute area coverage, we generate instances with the Voronoi approach and with the radius approach. For the values of the coverage radius of technologies, we consider instances with a radius of 1km and 3km for the Hauts-de-Seine and a radius of 10km and 30km for Mayenne. By varying the approaches to compute area coverage, we will generate area coverage graphs with different densities.

B. *Preliminary results*

Computations are made on a processor AMD Ryzen 5 PRO 5675U with Radeon Graphics clocked at 2.30 GHz. The code is written in Python 3.9.13, with the use of the optimization package Pyomo 6.1.2. The solver used is CPLEX 22.1.1.0 with default branch-and-bound algorithm. We set the time limit for solving to 1800 seconds. We assume that competitors' deployment strategies for the latest technology, 5G, will mirror those of historical technologies, 3G and 4G. Utilizing ANFR data, we formulate the deployment strategies for both legacy technologies for each competitor. These strategies are then consolidated into two overarching scenarios: one in which all competitors deploy according to the 3G model, and another where they emulate the 4G model.

We denote by P^{MIS}_{3G} and P^{MIS}_{4G} the models dedicated to the study of each scenario. The objective function is to

TABLE I
PRELIMINARY RESULTS FOR P^{MIS}.

Instance				P_{3G}^{MIS}		P_{4G}^{MIS}	
Id	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{S}_\tau\rvert$	Density	Sol_{3G}	f-gap	Sol_{4G}	f-gap
1	96	96	0.010	68 793 513	0	53 391 044	0
2	616	871	0.004	1 706 044	0	1 170 026	0
3	616	871	0.023	1 125 427	0	709 071	0
4	616	50	0.022	3 820 783	0	3 680 440	0
5	278	194	0.008	382 348	0	234 640	0
6	278	194	0.065	272 691	0	154 191	0
7	278	194	0.401	151 458	0	115 966	0
8	278	50	0.022	987 501	0	717 521	0

maximize the number of subscribers to the newest offer at the end of time horizon. We test our models on multiple instances and outline the solutions and final gaps in Table I. We notice from Table I that the optimal solutions, corresponding to the number of subscribers to the newest offer at the end of the time horizon, obtained by P_{3G}^{MIS} are on average 28% (i.e. (Sol_{4G} - Sol_{3G})/Sol_{3G}) higher than the optimal solutions obtained by P_{4G}^{MIS}. In other words, if competitors deploy the new technology, 5G, as they deployed the 3G it would be 28% more beneficial for OFR than if they emulate 4G deployments. From a business perspective, MNOs often learn from their previous deployment experiences to optimize their future ones. Historically, 4G technology had an overall better deployment than 3G because of the experience MNOs have gained. When it comes to density, we see that the optimal number of subscribers to the newest offer obtained on more dense instances are consistently lower than those attained in less dense instances. This outcome can be explained by one of the hypothesis taken to construct P^{MIS}. Specifically, we assumed that if a competitor covers an area, it can serve its users through the sites covering it without accounting for capacity constraints. In dense instances, deploying a site means covering more geographical areas, leading to an implicit assumption that competitors can serve more users than their actual capacities allow, which contributes to the observed difference in optimal solutions. This dynamic highlights the importance of better considering capacity limitations in future works.

IV. Concluding remarks and perspective

In this work, we studied a variant of the Mobile Investment Strategies (MIS) problem under competition, focusing on the MNO OFR perspective within the French market. We began by providing background information and identifying key levers influencing mobile investments. Then, we proposed a MINLP formulation, denoted as P^{MIS}, which was later linearized to model it. To evaluate the practical applicability of our model, we developed a methodology for identifying and extracting real-world datasets to construct realistic instances. Our analysis considered *static* and *deterministic* competition scenarios, allowing P^{MIS} to be interpreted as a best response to a known competitor deployment scheme. Additionally, we prioritized maximizing the number of users rather than minimizing costs to emphasize market positioning. Preliminary results demonstrated significant improvements in our objective function when varying competitor strategies, underscoring the importance of incorporating competition into the modeling process. Future directions include expanding the testbed with sensitivity analyses. Moreover, current assumptions consider competitors' deployment strategies as fixed and known in advance; extending the framework to include competitors' responses would enable the development of a *dynamic* competition model.

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