Hyperexponential ILF-Based Control for Synchronous Motor Using Model-Free Approach

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***Abstract*— This paper is addressed to a problem of synthesis of hyperexponential control algorithm based on Implicit Lyapunov Function (ILF) method using model-free technique for synchronous motors. The nonlinear motor model under consideration corresponds to a non-salient synchronous motor with surface–mounted permanent magnets installed in the rotor. The model-free approach is applied to replace the complex nonlinear motor model with an ultra-local model with simple dynamics. Then hyperexponential control law is formulated using the theorem with linear matrix inequalities. The effectiveness of the approach is demonstrated via realistic numerical simulations with delays in control channel and noised measurements.**

I. INTRODUCTION

Synchronous motors with permanent magnets are used in a wide range of servo applications in industry due to their simple construction, high efficiency and power factor. The highly nonlinear dynamics and often imprecisely known parameters of synchronous motors necessitate advanced control methods, especially when high performance is required. This task becomes especially important under complex operating conditions, which often include unknown external disturbances, measurement noise, control delays and unaccounted dynamics of the technical system.

Classical control methods, for example, a widely used method for regulating AC motors in industry Field-Oriented Control (FOC), often experience difficulties in mentioned complex operating conditions and require precise motor parameters for accurate and efficient operation. Though FOC provide some robust properties against measurement noises and parameter uncertainty, using inaccurate parameter values degrades system performance, slows speed and torque responses, and introduces errors [1], [2].

When dealing with systems characterized by significant parametric, structural, and signal uncertainties, model-free control methods often become preferable [3]–[7]. While empirically tuned PID controllers are common in practice for such scenarios, they may not consistently deliver high performance.

This paper adopts the model-free approach from [3], which substitutes a complex (and unknown) mathematical model with an ultra-local model. This ultra-local model represents the system as a controlled chain of integrators subject to matched additive disturbances, encompassing the system's unknown dynamics. Consequently, the task of model-free control design is simplified to the synthesis of robust feedback for this chain of integrators, including estimation and compensation of the unknown dynamics.

Effective feedback synthesis and tuning are critical for ensuring robustness against matched disturbances, measurement noise, and input delay, while simultaneously achieving high accuracy and fast convergence. A number of papers are addressed to this purpose and explore various techniques including PID control [3], sliding mode control [8], fuzzy logic [9], and neural networks [10].

Fast control methods aim to achieve convergence rates exceeding those of linear systems, surpassing any exponential convergence. This category encompasses hyperexponential methods (asymptotic, see e.g., [11]–[13]) and finite-time/fixed-time methods (non-asymptotic, e.g., [14]–[17]). These methods are particularly advantageous for time-critical systems. Finite-time convergence is also important for fast estimation of state variables and parameters of AC motors which is required to ensure high efficiency of motor control (see, for example, [18]–[20]).

Beyond rapid convergence, fast control methods often exhibit valuable robustness properties not found in classical approaches, such as the ability to compensate for non-Lipschitz disturbances. This paper investigates ILF-based hyperexponential control [11] for integrator chains. The combination of fast convergence and robustness against measurement noise, disturbances, and delays makes this method an attractive candidate for integration with the model-free approach.

The remaining of the paper is organized as follows. The nonlinear model of the synchronous motor with permanent magnets and the problem statement are given in Section II. Section III presents the basic definitions and theorem of the hyperexponential stability. ILF-based model-free approach and some control techniques adapting presented method to motor regulation are described in Section IV. Representative simulation results that show the efficiency of the proposed algorithm are given in Section V. The paper is wrapped-up with concluding remarks in Section VI.

II. MOTOR MODEL AND PROBLEM FORMULATION

First, we introduce the basic notations that are applied in this paper.

Notation 1.

1 $\mathbb{R}$ and $\mathbb{R}_+$ are the sets of real and real positive numbers, respectively.

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2 $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ are the n and $n\times n$ dimensional Euclidean spaces with the vector norm $\|\cdot\|$.
3 $\mathbb{S} := [0,2\pi)$ denotes the set of values for the rotor angular position in radians. $\mathbb{N}$ denotes the set of natural numbers.
4 $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix.
5 $\overline{1,m}$ denotes a sequence of integer numbers $1,...,m$.
6 $\mathrm{diag}\{\lambda_i\}_{i=1}^n$ is the diagonal matrix with elements λ_i, $i=\overline{1,n}$.
7 Relations of the form $P>0$ (<0; ≥ 0; ≤ 0) for the matrix $P\in\mathbb{R}^{n\times n}$ mean that P is symmetric and positive (negative) definite (semidefinite).

In this paper, we use the classical two–phase model of the unsaturated, non–salient PMSM from [21], [22]. In the stationary $\alpha-\beta$ frame the motor model is presented by the following equations

$$\dot{\lambda}_{\alpha\beta}(t) = v_{\alpha\beta}(t) - Ri_{\alpha\beta}(t), \tag{1}$$
$$J_m\dot{\omega}(t) = -B\omega(t) + \tau_e(t) - \tau_L(t), \tag{2}$$
$$\dot{\theta}(t) = \omega(t), \tag{3}$$

where $\lambda_{\alpha\beta}(t)\in\mathbb{R}^2$ is vector of the total flux, $i_{\alpha\beta}(t)\in\mathbb{R}^2$ is the vector of the stator currents, $v_{\alpha\beta}(t)\in\mathbb{R}^2$ is the vector of the stator voltages, $R\in\mathbb{R}_+$ is the resistance of the stator windings, $J_m\in\mathbb{R}_+$ is the rotor inertia, $\theta(t)\in\mathbb{S}$ is the rotor angular position, $\omega(t)\in\mathbb{R}$ is the rotor angular speed, $B\in\mathbb{R}$, $B\geq 0$ is the viscous friction coefficient, $\tau_L(t)\in\mathbb{R}$ is the external load torque, $\tau_e(t)\in\mathbb{R}$ is the torque of electrical origin, given by

$$\tau_e(t) = n_p i_{\alpha\beta}^\top(t)\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\lambda_{\alpha\beta}(t), \tag{4}$$

where $n_p\in\mathbb{N}$ is the number of pairs of poles.

The total flux of the motor with surface–mounted permanent magnets satisfies

$$\lambda_{\alpha\beta}(t) = Li_{\alpha\beta}(t) + \lambda_m\begin{pmatrix}\cos(n_p\theta(t))\\ \sin(n_p\theta(t))\end{pmatrix}, \tag{5}$$

where $L\in\mathbb{R}_+$ is the stator inductance and $\lambda_m\in\mathbb{R}_+$ is the constant flux due to permanent magnets.

Here we consider the following assumptions applied to the motor model.

A1 The currents $i_{\alpha\beta}(t)$ of the stator windings, the rotor position $\theta(t)$ and the rotor speed $\omega(t)$ are *measurable* signals. The total flux $\lambda_{\alpha\beta}(t)$, the torque of electrical origin $\tau_e(t)$ and the angular acceleration of the rotor $\dot{\omega}(t)$ are *immeasurable*.
A2 The signals $v_{\alpha\beta}(t)$ and $\tau_L(t)$ are such that the motor model (1)-(5) is forward complete and all signals are bounded $\forall t\geq 0$.
A3 The load torque $\tau_L(t)$ is *unknown*.
A4 The *only known* motor parameter of the PMSM is the number of pole pairs n_p.

Remark 1 Assumptions A1 and A3 are typical for many industrial applications and define the set of sensors that perform the corresponding measurements. Assumption A2 provides realistic conditions for the motor model that are consistent with practical implementation. Finally, Assumption A4 refers to the complex case of motor control synthesis, in which all motor parameters (except n_p) are unknown.

The objective is to develop hyperexponential ILF-based stabilization algorithm for PMSM model (1)–(5) using model-free control technique under assumptions A1–A4.

III. BASIC DEFINITIONS AND THEOREM OF THE HYPEREXPONENTIAL STABILITY

In this section, as preliminaries we present the basic definitions and theorem of the hyperexponential stability including robust hyperexponential control. Consider the following system

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0, \tag{6}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, and $f:\mathbb{R}^n\to\mathbb{R}^n$ is a vector field which satisfies $f(0)=0$. If f is discontinuous with respect to x, the solutions of (6) are understood in the sense of Filippov [23] via the corresponding differential inclusions. Suppose that the system (6) has unique solutions $\Phi(t,x_0)$ determined in forward time (at least locally).

Hyperexponential stability refers to the systems which convergence rate is greater than the rate of any stable linear system.

Introduce the vector $\alpha = (\alpha_0,\alpha_1,\cdots,\alpha_r)^\top\in\mathbb{R}_+^{r+1}$ with $r\in\mathbb{N}$. Then, the function of nested exponentials $\rho_{r,\alpha}:\mathbb{R}\to\mathbb{R}_+$ is defined recursively as follows:

$$\begin{aligned}&\rho_{0,\alpha}(z) = \alpha_0 z,\\ &\rho_{i,\alpha}(z) = \alpha_i\left(e^{\rho_{i-1,\alpha}(z)}-1\right), \quad i=\overline{1,r}.\end{aligned} \tag{7}$$

For the sake of completeness of the article, we provide here two definitions from [11].

Definition 1 [11] *The origin of the system (6) is hyperexponentially stable of degree $r\in\mathbb{N}$ if there exists $\varphi\in\mathcal{K}_\infty$ and $\alpha\in\mathbb{R}_+^{r+1}$ such that the inequality*

$$\|\Phi(t,x_0)\| \leq \varphi(\|x_0\|)e^{-\rho_{r,\alpha}(t)} \tag{8}$$

is satisfied for all $t\geq 0$ and $x_0\in D$, where $D\subset\mathbb{R}^n$ is an open neighborhood of the origin. If $D=\mathbb{R}^n$, then the origin of the system (6) is said to be globally hyperexponentially stable of degree r.

Therefore, in the sense of hyperexponential stability the rate of convergence increases as $\Phi(t,x_0)\to 0$. The first definition means that the convergence of the system is hyperexponential both near and far from the origin.

Hyperexponential stability can be characterized separately at infinity and near the origin, as described in [11]:

Definition 2 [11] *The system (6) is said to be*
• *hyperexponentially stable of degree $r\in\mathbb{N}$ at infinity, if it is globally asymptotically stable, and (8) is satisfied for all $x_0\in\mathbb{R}^n : \|x_0\|\geq 1$;*
• *hyperexponentially stable of degree $r\in\mathbb{N}$ at origin, if it is globally asymptotically stable, and (8) is satisfied for all $x_0\in\mathbb{R}^n : \|x_0\|\leq 1$.*

Thus, Definition 2 implies that hyperexponential stability is described at infinity and near the origin in a separate way.

Note, that there is a special case of hyperexponential stability that results in a finite/fixed-time stability [24], [14]. In this case, the system transients converge in a finite time as established in [15].

To establish the robust hyperexponential stabilization technique consider the following system

$$\dot{x}(t) = Ax(t) + bu(t) + d(t,x(t)), \tag{9}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the control input, $d : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is the function that can include external additive disturbances, nonlinearities and uncertainties of the system, $A \in \mathbb{R}^{n\times n}$ is the matrix that defines the dynamic properties of the system and $b \in \mathbb{R}^n$ is the vector of control inputs of the form

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

Here we use the Implicit Lyapunov Function (ILF) method that can be applies to different types of stability (for example, see [11], [15], [25], [26]).

Consider the implicitly defined Lyapunov function given by

$$Q_1(V,x) = V^{-2} x^\top D_1(\sigma(V)) P D_1(\sigma(V)) x - 1, \tag{10}$$

$$Q_2(V,x) = V^{-2} x^\top D_2\left(\sigma^{-1}\left(\frac{1}{V}\right)\right) P D_2\left(\sigma^{-1}\left(\frac{1}{V}\right)\right) x - 1, \tag{11}$$

where $\sigma(V) = 1 - \ln V$, $D_1(\lambda) = \mathrm{diag}\{\lambda^{\eta_i}\}_{i=1}^n$, $\eta_i = 1+n-i$, $D_2(\lambda) = \mathrm{diag}\{\lambda^{\kappa_i}\}_{i=1}^n$, $\kappa_i = i$ for $\lambda \in \mathbb{R}_+$ and $0 < P \in \mathbb{R}^{n\times n}$.

Also, denote two diagonal matrices $H_1 = \mathrm{diag}\{\eta_i\}_{i=1}^n$ and $H_2 = \mathrm{diag}\{\kappa_i\}_{i=1}^n$.

Using the Implicit Lyapunov Function (ILF) theorem, in the paper [11] a hyperexponential control law is proposed for the system (9) with $d \equiv 0$. In [27], the robust properties of the system (9), (14) with respect to disturbances and measurement noise were studied, demonstrating high performance. The following theorem establishes this result.

Theorem 1 [27] *Let the LMI*

$$\begin{aligned} AX + XA^\top + bY + Y^\top b^\top + \gamma_j(2X + XH_j + H_jX) \\ + \alpha_j X + R_j \le 0, \end{aligned} \tag{12a}$$

$$XH_j + H_jX > 0, \quad X > 0, \qquad j = 1,2 \tag{12b}$$

be feasible with respect to $X \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{1\times n}$ for some fixed numbers $\gamma_j, \alpha_j \in \mathbb{R}_+$ and some $R_j \in \mathbb{R}^{n\times n}$, $R_j > 0$, and the disturbance function d satisfies

$$\begin{aligned} d^\top D_1(\sigma(V)) R_1^{-1} D_1(\sigma(V)) d \le \alpha_1 V^2 \sigma^2(V) \\ \text{if } x^\top Px < 1, \\ d^\top D_2\left(\sigma^{-1}\left(\tfrac{1}{V}\right)\right) R_2^{-1} D_2\left(\sigma^{-1}\left(\tfrac{1}{V}\right)\right) d \le \alpha_2 V^2 \sigma^2\left(\tfrac{1}{V}\right) \\ \text{if } x^\top Px \ge 1. \end{aligned} \tag{13}$$

Then the control

$$u(V,x) = \begin{cases} KD_1(\sigma(V))x & \text{for } x^\top Px < 1, \\ \sigma^{n+1}(V^{-1}) KD_2(\sigma^{-1}(V^{-1}))x & \text{for } x^\top Px \ge 1, \end{cases} \tag{14}$$

where $V \in \mathbb{R}_+ : Q_1(V,x) = 0$ for $x^\top Px < 1$, $V \in \mathbb{R}_+ : Q_2(V,x) = 0$ for $x^\top Px \ge 1$, $K = YP$, $P = X^{-1}$, hyperexponentially stabilizes the system (9) with degree 1.

IV. ILF-BASED MODEL-FREE CONTROL

In this section we present the main steps of synthesis of model-free control and its adaptation to the motor regulation. First, consider the nonlinear system

$$\dot{q}(t) = f(q(t), \tilde{u}(t), \tau_d(t, q(t))),\ q(0) = q_0, \tag{15}$$

$$y(t) = h(x(t)), \tag{16}$$

where $q(t) \in \mathbb{R}^n$ is the state vector, $f(\cdot) : \mathbb{R}^{\tilde{n}} \times \mathbb{R} \times \mathbb{R}^k \to \mathbb{R}^{\tilde{n}}$ is a nonlinear function that satisfies $f(0) = 0$, $\tilde{u}(t) \in \mathbb{R}$ is the control signal, $\tau_d : \mathbb{R} \times \mathbb{R}^{\tilde{n}} \to \mathbb{R}^k$ represents disturbances applied to the system and $y(t) \in \mathbb{R}$ is the system output.

The task is to stabilize the output of the system (15)–(16) at the origin implementing the framework of the model-free control method given by [3]. Following this technique, we replace the *nonlinear* model (15)–(16) by an *ultra-local* system

$$y^{(n)}(t) = F(t) + \eta\tilde{u}(t). \tag{17}$$

where $y^{(n)}(t)$ is the n-th order derivative of the output variable $y(t)$, $F(t) \in \mathbb{R}$ is the total lumped unknown dynamics of the model (15)–(16) and $\eta \in \mathbb{R}_+$ is a design parameter.

Remark 2 The choice of order of output derivative $n \in \mathbb{N}$ depends on the system dynamics and stabilization task. For example, if the output $y(t)$ represents the position of some technical system (i.e., joint of a robotic manipulator) then the dynamics can be modelled applying the Newton's second law and the acceleration $y^{(2)}(t)$ can be used in (17). The rationale of choosing the design parameter $\eta \in \mathbb{R}_+$ comes from comparing the magnitudes of signals $y^{(n)}(t)$ and $\eta\tilde{u}(t)$. The unknown dynamics presented by the function $F(t) \in \mathbb{R}$ includes the disturbance $\tau_d(t, q(t))$ and can also include other unaccounted system uncertainties.

A number of different approaches has been proposed for online estimation of the function $F(t)$. For instance, see [3], [28], [29]. The problem of immeasurable derivatives of the output variable $y(t)$ can be solved applying various differentiators, that provide fast and accurate convergence [30]–[33].

The paper [3] proposes a simple estimation technique for the total lumped dynamics $F(t)$ using a sufficiently small delay $d_{mf} \in \mathbb{R}_+$:

$$F(t) \approx \hat{F}(t) = F(t - d_{mf}) = y^{(n)}(t) - \eta u(t - d_{mf}).$$

From the practical point of view, one of the possible choices is to set $d_{mf} \in \mathbb{R}_+$ equal to the sampling time of the control algorithm.

Here we apply the hyperexponential differentiator from [32] in the ILF-based control method to get fast estimation of $y^{(\nu)}$, $\nu = \overline{1,n}$. In particular, the latter approach is used to get the estimate of the rotor angular acceleration $\dot{\omega}(t) \in \mathbb{R}$ that appears in equation (2).

Once the function $F(t)$ and derivatives $y^{(\nu)}$, $\nu = \overline{1,n}$ are reconstructed, the control can be calculated as follows

$$\tilde{u}(t) = \frac{-\hat{F}(t) + u(t)}{\eta}. \tag{18}$$

Replacing $\tilde{u}(t)$ in (17) by (18) one can get the system of the form (9) with $x_1(t) = y(t)$ and $d(t,x(t)) = (0\, 0\, \cdots\, d_n(t,x(t)))^\top$. The function $d_n(t,x(t))$ that represents disturbances, system nonlinearities and uncertainties is given by estimation errors of the total lumped unknown dynamics $F(t)$:

$$d_n(t,x(t)) = F(t) - \hat{F}(t).$$

In this way, the application of the described model-free approach results in the task of controlling the system (9). For the latter task we propose to use control method (14) from Theorem 1 which ensures improved convergence rate resulting in hyperexponential stability with robustness properties against disturbances.

Next, we present several steps of the transformations of the motor model (1)-(5) for application of the developed hyperexponential control algorithm using ILF-based model-free technique. Here we follow the main goal of classical field-oriented controller, namely, to produce the maximum torque with a given current reference minimizing the losses.

First, translate the motor model from the stationary frame (α,β) presented by equations (1)–(5) to the rotating (d,q)-frame associated with the magnetic flux. Applying forward Park transformation

$$\begin{bmatrix} f_d \\ f_q \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \\ -\sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix}. \tag{19}$$

to (5) we get

$$\lambda_{dq} = Li_{dq} + \lambda_m \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{20}$$

Next, apply Park transformation to (1) and differentiate (20). Combining the result, one can obtain

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \end{bmatrix} + L \begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \end{bmatrix} + \begin{bmatrix} -n_p\omega L i_q \\ n_p\omega(Li_d + \lambda_m) \end{bmatrix}. \tag{21}$$

Notice, that the dynamics of the motor in rotating frame is nonlinear and contains cross-couplings along the d and q axes.

The torque generated by motor in (d,q)-frame is given by

$$\tau_e = n_p(\lambda_d i_q - \lambda_q i_d), \tag{22}$$

and for the non-salient PMSM we form

$$\tau_e = n_p \lambda_m i_q. \tag{23}$$

Thus, replacing τ_e in equation (2) by (23), one can obtain

$$J_m\dot{\omega}(t) \;=\; -B\omega(t) + n_p\lambda_m i_q(t) - \tau_L(t). \tag{24}$$

Expressing $i_q(t)$ from (21) and substituting the result into (24), we get

$$\dot{\omega}(t) = -\frac{B}{J_m}\omega(t) - \frac{n_p\lambda_m}{J_m R}\left(L\frac{di_d}{dt} + n_p\omega(Li_d + \lambda_m)\right) - \frac{\tau_L(t)}{J_m} + \frac{n_p\lambda_m}{J_m R} v_q. \tag{25}$$

The latter equation shows that the motor can be transformed to the ultra-local system (17), where the system output $y(t)$ is given by the rotor position $\theta(t)$, the unknown dynamics $F(t)$ is represented by the nonlinear dynamics of the motor, viscous friction forces and external load torque, and the control signal $\tilde{u}(t)$ is the voltage $v_q(t)$:

$$y^{(n)}(t) = \dot{\omega}(t) = \ddot{\theta}(t),\ n = 2, \quad \tilde{u}(t) = v_q(t),\ \eta = \frac{n_p\lambda_m}{J_m R},$$
$$F(t) = -\frac{1}{J_m}\left(B\omega + \frac{n_p\lambda_m}{R}\left(L\frac{di_d}{dt} + n_p\omega(Li_d + \lambda_m)\right) + \tau_L\right). \tag{26}$$

To ensure independent current control with the torque given by (23) we set the reference current $i_d^*(t) = 0$ and form a simple proportional-integral (PI) controller for $v_d(t)$ without compensation of nonlinear term $-n_p\omega(t)Li_q(t)$:

$$v_d(t) = PI(i_d^*(t) - i_d(t)) = -K_p i_d(t) - K_i \int i_d(t)dt. \tag{27}$$

where $K_p \in \mathbb{R}_+$ and $K_i \in \mathbb{R}_+$ are proportional and integral gains, respectively.

Thus, the hyperexponential control (14) and model-free technique (18) are applied to stator control voltage $v_q(t)$.

V. SIMULATION RESULTS

The objective of simulations is to verify the efficiency of the presented control algorithm via numerical simulations in the presence of measurement noises and delay of the control input. PMSM under the test is the motor BMP0701F from [34] which parameters are listed in the Table I.

Here we consider the case when the motor rotates at the speed $\omega(0) = 10$ rad/sec at the initial time moment and other initial states are zero. The task is to stabilize the motor speed at zero.

TABLE I

MOTOR DATA

Parameter (units)	Value
Number of pairs of poles n (–)	5
Stator inductance L (mH)	40.03
Stator resistance R (Ω)	8.875
Drive inertia J (kgm^2)	60×10^-6
Permanent magnet flux λ_m (Wb)	0.2068

Solving LMI from Theorem 1 we obtain the following design parameters for hyperexponential stabilization algorithm:

$$P = \begin{bmatrix} 1.4245 & 0.6383 \\ 0.6383 & 2.9718 \end{bmatrix}, \quad K = \begin{bmatrix} -0.2670 \\ -0.3687 \end{bmatrix}.$$

The design gains for the model-free approach including the small delay d_{mf} and the scaling coefficient η: $d_{mf} = 0.01$ sec, $\eta = 0.1$.

For the PI controller that is used only for i_d current component we apply without tuning: $K_p = 100$ and $K_i = 20$.

The load torque applied to the motor shaft is set by two pulses with a duration of 0.05 sec. The first positive pulse $\tau_L = 1$ Nm is applied at $t = 2$ sec and the second negative one $\tau_L = -0.5$ Nm at $t = 3$ sec. During the remaining time intervals, zero load is used.

The transients of the motor currents $i_{\alpha\beta}(t)$ in the stationary frame are illustrated in Fig. 1. In the test, we consider the case with sufficiently high measurement noises of currents $\delta_{i_{\alpha\beta}}(t)$ which include constant biases and harmonic signals

$$\delta_{i_{\alpha\beta}}(t) = \begin{bmatrix} 0.3+0.1\sin(100t) \\ -0.2+0.2\sin(50t) \end{bmatrix}.$$

Moreover, the delay of 0.05 sec is applied for the ILF-based control signal $u(t)$ that is given by (14) in Theorem 1 making the control problem more challenging.

Fig. 2 shows the transient process of the rotor speed $\omega(t)$. As seen from the figure, the rotor speed converges to zero fast at the initial time interval demonstrating monotonic behavior. When when the torque is applied to the motor, the rotor speed exhibits a relatively small overshoot and then converges to zero with a high rate without oscillations. Thus, the proposed control algorithm demonstrates good efficiency and robust properties against the measurement noises and delay in the control channel.

Figs. 3 and 4 show the control voltages in the rotating frame $v_{dq}(t)$. And the ILF-based control signal $u(t)$ is given in Fig. 5. One can conclude, that the behavior of the latter signal is similar to the rotor speed $\omega(t)$ in the inverse sense that is reasoned by efficient compensation of the total lumped unknown dynamics $F(t)$ by the model-free control.

For comparison, Fig. 6 shows the behavior of the classical FOC [22] in speed control mode. The design gains for the PI current and speed controllers: $K_p = 100$, $K_i = 20$ and $K_{p\omega} = 25$, $K_{i\omega} = 10$, respectively. As seen from the figure, the motor operation is unstable in the presence of noises and delay and the transient behavior of the rotor speed demonstrates high oscillations even using the exact values of the motor parameters.

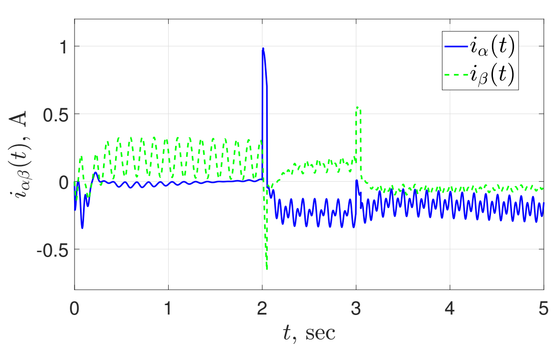

Fig. 1. Transients of stator currents $i_{\alpha\beta}(t)$

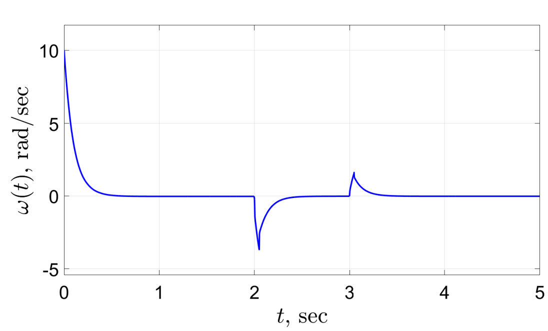

Fig. 2. Transients of the rotor speed $\omega(t)$

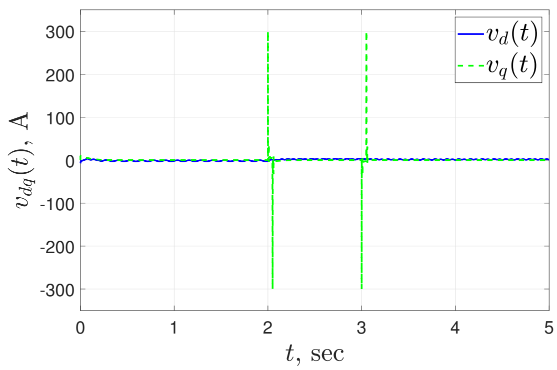

Fig. 3. Control voltages $v_{dq}(t)$

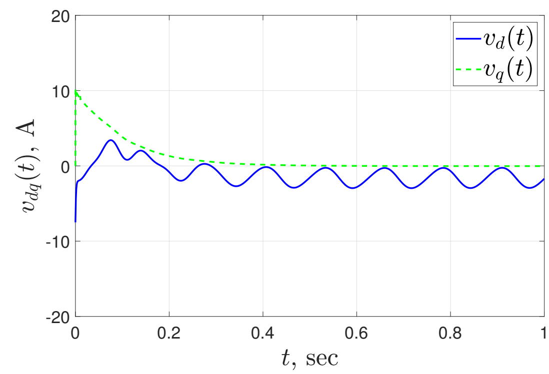

Fig. 4. Control voltages $v_{dq}(t)$ in the time interval $t \in (0;\, 1)$ sec

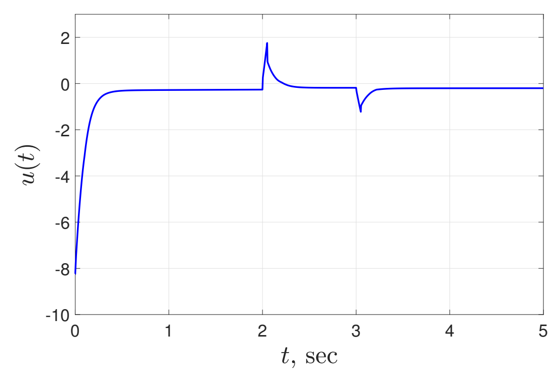

Fig. 5. ILF-based control signal $u(t)$

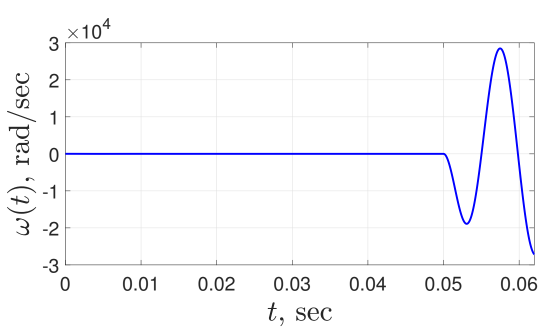

Fig. 6. Transients of the rotor speed $\omega(t)$ for the classical FOC

VI. CONCLUSIONS

In this paper, the hyperexponential control approach based on Implicit Lyapunov Function (ILF) method and model-free control framework are presented. Adding some techniques the proposed algorithm is adapted for application to the non-salient permanent magnet synchronous motor that has non-linear dynamic model. One of the key features of the developed algorithm is that all motor parameters are assumed to be unknown. Following model-free control, the non-linear motor model is replaced by a simplified ultra-local model with compensation for the total lumped unknown dynamics including disturbances and system uncertainties. A hyperexponential control law is then derived using a theorem based on linear matrix inequalities. The numerical simulations with the realistic operating conditions of the motor demonstrate high efficiency of the proposed composite method ensuring fast stabilization of the motor speed with good transient behavior. Moreover, the proposed approach provides considerable robust properties against measurement noises and delays in the control channel ensuring stable motor operation in the case when the classical field-oriented controller is unreliable. Further directions for investigations include experimental study of the effectiveness of the proposed approach and profound tuning of the design parameters.

REFERENCES

[1] S. Maiti, C. Chakraborty, Y. Hori, and M. Ta, "Model reference adaptive controller-based rotor resistance and speed estimation techniques for vector controlled induction motor drive utilizing reactive power," *Industrial Electronics, IEEE Transactions on*, vol. 55, pp. 594 – 601, 03 2008.

[2] P. Castaldi, W. Geri, M. Montanari, and A. Tilli, "A new adaptive approach for on-line parameter and state estimation of induction motors," *Control Engineering Practice*, vol. 13, no. 1, pp. 81–94, 2005.

[3] M. Fliess and C. Join, "Model-free control," *International Journal of Control*, vol. 86, no. 12, pp. 2228–2252, 2013.

[4] N. Killingsworth and M. Krstic, "PID tuning using extremum seeking: online, model-free performance optimization," *IEEE Control Systems Magazine*, vol. 26, no. 1, pp. 70–79, 2006.

[5] M. Bilal Kadri, *Model free adaptive fuzzy control: Beginners approach*. Saarbrucken: VDM Verlag, 2009.

[6] J. Spall and J. Cristion, "Model-free control of nonlinear stochastic systems with discrete-time measurements," *IEEE Transactions on Automatic Control*, vol. 43, no. 9, pp. 1198–1210, 1998.

[7] M. FLIESS and C. JOIN, "Model-free control and intelligent PID controllers: Towards a possible trivialization of nonlinear control?" *IFAC Proceedings Volumes*, vol. 42, no. 10, pp. 1531–1550, 2009, 15th IFAC Symposium on System Identification.

[8] H. Wang, X. Ye, Y. Tian, G. Zheng, and N. Christov, "Model-free–based terminal smc of quadrotor attitude and position," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 5, pp. 2519–2528, 2016.

[9] G. I. Y. Mustafa, H. Wang, and Y. Tian, "Model-free adaptive fuzzy logic control for a half-car active suspension system," *Studies in Informatics and Control*, 2019.

[10] Y. Chen, J. Liu, H. Wang, Z. Pan, and S. Han, "Model-free based adaptive rbf neural network control for a rehabilitation exoskeleton," in *2019 Chinese Control And Decision Conference (CCDC)*, 2019, pp. 4208–4213.

[11] K. Zimenko, D. Efimov, and A. Polyakov, "On hyperexponential stability and stabilization," *Automatica*, vol. 173, p. 112078, 2025.

[12] J. Wang, K. Zimenko, D. Efimov, and A. Polyakov, "On hyperexponential stabilization of a chain of integrators in continuous and discrete time," *Automatica*, vol. 166, p. 111723, 2024.

[13] M. Inoue, T. Wada, T. Asai, and M. Ikeda, "Non-exponential stabilization of linear time-invariant systems by linear time-varying controllers," *The 50th IEEE Conference on Decision and Control*, pp. 4090–4095, 2011.

[14] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[15] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time stabilization: Implicit Lyapunov function approach," *Automatica*, vol. 51, pp. 332–340, 2015.

[16] K. Zimenko, A. Polyakov, D. Efimov, and W. Perruquetti, "Robust feedback stabilization of linear MIMO systems using generalized homogenization," *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5429–5436, 2020.

[17] J. A. Moreno, *Lyapunov-Based Design of Homogeneous High-Order Sliding Modes*. Cham: Springer International Publishing, 2018, pp. 3–38.

[18] A. Pyrkin, A. Bobtsov, R. Ortega, A. Vedyakov, D. Cherginets, A. Ovcharov, D. Bazylev, and I. Petranevsky, "Robust nonlinear observer design for permanent magnet synchronous motors," *IET Control Theory & Applications*, vol. 15, no. 4, pp. 604–616, 2021.

[19] D. Bazylev, "Finite time parameter estimation algorithm for salient pmsm," in *2022 European Control Conference (ECC)*, 2022, pp. 303–308.

[20] A. Ovcharov, A. Pyrkin, A. Bobtsov, D. Bazylev, R. Ortega, and A. Vedyakov, "Finite time observer for induction motors based on drem algorithm," in *2020 European Control Conference (ECC)*, 2020, pp. 1318–1323.

[21] P. C. Krause, "Analysis of electric machinery," *McGraw Hill, New York*, 1986.

[22] K. H. Nam, "AC motor control and electrical vehicle applications," *CRC press*, 2018.

[23] A. Filippov, *Differential equations with discontinuous right-hand sides*. Kluwer, Dordrecht, 1988.

[24] Y. Orlov, "Finite time stability and robust control synthesis of uncertain switched systems," *SIAM Journal of Control and Optimization*, vol. 43(4), pp. 1253–1271, 2004.

[25] V. I. Korobov, "A solution of the problem of synthesis using a controllability function," in *Doklady Akademii Nauk*, vol. 248, no. 5. Russian Academy of Sciences, 1979, pp. 1051–1055.

[26] J. Adamy and A. Flemming, "Soft variable-structure controls: a survey," *Automatica*, vol. 40, no. 11, pp. 1821–1844, 2004.

[27] K. Zimenko, D. Efimov, and A. Polyakov, "Robust hyperexponential control for linear systems," *The 23rd European Control Conference*, accepted.

[28] M. Fliess, C. Join, and H. Sira-Ramirez, "Non-linear estimation is easy," *International Journal of Modelling, Identification and Control*, vol. 4, no. 1, pp. 12–27, 2008.

[29] D. Liu, O. Gibaru, and W. Perruquetti, "Error analysis of jacobi derivative estimators for noisy signals," *Numer Algor*, vol. 58, pp. 53–83, 2011.

[30] M. Angulo, J. Moreno, and L. Fridman, "Robust exact uniformly convergent arbitrary order differentiator," *Automatica*, vol. 49(8), pp. 2489–2495, 2013.

[31] E. Cruz-Zavala, J. Moreno, and L. Fridman, "Uniform robust exact differentiator," *IEEE Transactions on Automatic Control*, vol. 56(11), pp. 2727–2733, 2011.

[32] J. Wang, K. Zimenko, A. Polyakov, and D. Efimov, "An exact robust high-order differentiator with hyperexponential convergence," *IEEE Transactions on Automatic Control*, vol. 70, no. 1, pp. 627–634, 2025.

[33] A. Levant, "Robust exact differentiation via sliding mode technique," *Automatica*, vol. 34, pp. 379–384, 1998.

[34] Schneider Electric, "Bmp synchronous motor, motor manual, v1.00, 12.2012," 2012.