Dynamic pricing to control stochastic retail demand: Near-Optimal Weight Functions for Large Lots of Perishable Product Considering Leftovers*

Yu Cao and Anna V. Kitaeva

Abstract—**This research investigates an inventory system employing a fixed-order quantity policy for managing a perishable product with deterministic shelf life over a finite planning horizon, assuming no replenishment during the sales period. The demand follows a non-homogeneous compound Poisson distribution, where the intensity exhibits dual dependence on both current inventory availability and an unspecified temporal weight function while controlled by retail price. Through the application of diffusion approximation of the inventory level and incorporating intensity-price dependence, we derive a practical approximation for the optimal weight function that remains computationally efficient even for substantial lot sizes. The case of leftovers' salvage has been considered. For two types of near-optimal weight functions the expected revenues are obtained, and numerical examples of their maximization with respect to an unknown constant are presented.**

I. Introduction and Problem Statement

Contemporary e-commerce platforms demonstrate particular interest in dynamic pricing strategies, as technological advancements have virtually eliminated menu costs. This approach has gained significant traction in retail grocery sectors through expiration date-based pricing implementations; see [1].

Recent studies [2-3] have examined the ecological and societal consequences of managing perishable goods, highlighting how dynamic pricing affects both revenue generation and product waste reduction. Empirical evidence [4-5] further demonstrates that implementing dynamic pricing strategies in retail settings can substantially enhance profitability. Dynamic pricing practices are particularly useful when demand is both price sensitive and stochastic. The literature contains comprehensive reviews [6-11] documenting thorough investigations into production and inventory systems handling perishable goods.

Here, we present a generalization of the retail price control models in stochastic environment proposed and studied in [12–14]. These three papers deal with the products that need to be sold before a certain point in time. It is assumed that demand exhibits high price elasticity, demonstrating marked responsiveness to pricing variations.

In [12], a stochastic dynamic price control model is proposed, which allows us, triggering purchases, to sell all the perishable product at hand during the period almost surely.

In this paper, we derive the equation for the optimal weight function, which maximizes the expected revenue, and consider its approximate solution as a near-optimal weight function. This near-optimal weight function almost surely implies leftovers depending on the range of the unknown constant it contains. Firstly, the core optimization objective focuses on expected revenue maximization through the constant considering nonzero salvage value. Secondly, we modify the optimal weight function approximation so that the price at the beginning of the sales period becomes close to the base price and obtain the expected revenues in case of possible leftovers. For both functions, the optimization task can be solved only numerically. The corresponding numerical examples are given. The sales process's stochastic properties and associated expected revenues are analytically derived via diffusion approximation of the inventory process.

Let us introduce the model's assumptions and notations. We consider a supply chain network comprising one vendor and multiple buyers, the vendor is a monopolist and seeks to maximize the revenue, ordering fixed lot Q_0 per unit price d and selling it within a fixed time period T.

If leftovers happen, products are sold on the secondary market at salvage value per unit s, $s < d$, or are disposed, in this case, $s < 0$.

The demand follows a compound Poisson process with price-dependent intensity $\lambda(c)$, where $c = c(t)$ denotes the unit selling price. Individual order quantities are modeled as i.i.d. continuous random variables with the first and second moments a_1 and a_2 respectively. The system operates under a no-replenishment policy throughout the planning horizon.

As evidenced by the results presented in [15], we assume that the stock level process $Q(\cdot)$ is characterized by a stochastic differential equation as follows:

*Yu Cao is with the National Research Tomsk State University, Tomsk, CO 634050 RUS (corresponding author phone: +7 (960) 977 7259; e-mail: ch.cy@stud.tsu.ru).

Anna V. Kitaeva is with the National Research Tomsk State University, Tomsk, CO 634050 RUS (e-mail: kit1157@yandex.ru).

$$dQ(t) = -a_1\lambda(c(t))dt + \sqrt{a_2\lambda(c(t))}dw(t),$$

where $w(\cdot)$ is the Wiener process.

Let us consider the following model of the intensity of the customers' flow control through the dynamic price:

$$a_1\lambda(c(t)) = \frac{Q(t)}{T\varphi(t/T)}, \quad (1)$$

where $\varphi(\cdot)$ is an unknown weight function, $t \in [0,T]$. We will call control model (1) as a general one.

II. Near-optimal Weight Function for a Large Lot Size

A. Expectation and variance of the stock level process

Thus, the stock level follows the stochastic differential equation:

$$dQ(t) = -\frac{Q(t)}{T\varphi(t/T)}dt + \sqrt{\frac{a_2}{a_1}\frac{Q(t)}{T\varphi(t/T)}}dw(t). \quad (2)$$

The expectation and variance

$$E\{Q(t)\} = \bar{Q}(t) = Q_0 \exp\{-\psi(t/T)\}, \quad (3)$$

$$Var\{Q(t)\} = \frac{a_2 Q_0}{a_1}\exp\{-\psi(t/T)\}\left(1-\exp\{-\psi(t/T)\}\right), \quad (4)$$

where $\psi(z) = \int_0^z \varphi^{-1}(x)dx.$

B. The expected revenue and its maximization

We adapt linear the intensity-of-price dependence

$$\lambda(c) = \lambda_0 - \lambda_1\frac{c(t)-c_0}{c_0}, \quad (5)$$

where c_0 is a stationary (basic) price corresponding stationary intensity λ_0, and parameter $\lambda_1 > 0$ quantifies the responsiveness of the demand intensity $\lambda(\cdot)$ to deviations of the relative price from its stationary level.

From (1) and (5) we get $c(t) = c_0\left(1+\frac{\lambda_0}{\lambda_1} - \frac{Q(t)}{a_1\lambda_1 T\varphi(t/T)}\right)$.

The expected revenue over the cycle

$$\bar{S} = \int_0^T E\{a_1c(t)\lambda(t)\}dt = \frac{c_0Q_0}{\lambda_1}\Bigg[(\lambda_0+\lambda_1)\int_0^1 e^{-\psi(z)}\psi'(z)dz -$$

$$-\frac{1}{a_1T}\left(Q_0-\frac{a_2}{a_1}\right)\int_0^1 e^{-2\psi(z)}\psi'^2(z)dz - \frac{a_2}{a_1^2T}\int_0^1 e^{-\psi(z)}\psi'^2(z)dz\Bigg].$$

Taking into account that $\int_0^1 e^{-\psi(z)}\psi'(z)dz = 1-e^{-\psi(1)}$, we get

$$\bar{S} = \frac{c_0Q_0}{\lambda_1}\Big[(\lambda_0+\lambda_1)\left(1-e^{-\psi(1)}\right) -$$

$$-\frac{1}{a_1T}\left(Q_0-\frac{a_2}{a_1}\right)\int_0^1 e^{-2\psi(z)}\psi'^2(z)dz - \frac{a_2}{a_1^2T}\int_0^1 e^{-\psi(z)}\psi'^2(z)dz\Bigg].$$

Let us formulate the optimization problem

$$\int_0^1\left[(a_1Q_0/a_2-1)e^{-2\psi(z)} + e^{-\psi(z)}\right]\psi'^2(z)dz \quad \Rightarrow \min_{\psi(\cdot)}$$

subject to $\psi(0)=0$.

Following Euler-Lagrange equation, optimal function $\psi(\cdot)$ satisfies

$$Ae^{-\psi}(\psi''-\psi'^2) + \left(\psi''-\frac{1}{2}\psi'^2\right) = 0, \quad (6)$$

where coefficient $A = a_1Q_0/a_2 - 1$.

Coefficient $A \gg 1$, because Q_0 is usually large, so neglecting the last term in (6), we get $\psi''-\psi'^2=0$. It follows that near-optimal weight function $\varphi(z) = C - z$, where C is a constant.

If $C > 1$, then the leftovers are possible. The expected stock level at the end of the period $\bar{Q}(T) = Q_0(1-1/C)$.

Fig. 1 shows the plots of the optimal weight functions for different values of coefficients A and $C > 1$. Black lines represent the exact solutions, whereas red lines indicate the approximate solutions. For all C values and $A = 100$ the exact and approximate solutions are close to each other. The difference between the solutions increases by the end of the cycle.

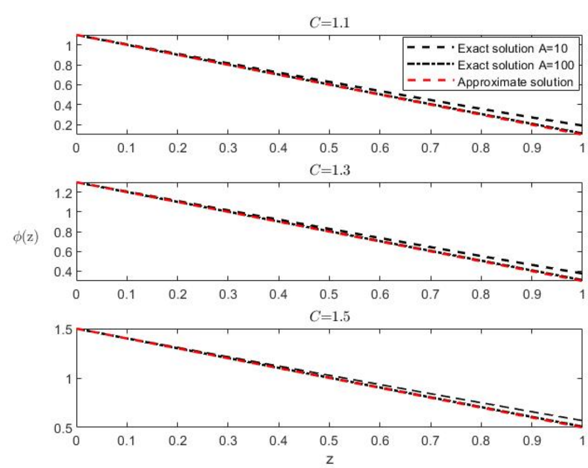

Figure 1. Optimal weight function $\varphi(\cdot)$ dependence on $t/T = z$ for the exact and approximate solutions.

Consider the case of leftovers, that is, $C > 1$. Let the salvage value per unit $s = \eta d$, where coefficient $-1 \le \eta \le 1$ reflects the product's deterioration throughout the period, like as in [16]. The expected salvage value $S_s = \eta d Q_0 \left(\frac{C-1}{C}\right)$. Then the expected revenue of general model for $C > 1$

$$\overline{S}_{C>1\_general} = \frac{c_0 Q_0}{\lambda_1}\left[\frac{(\lambda_0+\lambda_1)}{C} - \frac{1}{a_1 C^2 T}\left(Q_0 - \frac{a_2}{a_1}\right) + \right. \tag{7}$$

$$\left. + \frac{a_2}{a_1^2 CT}\ln\left(1-\frac{1}{C}\right)\right] + \eta d Q_0\left(1-\frac{1}{C}\right).$$

III. Lot Weighted Control Model

In this section, we examine the specified form of price-dependent intensity.

$$a_1 \lambda(c(t)) = \frac{CQ(t)}{CT-t}, \tag{8}$$

where $C > 0$ is a constant, $t < CT$. We will refer to this model as a lot-weighted one.

Then the stock level process is described by the following stochastic differential equation:

$$dQ(t) = -\frac{CQ(t)}{CT-t}dt + \sqrt{\frac{a_2}{a_1}\frac{CQ(t)}{CT-t}}dw(t). \tag{9}$$

Below we consider probabilistic characteristics of the stock level process and give the expression of the expected revenue for $C > 1$.

A. Expectation and variance of the stock level process

Expectation and variance of $Q(t)$

$$E\{Q(t)\} = Q_0 g(t),\ Var\{Q(t)\} = \frac{a_2}{a_1}Q_0 g(t)(1-g(t)),$$

where $g(t) = \left(1-\frac{t}{CT}\right)^C$, $t \le CT$.

Note that $E\{Q(T)\} = Q_0(1-1/C)^C$ for $C \ge 1$.

B. Probability density function of the stock level process

Applying Itô's lemma to equation (9) yields:

$$d\exp(-pQ(t)) = p\frac{CQ(t)}{CT-t}\exp(-pQ(t))\left(1+\frac{a_2}{2a_1}p\right)dt -$$

$$-p\exp(-pQ(t))\sqrt{\frac{a_2}{a_1}\frac{CQ(t)}{CT-t}}dw(t).$$

After averaging, we have

$$\frac{(CT-t)}{C}\frac{\partial\Phi}{\partial t} + p\left(1+\frac{a_2}{2a_1}p\right)\frac{\partial\Phi}{\partial p} = 0, \tag{10}$$

where $\Phi(p,t) = E\{\exp(-pQ(t))\}$.

From (10) we get

$$\Phi(p,t) = \varphi\left(\frac{p(CT-t)^C}{p+\beta}\right) = \exp\left(-\frac{\beta p g(t)}{p+\beta-pg(t)}Q_0\right),$$

where $\varphi(\cdot)$ is an unknown function and parameter $\beta = 2a_1/a_2$.

Using inverse Laplace transform we obtain the density function

$$f(q,t) = \exp\left(\frac{-\beta Q_0 g(t)}{1-g(t)}\right)\times \tag{11}$$

$$\times\left(\delta(q) + \beta\exp\left(\frac{-\beta Q_0}{1-g(t)}\right)\frac{\sqrt{g(t)Q_0/q}}{1-g(t)}I_1\left(\frac{2\beta\sqrt{g(t)Q_0 q}}{1-g(t)}\right)\right),$$

where $I_1(\cdot)$ is the first order modified Bessel function of the first kind and $\delta(\cdot)$ represents the Dirac delta distribution. This result provides some other characteristics of the sales process.

C. Distribution of the selling period's duration

Denote τ the first occurrence time of the crossing of zero level by $Q(\cdot)$ subject to $Q(0) = Q_0$. That is, τ denotes the duration required to complete the sale of lot Q_0. The cumulative distribution function of τ is the multiplayer before $\delta(\cdot)$ function in (11).

The cumulative distribution function of τ is given by:

$$F_\tau(t) = P(\tau \le t) = \exp\left(-\beta Q_0\frac{g(t)}{1-g(t)}\right). \tag{12}$$

The expected duration of the selling period

$$E\{\tau\} = \int_0^T (1-F_\tau(t))dt = T\left(1 - C\int_{1-1/C}^{1}\exp\left(-\beta Q_0\frac{z^C}{1-z^C}\right)dz\right).$$

For $\beta Q_0 >> 1$

$$E\{\tau\} \approx T\left(1 - C\int_{1-1/C}^{1}\exp\left(-\beta Q_0 z^C\right)dz\right) =$$

$$= T\left(1 - \frac{\Gamma(1/C, \beta Q_0 g(T))}{(\beta Q_0)^{1/C}}\right),$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function.

D. The expected revenue and its optimization

Let us find the expected revenue for the lot-weighted model in case of a large lot size.

Define c_0 the stationary (or basic) price that satisfies equation $a_1\lambda(c_0) = Q_0 / T$. Represent the deviations from the stationary price by $\Delta c(t) = c(t) - c_0$.

Employing the Taylor series expansion, we derive $a_1\lambda(c(t)) = CQ(t)/(CT-t) = a_1\lambda(c_0) + a_1\lambda'(c_0)\Delta c(t) + \ldots$ and $\Delta c(t) \approx \frac{C}{a_1\lambda'(c_0)}\left(\frac{Q(t)}{CT-t} - \frac{Q_0}{CT}\right)$.

Let us find conditional expectation $E\{Q(t) \mid Q(t) > 0\}$.

$$E\{Q(t)\} = (1-\pi_0(t))E\{Q \mid Q(t) > 0\} = Q_0 g(t),$$

where $\pi_0(t) = P(\tau \le t) = \exp\left(-\beta Q_0 \frac{g(t)}{1-g(t)}\right)$.

Consequently, $E\{Q(t) \mid Q(t) > 0\} = \frac{Q_0 g(t)}{(1-\pi_0(t))}$,

$$E\left\{\frac{Q}{CT-t} - \frac{Q_0}{CT}\middle| Q(t) > 0\right\} = \frac{Q_0 g(t)}{(CT-t)(1-\pi_0(t))} - \frac{Q_0}{CT},$$

$$E\{\Delta c(t) | Q(t) > 0\} = \frac{CQ_0 g(t)}{a_1\lambda'(c_0)(CT-t)(1-\pi_0(t))} - \frac{Q_0}{a_1\lambda'(c_0)T}.$$

If $C > 1$, the expected revenue for $\beta Q_0 >> 1$

$$\begin{aligned}\bar{S}_{C>1\_lot-weighted} &= \frac{Q_0(\lambda(c_0) + c_0\lambda'(c_0))}{\lambda'(c_0)} - \\ &- \frac{Ta_1\lambda^2(c_0)}{\lambda'(c_0)}\left(1 - \frac{\Gamma(1/C, \beta Q_0 g(T))}{(\beta Q_0)^{1/C}}\right) + \\ &+ \left(\eta d - \frac{(\lambda(c_0) + c_0\lambda'(c_0))}{\lambda'(c_0)}\right) Q_0 g(T).\end{aligned} \tag{13}$$

Using linear approximation (5) and substituting $Q_0 / T = a_1\lambda_0$, we can rewrite (13) as follows

$$\begin{aligned}\bar{S}_{C>1\_lot-weighted} &= \left(\eta d + \frac{c_0(\lambda_0 - \lambda_1)}{\lambda_1}\right) Q_0 (1-1/C)^C + \\ &+ c_0 Q_0 \left(1 - \frac{\lambda_0}{\lambda_1}\frac{\Gamma\left(1/C, \beta Q_0(1-1/C)^C\right)}{(\beta Q_0)^{1/C}}\right).\end{aligned} \tag{14}$$

Tending C to one from above we get the same result as in [12].

IV. Numerical Illustration

The expected revenues (7) and (14) demonstrate concavity with respect to C. We use simulations to validate the general and lot-weighted models. The non-homogeneous Poisson process is generated via thinning method, with 1,000 replications per configuration and simulation results are reported as means across iterations. Fig.2 illustrates that the theoretical weighted expected revenues align well with the simulation results across varying parameters.

Under condition $\lambda_0 T a_1 = Q_0$, the relative revenues of the two models depend on three dimensionless system parameters: λ_0/λ_1, βQ_0, and $\eta d / c_0$. Fig. 3 depicts the numerical results for relative revenues dependence on for general (black line) and lot-weighted (red line) models, distribution of purchases is uniform,

$$\begin{aligned}\bar{S}_{C>1\_general} / c_0 Q_0 &= \frac{1}{C}\left(1 + \left(1 - \frac{1}{C}\right)\frac{\lambda_0}{\lambda_1}\right) + \frac{\eta d}{c_0}\left(1 - \frac{1}{C}\right) + \\ &+ \frac{2\lambda_0}{C\beta Q_0 \lambda_1}\left(\frac{1}{C} + \ln\left(\frac{C-1}{C}\right)\right),\end{aligned}$$

$$\begin{aligned}\bar{S}_{C>1\_lot-weighted} / c_0 Q_0 &= 1 - \frac{\lambda_0}{\lambda_1}\frac{\Gamma\left(1/C, \beta Q_0(1-1/C)^C\right)}{(\beta Q_0)^{1/C}} + \\ &+ (\eta d / c_0 + \lambda_0/\lambda_1 - 1)(1-1/C)^C.\end{aligned}$$

The results demonstrate that increasing λ_0/λ_1 and βQ_0 leads to significant revenue improvements and shifts in both revenues and coefficients C required to achieve revenues' maximum, as shown in the first three subplots. Notably, the lot-weighted model exhibits greater robustness to C deviations, particularly for negative salvage values.

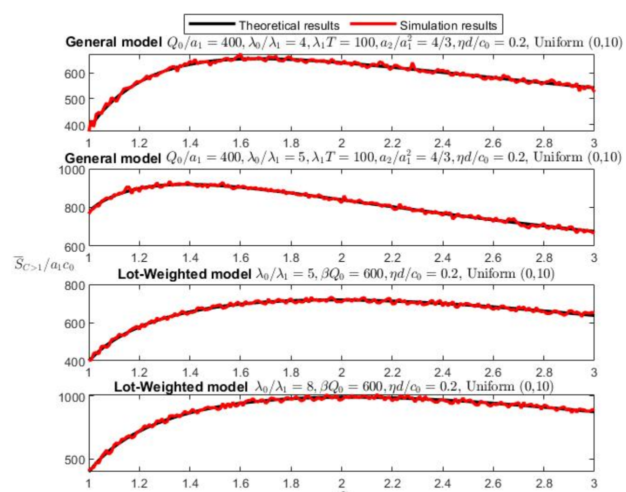

Figure 2. Theoretical models verification.

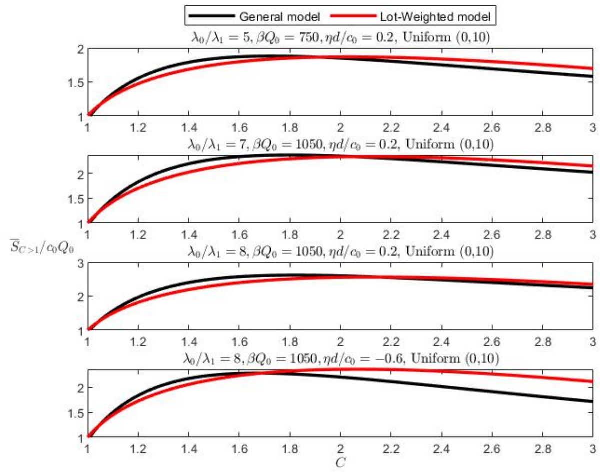

Figure 3. Relative revenues dependence on *C*.

Let us now investigate the impact of three dimensionless system parameters on both the revenues and corresponding optimal coefficients *C* for the models.

In Tab. 1 and Fig. 4, numerical results of the maximal relative revenues calculation for the general model and the lot-weighted one are presented under different sets of dimensionless system parameters.

Both models attain similar peak revenues under baseline conditions ($\beta Q_0 = 1050,\ \eta d / c_0 = -0.6$). The lot-weighted model outperforms the general one in two scenarios: (1) when the lot size is substantially increased ($\beta Q_0 > 600$), or (2) when utilization cost is high (implied by $\eta d / c_0 = -0.6$). Three-dimensional modeling demonstrates the advantages and disadvantages of the two models more effectively as shown in Fig. 5.

Fig. 6 depicts the numerical results for optimal *C* values with respect to variations of three dimensionless system parameters.

In contrast to parameters λ_0 / λ_1 and $\eta d / c_0$, parameter βQ_0 exhibits an inverse influence on the two models — increasing βQ_0 slightly reduces optimal *C* values for the general model, while increases the value for the lot-weighted model. The general model exhibits superior stability under large lot-size conditions. Specifically, maintaining the coefficient *C* slightly below 1.7 is sufficient to achieve near-optimal revenue. This contrasts with the lot-weighted model, which requires finer calibration of *C* to optimize the inventory system's performance.

TABLE I. OPTIMAL RELATIVE REVENUES

λ_0 / λ_1	5	8	8	8
βQ_0	750	750	1050	1050
$\eta d / c_0$	0.2	0.2	0.2	-0.6
$\overline{S}_{C>1\_general} / c_0 Q_0$	1.88	2.61	2.62	2.28
$\overline{S}_{C>1\_lot-weighted} / c_0 Q_0$	1.87	2.51	2.56	2.35

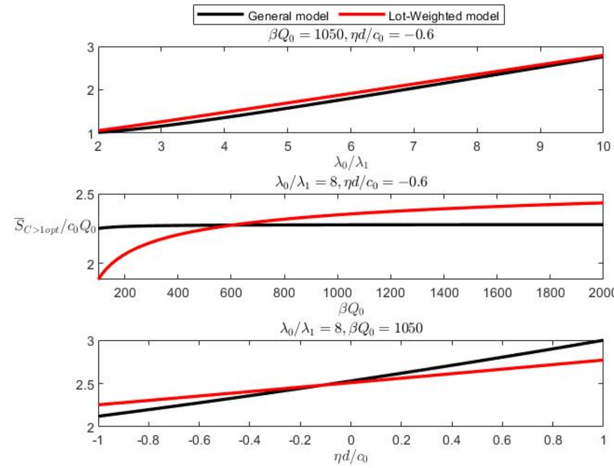

Figure 4. Optimal relative revenues dependence on $\lambda_0 / \lambda_1,\ \beta Q_0,\ \eta d / c_0$.

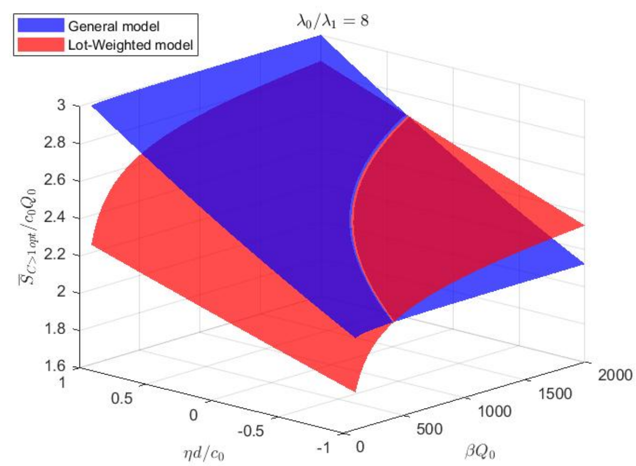

Figure 5. Optimal relative revenues dependence on $\beta Q_0,\ \eta d / c_0$.

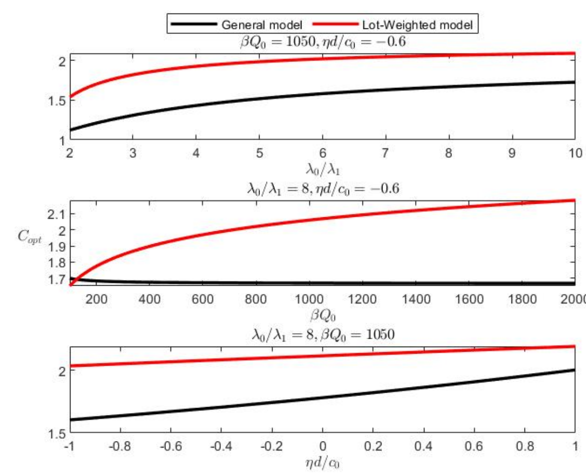

Figure 6. C_{opt} dependence on $\lambda_0 / \lambda_1,\ \beta Q_0,\ \eta d / c_0$.

V. CONCLUSION

For large lot sizes, we describe the stock level process driven by a compound Poisson demand with a customer flow intensity proportional to the stock level and a time-dependent weight function using a diffusion approximation. We get the near-optimal weight function for large lot sizes in closed form using a linear intensity-of-price dependence. This weight function depends on an unknown constant. The modified, lot-weighted function, also depending on a constant, is presented to ensure more stable operation of the inventory system. Optimization with respect to the constants under the salvage values consideration can be done only numerically, the revenues functions are concave.

The numerical analysis demonstrates that while both models achieve comparable peak revenues under baseline conditions, their performance diverges under parameters variations. The lot-weighted model excels in scenarios with large lot sizes or high utilization costs, yielding higher optimal revenues. In contrast, the general model exhibits greater stability and robustness, requiring only minimal tuning C to sustain near-optimal performance when only βQ_0 is changed. These findings highlight a trade-off: the lot-weighted model offers superior revenue potential under specific constraints, whereas the general model provides broader parametric insensitivity.

Our future research will focus on applying these two models, either individually or in combination, to diverse real-world scenarios to assist retailers maximize profits.

REFERENCES

[1] Aschemann-Witzel, J, "Consumer perception and preference for suboptimal food under the emerging practice of expiration date-based pricing in supermarkets," *Food Quality and Preference*, vol. 63, pp. 119–128, 2018.

[2] Adenso-Díaz, B., Lozano, S., and Palacio, A, "Effects of dynamic pricing of perishable products on revenue and waste," *Applied Mathematical Modelling*, vol. 45, pp. 148–16, 2017.

[3] Tekin, P., and Erol, R, "A new dynamic pricing model for the effective sustainability of perishable product life cycle," *Sustainability*, vol. 9, no. 8, pp. 1330, 2017.

[4] Zhao, W., and Zheng, Y.-S, "Optimal dynamic pricing for perishable assets with nonhomogeneous demand," *Management Science*, vol. 46, pp. 375−388, 2000.

[5] Sahay, A, "How to reap higher profits with dynamic pricing," *MIT Sloan Management Review*, vol. 48, no. 4, pp. 53–60, 2007.

[6] Raafat, F, "Survey of literature on continuously deteriorating inventory models," *Journal of the Operational Research Society*, vol. 42, pp. 27–37, 1991.

[7] Goyal, S. K., and Giri, B. C, "Recent trends in modelling of deteriorating inventory," *European Journal of Operational Research*, vol. 134, pp. 1–16, 2001.

[8] Karaesmen, I.Z., Scheller–Wolf, A., and Deniz, B, "Managing perishable and aging inventories: review and future research directions," In *Kempf K. G. et al. (eds.), Planning Production and Inventories in the Extended Enterprise*, Springer, New York, 2011, vol. 1, pp. 393–436.

[9] Bakker, M., Riezebos, J., and Teunter R, "Review of inventory systems with deterioration since 2001," *European Journal of Operational Research*, vol. 221, no. 2, pp, 275–284, 2012.

[10] Pahl, J., and Voß, S, "Integrating deterioration and lifetime constraints in production and supply chain planning: A survey," *European Journal of Operational Research*, vol. 238, no. 3, pp. 654–674, 2014.

[11] Janssen, L., Claus, T., and Sauer, J, "Literature review of deteriorating inventory models by key topics from 2012 to 2015," *International Journal of Production Economics*, vol. 182, pp. 86–112, 2016.

[12] Kitaeva, A.V., Stepanova, N.V., and Zhukovskaya, A.O, "Zero Ending Inventory Dynamic Pricing Model under Stochastic Demand, Fixed Lifetime Product, and Fixed Order Quantity," *IFAC-PapersOnLine*, Berlin, Germany, August 2019, vol. 52, no. 13, pp. 2482–2487.

[13] Kitaeva, A.V., Stepanova, N.V., and Zhukovskaya, A.O, "Profit Optimization for Zero Ending Inventories Dynamic Pricing Model under Stochastic Demand and Fixed Lifetime Product," *IFAC-PapersOnLine*, Berlin, Germany, July, vol. 202053, no. 2, pp. 10505–10510.

[14] Kitaeva, A.V., Stepanova, N.V., and Zhukovskiy, O.I, "Profit Optimization with a Power-Law Adjustable Coefficient for Zero Ending Inventories Dynamic Pricing Model, Stochastic Demand, and Fixed Lifetime Product," *IFAC-PapersOnLine*, Nantes, France, June, vol. 202255, no. 10, pp. 1793–1797.

[15] Kitaeva, A. V., Zhukovskaya, A. O., and Zmeev, O. A, "Compound Poisson demand with price-dependent intensity for fast moving items: price optimisation and parameters estimation," *International Journal of Production Research*, vol. 55, no. 14, pp. 4153–4163, 2017.

[16] Chintapalli, P, "Simultaneous pricing and inventory management of deteriorating perishable products," *Annals of Operations Research*, vol. 229, pp. 287–301, 2015.