FRQI Pairs method for image classification using Quantum Recurrent Neural Network

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***Abstract*— This study presents the Flexible Representation for Quantum Images (FRQI) Pairs method, a novel approach that leverages Quantum Recurrent Neural Networks (QRNN) for image classification. The proposed method achieves an accuracy of 74.60% on the full Modified National Institute of Standards and Technology (MNIST) handwritten digit data set, demonstrating its effectiveness in handling quantum encoded data for classification tasks.**

By reducing the size of the QRNN by the exponential factor, the FRQI Pairs method highlights the potential of integrating quantum computing principles with neural network architectures, offering a promising direction for advancing quantum machine learning.

The research evaluates the FRQI Pairs method against existing quantum and classical models, demonstrating its competitive performance against other state-of-the-art approaches and showing potential for future advancements in the field. This research opens avenues for further exploration of quantum preprocessing and hybrid model architectures, marking a step forward in the application of quantum machine learning.

I. Introduction

Quantum image processing is a promising field in quantum computing. Starting with Venegas-Andraca and Bose's [1] qubit lattice representation for quantum image encoding, the description of quantum images came right after this [2]. Quantum states describe patterns for two main reasons: to improve classification efficiency [3] and to provide valuable models for traditional issues [4]. The Flexible Representation for Quantum Images (FRQI) was first proposed in [5] and was further developed in [6]. This research focuses on quantum image encoding and quantum machine learning classification methods applied to the MNIST (Modified National Institute of Standards and Technology) dataset.

The field of quantum machine learning is expanding rapidly, and new methods emerge, sometimes inspired by the classical machine learning methods or some developed purely for quantum computers. Some of the recent methods are inspired by traditional machine learning techniques, e.g., quantum state vector machine (QSVM) [7], [8], [9], [10], quantum k-nearest neighbors (QKNN) [11], [12], [13], and quantum nearest mean classifier (QNMC) [14], [15], [16]. In the family of deep methods: variational quantum circuits (VQC) [17], [18], [19] inspired by classical neural networks, quantum tensor networks (QTN) [20], [21], [22], quantum convolutional neural networks (QCNN) [23], [24], [25], [26], [27], random quantum neural networks (RQNN) [28] and quantum recurrent neural networks (QRNN) [29], [30].

Among the last group, there appears to be no agreement on how to phrase the architecture name, namely the words *quantum* and *recurrent* appear in both combinations, resulting in the recurrent models being referred to as QRNN or RQNN. The authors of this paper will use the abbreviation QRNN for recurrent networks and RQNN for random networks.

The paper is organized as follows. Section II describes briefly the MNIST dataset and the theoretical background on existing methods utilized, namely FRQI and QRNN. Section III explains the experimental setup and the newly proposed FRQI Pairs method together with the results of the experiments. Section IV compares the results of Section III with the state of the art. Section V summarizes the most important advantages and disadvantages of the proposed method together with the suggestions for the future improvements.

II. Subject

The prototyping results of this paper are based on the thesis [30]. The work proposes a novel approach to the classification of quantum-encoded images using QRNN [29] with the input data encoded in a quantum way, using the FRQI [5]. The use of quantum encoding allows for a futuristic assumption that the classification is performed on some universal quantum computer where the encoded data is stored in a quantum memory.

However, such devices are not available in the Noisy Intermediate Scale Quantum (NISQ) [31] era, and the disposal of the data preprocessing steps required every time an image is loaded into the quantum system would significantly reduce the classical processing overhead.

A. MNIST Database

The Modified National Institute of Standards and Technology (MNIST) database contains handwritten digits stored in the form of 28×28 pixel images with $0...255$ values representing the pixel intensity. A scaled-down sample image is shown in Fig. 1. The dataset has been chosen due to its wide use for benchmark purposes, hence allowing for broad comparison with classical and quantum methods. The recent criticism of MNIST benchmarks for quantum machine learning algorithms [32] suggests that further research is needed to test the approach on different datasets and from other perspectives.

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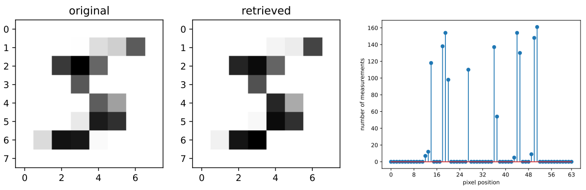

Fig. 1. Comparison of the scaled image from MNIST database (left) and its retrieved version from FRQI encoding (center). Aggregated results of $10_0 00$ measurements of the FRQI representation of the image (right).

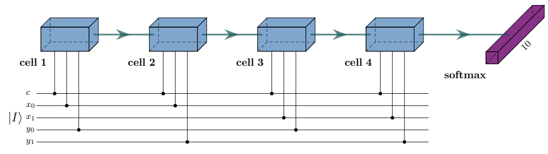

Fig. 2. Schematic diagram of the FRQI Pairs Model [30] for 4×4 image. The approach uses all combinations of $\{c, x, y\}$ qubits as inputs to QRNN cells, where x, y correspond to the X and Y qubits for position encoding.

B. Flexible Representation of Quantum Images

The FRQI method proposed in [5] allows efficient storage of single channel images using only $\mathcal{O}\left(\lceil \log_2 n \rceil\right)$ qubits, where n is roughly the side length of the image. The method also allows easy manipulation of the image properties, thus placing it and its descendants as a versatile image storing and manipulation method for a quantum computer. The method has been improved to support multichannel images in further work [33], [34], [35]. Since the MNIST dataset used for classification includes only grayscale images, the FRQI method for encoding has been used for simplicity.

The method encodes the position of the image pixel using $\lceil \log_2 W \rceil$ and $\lceil \log_2 H \rceil$ qubits for the width and height of the image, respectively, plus an additional single qubit for color value encoding $\nu = \lceil \log_2 W \rceil + \lceil \log_2 H \rceil + 1$. Assuming that the image is bounded by a square envelope of dimension 2^n, the resulting number of qubits is

$$\nu = 2n + 1. \tag{1}$$

We can describe the resulting FRQI image state $|I\rangle$ as

$$|I(\boldsymbol{\theta})\rangle = \frac{1}{2^n} \sum_{x=0}^{2^{2n}-1} \left(\cos\theta_x |0\rangle + \sin\theta_x |1\rangle\right) \otimes |x\rangle\,, \tag{2}$$

where $\theta_x \in \left[0, \frac{\pi}{2}\right]$ and $x \in \left\{0, 1, \ldots, 2^{2n} - 1\right\}$. The original MNIST pixel values are integer values from the range $\{0, 1, \ldots 255\}$, thus they have to be uniformly scaled into $[0, \frac{\pi}{2}]$. An example of a scaled MNIST image, with its FRQI measurements and retrieved version, is shown in Fig. 1.

C. Quantum Recurrent Neural Networks

Bausch [29] presents the first QRNN model capable of performing complex tasks such as sequence learning and digit classification. QRNN applies an enhanced version of a quantum neuron presented in [36] with amplitude amplification to create a nonlinear activation function. The model was tested in various tasks, including memorization, sequence prediction, and classification of the MNIST digits data set, demonstrating its ability to handle high-dimensional training data.

The original implementation of QRNN shows results that have an impact on quantum machine learning, especially in the management of long-sequence data without the gradient vanishing problem typical of classical RNNs.

III. Methods

A. Examined Models

The thesis [30] introduces a quantum-specific approach to combine images encoded in a quantum way with QRNN [29]. The QRNN architecture has been selected due to its proven applicability in the image classification task. The research aims at verification of the performance with more sophisticated encoding methods. The novelty of the presented approach is the use of input images encoded by the FRQI method as described in Section II-B. The data set used for prototyping is the MNIST handwritten digits, scaled down from the original 28×28 to 8×8 for most cases.

The original work presents three models:

1) The single-cell model takes as input all the outputs of the FRQI encoding. Information passes through the cell only once.
2) The naive model, which is an enhanced version of the single-cell model with multiple repetitions of the cell. The author uses two-cell models in his thesis.
3) The FRQI Pairs model implements the architecture of a full QRNN, where each cell takes as an input combination of parameters responsible for coding color and position.

The author tested all three models with more than 40 different sets of parameters, such as the number of hidden layers in the cell, the size of the input image (in some cases, 28×28 images are used), and the optimizer learning rate.

FRQI encoding and QRNN have been implemented using Tensorflow Quantum [37] and Cirq [38] Python libraries, while model training has been performed using Keras [39]. All the missing details related to model training can be found in [30].

B. FRQI Pairs Model

The feature of this approach is the fact that each cell takes as its input two pieces of information (pairs):

- the qubits responsible for channel color intensity (e.g., for FRQI – single qubit for grayscale value) and
- the qubits responsible for encoding the pixel position (e.g., for 2D images – two qubits for coding the X-Y position).

The total number of cells depends on the number of combinations between x and y qubits, since each cell takes as input one of those combinations. If the image has a dimension of 2^n as defined in (1), the resulting number of cells can be

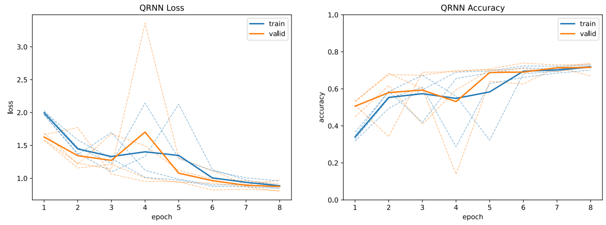

Fig. 3. Aggregated results of the four champion FRQI Pairs models training from [30]: loss function - categorical cross-entropy (left), accuracy (right).

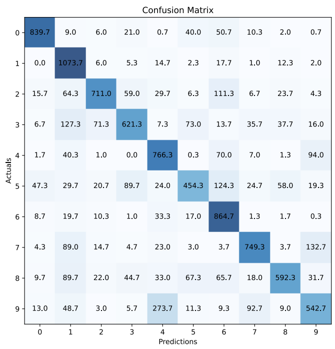

Fig. 4. Average confusion matrix of the four champion FRQI Pairs models.

calculated as

$$N = n^2. \tag{3}$$

For comparison, the number of cells in a direct implementation of a QRNN, where each cell takes one pixel of the same image, as in [29], requires an exponentially larger number of cells, compared to (3), that is $(2^n)^2 = 2^{2n}$. An example diagram of the FRQI Pairs model for a 4×4 image is shown on Fig. 2.

During the prototyping and tuning phase, increasing the number of QRNN working memory qubits had a positive impact on the test results, with the highest examined number of 4 qubits. Hence, the final QRNN parameters were set to 4 memory qubits and a single deep layer for each cell. The best model used 11 qubits in total (4 QRNN memory + 7 FRQI) and six cells, and its number of trainable parameters was 716 (636 in PQC, 80 for softmax).

From the training graphs presented in Fig. 3 it is clear that training and validation loss decrease steadily, so the model did not overfit. The model achieved a test accuracy of 74.6% with a confusion matrix of the test set presented in Fig. 4. It is visible that the model can generalize well and for each class most of the predictions lie on the diagonal of the confusion matrix. There appears to be an outstanding group where the models confused the actual digit nine with four. This could be improved by data preprocessing methods and also potentially by using higher-resolution images, so a model has more information to use for predictions.

IV. Comparative Analysis

In this section, the authors present the comparison of the results with papers that meet the following criteria:

TABLE I

Summary of the works used for comparison in Section IV.

Ref.	Dataset Modification	Image Encoding	Classifier	Test Accuracy
[19]	2000+1000 images[1], padded to 32×32, PCA	Amplitude, angle, hybrid encodings	VQDNN	Binary: 99.00%, 10-class: 80.00%
[27]	1000 images, scaled to 8×8	AQSP	QCNN	Binary: 96.65%
[28]	4-class dataset (24k+4k[1]), noised, classical deep embedding into 128 values	Amplitude encoding	RQNN	4-class: 97.20%[2]
[29]	Full dataset (60k+10k[1]), scaled to 10×10, 10-class only: PCA, t-SNE	Grayscale value X-gates binary representation	QRNN	Binary: 99.00%[2], 10-class: 95.00%[2]
[30]	Full dataset (60k+10k[1]), scaled to 8×8	FRQI	QRNN	10-class: 74.60%
[40]	800+200 images[1], scaled to 7×7	Continuous-variable encoding	QRNN	Binary: 85.00%
[41]	Binary: 5000+2100[1], 10-class: full dataset[3], padded up to 32×32	Amplitude encoding	QCNN	Binary: 96.30%, 10-class: 74.30%

1) the use of the MNIST dataset,
2) the use of accuracy metrics.

With the conditions fulfilled, it is possible to compare gathered publications with [30]. The summary of this section is presented in Tab. I.

The final results of [30] show that the proposed model is able to grasp the principles of the underlying data distributions. Its test accuracy is 74.6%. However, not ideal, its accuracy at the level of magnitude allowing reasonable comparison to three of the presented works, namely [40], [19] and [41].

The work presented in [40] implements QRNN, which was tested against a binary classification of the MNIST digits 3 and 6. The results confirm the QRNN utility for the classification of handwritten digits. However, they suggest that the classical LSTM model with a similar number of parameters performs better, what might suggest that the QRNN model is inefficient in image classification.

The paper [41] introduces QCNN that is a different family of models. The model was trained to classify all ten digits, and except for the 0-padding to 32×32 size, no initial data transformation was performed. The numbers of trainable parameters of the model from [30] and [41] are at a similar level: 716 and 379, respectively. For both models, the final accuracy is also similar: 74.6% and 74.3%, while the data used to train the final model have higher resolution in the case of [41]. Both results suggest the QRNN model might have a higher learning potential than what was achieved in [30], and may achieve even better results for full-sized images. From (1) and (3) we find that the 28×28 FRQI encoding needs 11 qubits, so the circuit will use 15 or more qubits, depending on the number of memory lanes, while the number of cells will increase to 25. The increase in the number of cells and qubits will result in a higher number of the model trainable parameters. However, it will also increase the model training capacity and potentially improve the results.

The authors of [19] presented a VQDNN model, which resembles a densely connected network. They managed to

[1]Training+test set sizes
[2]Maximum presented test accuracy for given task
[3]Sampled by 100 in each training epoch

drastically reduce the number of qubits required to operate the network, providing a solution that uses only ten qubits for 10-class problems. For the features extracted using the PCA, the authors experimented with amplitude and angle encoding, as well as their combination. The total number of parameters for the deepest model was 430. The depth of the model increases the model training capacity, and the best model achieved approximately 80% accuracy. The proposed model of the VQDNN network proved its potential for experimentation to replace the internals of the QRNN cell from [30].

The use of PCA as the feature extraction method should also be considered in the further development of FRQI-QRNN models, as its impact was shown in [29] and [19]. However, the method might not be directly applicable to the problem [30] is trying to solve, i.e., the quantum ML model operating on already quantum-encoded data. Despite the fact that the PCA method can help reduce the dimensionality of feature spaces, thus reducing the model complexity and allowing for higher performance, with the same number of trainable parameters–one can imagine that the data stored in future quantum memory would be encoded and compressed using different means. The applicability of the quantum version of PCA [42], [43] or other methods such as [44] to the feature preprocessing phase should also be explored.

Other works that use the amplitude encoding method for the classification of MNIST datasets are [28], [27]. They both present classifier types different from QRNN, but the authors managed to achieve high test accuracy on limited datasets. Random QNN [28] shows high robustness against noisy data. However, the image embedding is performed by a classical densely connected layer, which, similarly to PCA, offloads some part of the solution to the classical part.

The Approximation Quantum State Preparation (AQSP) method for image encoding uses a simulated quantum circuit to train the image representation and has a time complexity of only $\mathcal{O}(n)$ [27]. The authors also combine the AQSP method with the proposed QCNN framework. The design of a hybrid recurrent network with convolutional and pooling input layers might help make the FRQI-QRNN architecture less dependent on the input image size and reduce the complexity of the recurrent part.

V. Conclusions

The presented state-of-the-art methods trained to solve the MNIST classification task prove that the area of Quantum Machine Learning has the potential to solve real-life machine learning problems.

The new FRQI Pairs architecture, presented in [30], requires an exponentially lower number of recurrent cells compared to [29] (n^2 vs. 2^{2n} (3)) which may lead to shorter execution times thus a lower chance of decoherence, and a higher computational efficiency during the inference phase if applied to real-world problems.

An important aspect of the presented architecture is that it utilizes a well-grounded FRQI method as its input, which allows to use the method's data encoding advantage over the classical representation. This saves the processing time at the cost of slightly larger number of qubits, but without the need to leave the quantum realm to use the classical pixel values for qubit encoding. Assuming that in the future we would be able to persistently store data in quantum form, the FRQI Pairs method would be a solid starting point in the development of fully quantum neural networks.

Another important feature of the FRQI Pairs method is that it manages to train on the full MNIST dataset and has proven performance comparable to other methods such as those proposed in [43], [19]. Many other presented works struggle to capture the idea of the MNIST data base, that is a dataset for benchmarking machine learning models and ideas based on reliable and repeatable dataset. MNIST is meant to be taken as a whole, to enable direct comparison with numerous classical models.

Although areas were left to improve compared to [29], the method has the potential to be extended by various preprocessing routines [29], [44], [42], [43] and layer/model architectures [19], [41], [27]. The approach of the other works to the problem suggests that the research from [30] should also be extended to a binary image classification case for broader comparison possibilities.

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