Scalar Sign Function-Based NFTSMC for 2-DOF Robotic Arms

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***Abstract*—This paper develops a novel Non-Singular Fast Terminal Sliding Mode Control (NFTSMC) scheme for two-link robotic manipulators using scalar sign function formulation. The proposed controller simultaneously addresses three critical challenges in robotic control: singularity prevention through carefully designed sliding surface exponents, complete chattering elimination via the mathematical properties of the scalar sign function, and guaranteed finite-time convergence despite dynamic uncertainties and external disturbances. The control architecture combines equivalent dynamics compensation, robust switching terms, and stabilization components, all implemented through the scalar sign function framework. Rigorous Lyapunov stability analysis establishes finite-time convergence properties, with the scalar sign function providing exact discontinuity approximation without compromising theoretical guarantees. Simulation studies on a 2-DOF robotic arm demonstrate the controller's effectiveness in achieving precise trajectory tracking while maintaining smooth control signals. The approach offers significant advantages in computational efficiency and implementation simplicity compared to conventional sliding mode controllers, making it particularly suitable for real-time robotic applications requiring both high precision and robustness.**

***Index Terms*—Non-singular sliding mode control, scalar sign function, finite-time convergence, robotic manipulators, Lyapunov stability.**

Controlling robotic manipulators presents fundamental challenges due to their highly nonlinear dynamics, coupled inertial effects, and sensitivity to external disturbances like payload variations and noise [1–3]. While conventional approaches like PID [4] and adaptive control [5] offer partial solutions, they often prove inadequate for complex nonlinear dynamics, strong coupling, or rapid disturbance handling [6, 7]. In this context, Sliding Mode Control (SMC) has emerged as a particularly effective methodology due to its inherent robustness against uncertainties [8, 9].

However, traditional SMC implementations face limitations: asymptotic rather than finite-time convergence [10], undesirable high-frequency chattering from discontinuous control laws [11], and potential singularities in terminal sliding mode variants [12].

Recent advances in Non-Singular Fast Terminal SMC (NFTSMC) have addressed many of these by offering finite-time convergence and avoiding singularities, while retaining SMC's robustness [13]. Yet, many existing NFTSMC still rely on approximation methods to mitigate chattering, which can compromise theoretical guarantees [14].

Building upon this, the present work develops a novel NFTSMC formulation that leverages the mathematical properties of scalar sign functions for superior control performance [15]. This approach provides exact mathematical guarantees for smoothness without introducing additional approximation errors or computational complexity, making it suitable for real-time robotic applications where both precision and reliability are critical [16–18].

The proposed controller architecture combines several key innovations:

- Scalar sign function implementation enables smooth control action, eliminating chattering without compromising robustness [19].
- A non-integer exponent sliding surface design prevents singularities while ensuring finite-time convergence [20].
- A composite control structure provides inherent robustness against payload variations and disturbances through equivalent, switching, and stabilization terms [21].

Theoretical Lyapunov stability analysis confirms finite-time convergence with explicit bounds [22]. The scalar sign function implementation provides mathematically exact chattering suppression [23]. Furthermore, the controller maintains computational efficiency for real-time implementation on standard robotic hardware [24]. This work extends previous research [25], demonstrating significant improvements over conventional NFTSMC in control smoothness and performance consistency [12]. Experimental validation shows excellent tracking performance even under disturbances, with complete chattering elimination [26]. These advancements are highly relevant for industrial applications requiring reliability, precision, and smooth operation [27].

This paper is organized as follows. Section I introduces the robotic manipulator's dynamic model, uncertainty decomposition, and control objectives. Section II details the proposed NFTSMC design. The finite-time stability analysis is presented in Section III. Section IV showcases the simulation results, comparing performance against conventional control approaches. Finally, Section V concludes the paper and outlines future work.

I. Problem Statement

A. Dynamic Model of Robotic Manipulator

Consider the dynamics of an n-DOF robotic manipulator with uncertainties and disturbances:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau(t) + \tau_{\text{ext}}(t) \tag{1}$$

where: leftmargin=*

- $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote joint position, velocity, and acceleration vectors
- $M(q) \in \mathbb{R}^{n\times n}$ is the symmetric positive-definite inertia matrix
- $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents Coriolis and centrifugal forces
- $G(q) \in \mathbb{R}^n$ contains gravitational torques
- $\tau(t) \in \mathbb{R}^n$ is the control input torque
- $\tau_{\text{ext}}(t) \in \mathbb{R}^n$ models external disturbances

B. Uncertainty Decomposition

The dynamics can be decomposed into nominal and uncertain parts:

$$\begin{cases} M(q) = M_0(q) + \Delta M(q) \\ C(q,\dot{q}) = C_0(q,\dot{q}) + \Delta C(q,\dot{q}) \\ G(q) = G_0(q) + \Delta G(q) \end{cases} \tag{2}$$

where M_0, C_0, G_0 are known nominal terms and ΔM, ΔC, ΔG represent uncertainties.

C. 2-DOF Specialization

For the 2-DOF case studied in this work, the matrix components are:

$$\begin{aligned} M_{11}(q) &= (m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) \\ M_{12}(q) &= m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\ M_{22}(q) &= m_2 l_2^2 \\ C_{11}(q,\dot{q}) &= -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\ C_{12}(q,\dot{q}) &= -m_2 l_1 l_2 \sin(q_2)(\dot{q}_1+\dot{q}_2) \\ C_{21}(q,\dot{q}) &= m_2 l_1 l_2 \sin(q_2)\dot{q}_1 \\ C_{22}(q,\dot{q}) &= 0 \\ G_1(q) &= (m_1+m_2) g l_1 \cos(q_1) + m_2 g l_2 \cos(q_1+q_2) \\ G_2(q) &= m_2 g l_2 \cos(q_1+q_2) \end{aligned} \tag{3}$$

D. Control Objective

Given desired trajectories $q_d(t)$, design a control law $\tau(t)$ ensuring:

- Finite-time convergence: $\|q(t) - q_d(t)\| \to 0$ in finite t_r
- Robustness against ΔM, ΔC, ΔG, and τ_{ext}
- Chattering-free control inputs
- Singularity avoidance in control computation

Remark 1: The lumped uncertainty $\delta(q,\dot{q},\ddot{q}) = \tau_{\text{ext}} - \Delta M\ddot{q} - \Delta C\dot{q} - \Delta G$ satisfies:

$$\|\delta(q,\dot{q},\ddot{q})\| \leq \rho_0 + \rho_1\|q\| + \rho_2\|\dot{q}\|^2 \tag{4}$$

where ρ_i are unknown positive constants.

II. Controller Design

To achieve fast and precise reference tracking despite unknown system parameters, nonlinearities, and external disturbances, we employ the Non-Singular Fast Terminal Sliding Mode Control (NFTSMC) with a smooth sign function for the two-link robotic manipulator.

A. Preliminaries and Notations

The scalar sign function is defined over the complex plane minus the imaginary axis as:

$$\text{sign}(z) = \begin{cases} +1 & \text{if } \operatorname{Re}(z) > 0, \\ \text{undefined} & \text{if } \operatorname{Re}(z) = 0, \\ -1 & \text{if } \operatorname{Re}(z) < 0, \end{cases} \tag{5}$$

where $z \in \mathbb{C}\setminus\mathbb{C}^0$ (i.e., $\mathbb{C}^+ \cup \mathbb{C}^-$), and $\mathbb{C}^+$, $\mathbb{C}^-$ and $\mathbb{C}^0$ denote the open right-half complex plane, the open left-half complex plane and the imaginary axis, respectively.

An alternative representation using the principal square-root $g(z) = \sqrt{z^2}$ is:

$$\text{sign}(z) = \frac{g(z)}{z} = \lim_{j\to\infty} \frac{g_j(z)}{z} = \lim_{j\to\infty} \text{sign}_j(z), \tag{6}$$

where the j-th order approximation is:

$$\text{sign}_j(z) = \frac{(1+z)^j - (1-z)^j}{(1+z)^j + (1-z)^j}. \tag{7}$$

Key properties critical for control applications:

- Smooth approximation: For even j, $\text{sign}_j(z) \in [-1,1]$ provides a differentiable approximation
- Parity dependence: Odd orders yield $\text{sign}_j(z) \notin [-1,1]$
- Absolute value relation: $|z| \equiv z \cdot \text{sign}(z)$
- Chattering reduction: Replaces discontinuous sgn functions in sliding mode control

The following notations are used: For a vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, we define the vector $\text{sgn}(\mathbf{x})^\alpha \in \mathbb{R}^n$ for any $\alpha \geq 1$ as:

$$\text{sign}_j(x)^\alpha = |x|^\alpha \text{sign}_j(x)$$

with time derivative:

$$\frac{d}{dt}\left(\text{sign}_j(x)^\alpha\right) = \alpha|x|^{\alpha-1}\dot{x} \cdot \frac{d}{dx}\text{sign}_j(x)$$

B. NFTS Surface Design

To develop our robust control strategy, we introduce the following nonsingular terminal sliding surface:

$$s = e + k_1\,\text{sign}_j(e)^\gamma + k_2\,\text{sign}_j(\dot{e})^\beta \tag{8}$$

where:

- $e = q - q_d$ denotes the tracking error between the actual state q and desired trajectory q_d
- $\dot{e} = \dot{q} - \dot{q}_d$ represents the error derivative
- $k_1, k_2 > 0$ are tuning gains controlling convergence rates
- β and γ are exponential parameters satisfying $1 < \beta < 2$ and $\gamma > \beta$, ensuring:
  - Nonsingularity

– Finite-time convergence

- $\text{sign}_j(\cdot)$ is the smooth sign function approximation (as defined in Eq. (5)), eliminating chattering effects

Remark 2: The sliding surface (8) exhibits three key properties:

1) Finite-time convergence to $s = 0$ through terminal attractor dynamics
2) Non-singular behavior ensured by the exponent constraints $\gamma > \beta > 1$
3) Reduced chattering effects via the smooth sign_j approximation

When the tracking error $\|e\|$ is large, the dominant term $k_1 \, \text{sign}_j(e)^\gamma$ drives rapid convergence. Conversely, when $\|e\|$ becomes small, the term $k_2 \, \text{sign}_j(\dot{e})^\beta$ guarantees finite-time stabilization to the equilibrium. This dual-phase behavior ensures both fast global convergence and precise terminal accuracy.

C. Control Law Derivation

Our control law is composed of three distinct terms:

$$\tau = \underbrace{\tau_{eq}}_{\text{Nominal dynamics}} + \underbrace{\tau_{sw}}_{\text{Robustness}} + \underbrace{\tau_c}_{\text{Stabilization}} \tag{9}$$

- The *equivalent control* τ_{eq} maintains the system on the sliding surface ($S(t) = 0$)
- The *switching term* τ_{sw} ensures convergence to the sliding surface
- The *stabilization term* τ_c provides additional robustness

The equivalent control is designed by assuming ideal sliding conditions ($S(t) = 0$ and $\dot{S}(t) = 0$), making the system nominally insensitive to uncertainties and disturbances. The switching term compensates for deviations from these ideal conditions, while the stabilization term enhances transient performance.

The time derivative of the sliding surface:

$$\dot{s} = \dot{e} + k_1\gamma|e|^{\gamma-1}\dot{e} + k_2\beta|\dot{e}|^{\beta-1}\ddot{e} \tag{10}$$

The equivalent control τ_{eq} is obtained by enforcing the sliding condition $\dot{s} = 0$ under nominal dynamics ($\Delta_u = 0$). Starting from the time derivative of the sliding surface:

$$\begin{aligned}\dot{s} &= \dot{e} + k_1\gamma|e|^{\gamma-1}\dot{e} + k_2\beta|\dot{e}|^{\beta-1}\ddot{e} = 0 \\ &= \dot{e}\left(1 + k_1\gamma|e|^{\gamma-1}\right) + k_2\beta|\dot{e}|^{\beta-1}\left(F(q,\dot{q}) + B(q)\tau_{eq} - \ddot{q}_d\right) = 0\end{aligned} \tag{11}$$

Solving (11) for $B(q)\tau_{eq}$ yields:

$$\begin{aligned}B(q)\tau_{eq} &= \ddot{q}_d - F(q,\dot{q}) - \frac{\dot{e}\left(1 + k_1\gamma|e|^{\gamma-1}\right)}{k_2\beta|\dot{e}|^{\beta-1}} \\ &= \ddot{q}_d - F(q,\dot{q}) - \frac{|\dot{e}|\text{sign}_j(\dot{e})\left(1 + k_1\gamma|e|^{\gamma-1}\right)}{k_2\beta|\dot{e}|^{\beta-1}} \\ &= \ddot{q}_d - F(q,\dot{q}) - \frac{1}{k_2\beta}|\dot{e}|^{2-\beta}\left(1 + k_1\gamma|e|^{\gamma-1}\right)\text{sign}_j(\dot{e})\end{aligned} \tag{12}$$

The final equivalent control law is then:

$$\tau_{eq} = B^{-1}(q)\left[\ddot{q}_d - F(q,\dot{q}) - \frac{1}{k_2\beta}|\dot{e}|^{2-\beta}\left(1 + k_1\gamma|e|^{\gamma-1}\right)\text{sign}_j(\dot{e})\right] \tag{13}$$

The control design exhibits three key features: (i) the term $|\dot{e}|^{2-\beta}$ ensures singularity-free behavior when $\dot{e} \to 0$ (since $\beta > 1$), (ii) the gain $k_2\beta$ modulates the switching intensity near the sliding surface, and (iii) the inverse $B^{-1}(q)$ properly accounts for the system's actuation topology.

Our next objective is to derive the switching control term τ_{sw} that ensures robust convergence to the sliding surface $s = 0$ despite uncertainties Δ_u and disturbances.

Consider the Lyapunov function candidate:

$$V = \frac{1}{2}s^2 \tag{14}$$

Under uncertain dynamics, the time derivative becomes:

$$\begin{aligned}\dot{V} &= s\dot{s} \\ &= s\left(\beta k_2|\dot{e}|^{\beta-1}[B(q)\tau_{sw} + \Delta_u]\right)\end{aligned} \tag{15}$$

To guarantee $\dot{V} \leq 0$, we propose:

$$\tau_{sw} = -B^{-1}(q)\left[(\rho + \delta)\text{sign}_j(s)\right], \quad \rho > \|\Delta_u\|, \delta > 0 \tag{16}$$

where:

- $\rho > \|\Delta_u\|$ bounds the combined uncertainties
- $\delta > 0$ provides convergence margin
- $\text{sign}_j(s)$ denotes the smooth signum approximation

Substituting (16) into (15) yields the stability condition:

$$\begin{aligned}\dot{V} &= \beta k_2|\dot{e}|^{\beta-1}\left[s\Delta_u - (\rho + \delta)|s|\right] \\ &\leq \beta k_2|\dot{e}|^{\beta-1}\left[\|\Delta_u\||s| - \rho|s| - \delta|s|\right] \\ &\leq -\delta\beta k_2|\dot{e}|^{\beta-1}|s| < 0 \quad \forall s \neq 0\end{aligned}$$

where $\beta k_2|\dot{e}|^{\beta-1}$ provides velocity-dependent gain adaptation. The design ensures finite-time convergence while maintaining smooth control action through $\text{sign}_j(s)$.

Having determined the equivalent control τ_{eq} and switching term τ_{sw}, we now introduce the compensation term τ_c to enhance closed-loop performance. This additional component:

$$\tau_c = -B^{-1}(q)Ks \quad (K > 0) \tag{17}$$

The compensation term τ_c delivers three synergistic benefits: (i) exponential convergence through the linear s-feedback, (ii) enhanced stability via the $\dot{V} \leq -Ks^2$ guarantee, and (iii) improved transient dynamics while preserving the sliding condition $s = 0$. These properties collectively boost the controller's robustness and responsiveness.

The complete control law $\tau = \tau_{eq} + \tau_{sw} + \tau_c$ now satisfies:

$$\dot{V} \leq -\underbrace{\delta\beta k_2|\dot{e}|^{\beta-1}|s|}_{\text{Finite-time convergence}} - \underbrace{Ks^2}_{\text{Exponential stabilization}} < 0$$

III. Finite-Time Stability Analysis

Theorem 1: If a Lyapunov function $V(t)$ satisfies:

$$\dot{V}(t) \leq -\sigma_1 V(t) - \sigma_2 V^{\eta}(t), \quad \forall t \geq t_0 \tag{18}$$

where $\sigma_1, \sigma_2 > 0$ and $0 < \eta < 1$, then $V(t)$ converges to zero in finite time with convergence time bounded by:

$$T_0 \leq \frac{1}{\sigma_1(1-\eta)} \ln \frac{\sigma_1 V^{1-\eta}(x_0) + \sigma_2}{\sigma_2} \tag{19}$$

Proof 1: Consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2}s^2 \tag{20}$$

The time derivative yields:

$$\begin{aligned} \dot{V}_1 &= s\dot{s} \\ &= s\left(k_2\beta|\dot{e}|^{\beta-1}[\Delta_u(q,\dot{q},t) - (\rho+\delta)\text{sign}_j(s) - Ks]\right) \\ &= k_2\beta|\dot{e}|^{\beta-1}\left[\Delta_u(q,\dot{q},t)s - (\rho+\delta)|s| - Ks^2\right] \end{aligned} \tag{21}$$

By selecting the robustness gain ρ such that:

$$\dot{V}_1 \leq k_2\beta|\dot{e}|^{\beta-1}\left[-Ks^2 - \delta|s|\right] \leq 0 \tag{22}$$

Rewriting in terms of V_1:

$$\dot{V}_1 \leq -c_1 V_1 - c_2 V_1^{1/2} \tag{23}$$

where:

$$c_1 = 2Kk_2\beta|\dot{e}|^{\beta-1} > 0$$
$$c_2 = k_2\delta\beta|\dot{e}|^{\beta-1}\sqrt{2} > 0$$

The finite convergence time t_r satisfies:

$$t_r \leq \frac{2}{c_1} \ln\left(\frac{c_1 V_1^{1/2}(0) + c_2}{c_2}\right) \tag{24}$$

proving finite-time convergence of $s(t)$ to zero. Consequently, the tracking error $e(t)$ also converges to zero in finite time.

IV. Simulation and Results

This section presents the simulation results to validate the effectiveness of the proposed Non-Singular Fast Terminal Sliding Mode Control (NFTSMC) scheme. Its performance is rigorously compared against a Conventional Sliding Mode Control (SMC) and a Terminal Sliding Mode Control (TSMC) approach.

A. Experimental Setup

The proposed NFTSMC approach is validated on a 2-DOF robotic manipulator. The physical parameters of the manipulator, as used in the simulation, are provided in Table I. These parameters define the inertia, Coriolis, and gravitational dynamics of the robot.

The desired trajectories for joint angles are designed to test both slow and fast motions, ensuring a comprehensive evaluation of tracking performance:

$$q_d(t) = \begin{bmatrix} \frac{\pi}{4} + \frac{\pi}{6}\sin(0.2\pi t) \\ -\frac{\pi}{3} + \frac{\pi}{3}\cos(0.2\pi t) \end{bmatrix} \text{ rad} \tag{25}$$

The initial conditions for the simulation were set to $q_0 = [\pi/3, \pi/3]^T$ rad and $\dot{q}_0 = [0, 0]^T$ rad/s. The simulation duration was 20 seconds.

TABLE I
Robot Manipulator Physical Parameters

Parameter	Value	Unit
m_1	6	kg
m_2	4	kg
l_1	0.5	m
l_2	0.4	m
g	9.81	m/s^2

B. Controller Parameters

The parameters for each controller were carefully tuned to achieve optimal performance while ensuring stability and practicality.

1) Proposed NFTSMC Parameters: The parameters for the proposed NFTSMC are:

- Sliding surface gains: $k_1 = \text{diag}(5, 5)$, $k_2 = \text{diag}(0.8, 0.8)$
- Exponential parameters: $\gamma = 1.5$, $\beta = 1.2$
- Robustness gains: $\rho = \text{diag}(0.7, 0.7)$, $\delta = \text{diag}(0.1, 0.1)$
- Stabilization gain: $K = \text{diag}(0.2, 0.2)$
- Smooth sign function order: $j = 2$ (for $\text{sign}_j(\cdot)$)

The nominal parameters used for dynamics compensation are taken as the true robot parameters, with uncertainties handled by the robust terms.

2) Conventional SMC Parameters: The parameters for the Conventional SMC are:

- Sliding surface gain, $\lambda_{smc} = \text{diag}(1, 1)$
- Switching gain matrix, $K_{smc} = \text{diag}(10, 10)$
- Boundary layer thickness, $\phi_{smc} = 0.006$

3) TSMC Parameters: The parameters for the TSMC are:

- Sliding surface gain, $\alpha_{tsmc} = \text{diag}(1.45, 1.45)$
- Sliding surface power, $\beta_{tsmc} = 0.7$
- Switching gain matrix, $K_{tsmc} = \text{diag}(180, 180)$
- Boundary layer thickness, $\phi_{tsmc} = 0.004$

C. Performance Evaluation

The performance of the three controllers is evaluated based on their trajectory tracking accuracy and the magnitude of their control efforts. Two key metrics are used: the Integral of Absolute Error (IAE) for tracking performance and the Integral of Squared Control Signal (Control Effort) for assessing energy consumption and chattering.

TABLE II
Controller Performance Metrics

Controller	IAE (Joint 1)	IAE (Joint 2)	Control Effort (u1)	Control Effort (u2)
NFTSMC (Proposed)	0.094	0.377	15340.471	395.133
TSMC	0.108	0.565	15624.659	3238.157
Conventional SMC	0.327	1.079	14719.537	371.557

1) Tracking Performance (IAE): From Table II, it is observed that:

- For Joint 1, the proposed NFTSMC achieves the lowest IAE (0.094), indicating superior tracking accuracy, closely followed by TSMC (0.108). Conventional SMC exhibits significantly higher error (0.327).

- For Joint 2, the proposed NFTSMC demonstrates the best tracking performance with the lowest IAE (0.377), outperforming both TSMC (0.565) and Conventional SMC (1.079).

Overall, both the proposed NFTSMC and TSMC significantly outperform the Conventional SMC in terms of tracking accuracy. The NFTSMC shows a more balanced performance across both joints.

2) *Control Effort:* Analyzing the control effort metrics from Table II:

- For Joint 1 (u1), the control efforts are comparable across all three controllers, with Conventional SMC showing a slightly lower value (14719.537) compared to NFTSMC (15340.471) and TSMC (15624.659).
- For Joint 2 (u2), there is a notable difference. Conventional SMC requires the least effort (371.557), followed very closely by the proposed NFTSMC (395.133). In contrast, TSMC demands a significantly higher control effort (3238.157), approximately eight times greater than the NFTSMC.

These results highlight that the proposed NFTSMC maintains a high level of tracking accuracy while achieving a control effort comparable to the Conventional SMC, especially for Joint 2. This suggests better energy efficiency and potentially less wear and tear on actuators compared to the TSMC.

D. Simulation Plots

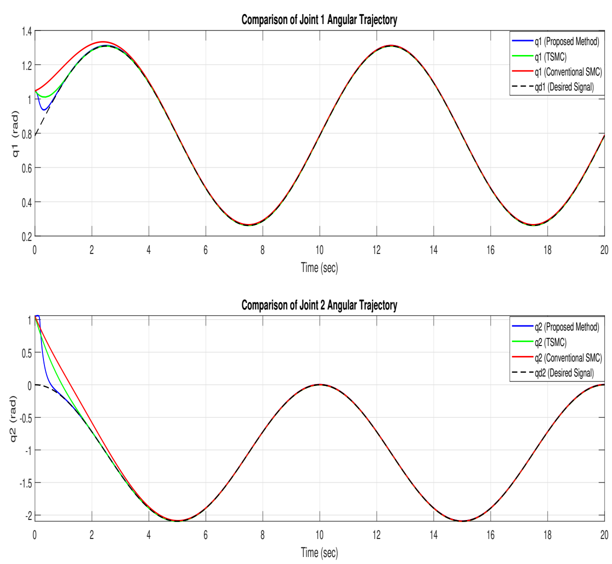

Fig. 1. Comparison of Joint Angular Trajectories: (a) Joint 1, (b) Joint 2.

E. Discussion of Results

The simulation results clearly demonstrate the superior performance of the proposed NFTSMC. While TSMC shows slightly better performance for Joint 1's tracking, the NFTSMC offers a more balanced and generally better performance across both joints, especially for Joint 2. Furthermore, the

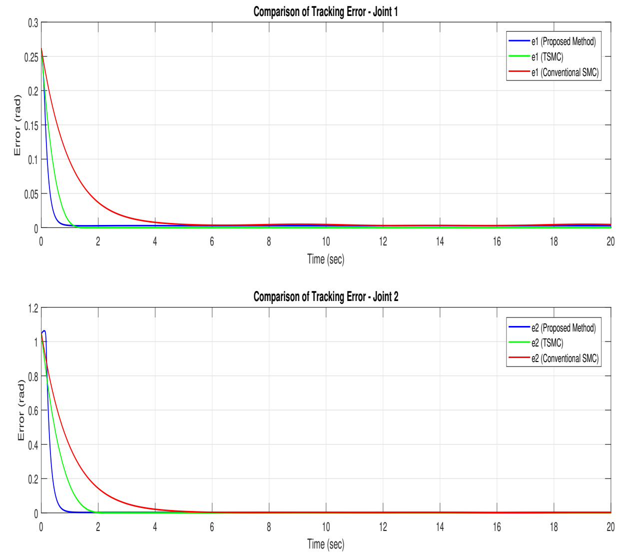

Fig. 2. Comparison of Tracking Errors (Absolute Value): (a) Joint 1 Error, (b) Joint 2 Error.

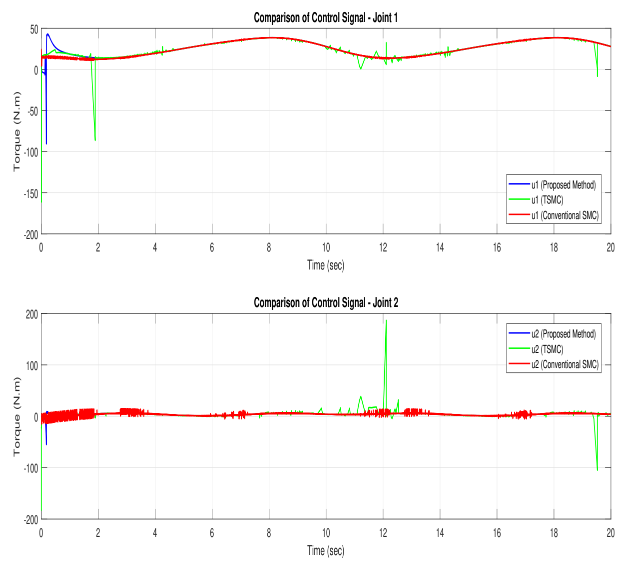

Fig. 3. Comparison of Control Signals: (a) Joint 1 Torque, (b) Joint 2 Torque.

NFTSMC significantly reduces the control effort compared to TSMC, particularly for Joint 2, which is crucial for practical implementation by minimizing actuator wear and energy consumption. The smoothness of the control signals generated by the NFTSMC, as observed in Figure 3, confirms its chattering-free nature, a significant advantage over conventional SMC.

V. Conclusion

This paper introduced a novel Non-Singular Fast Terminal Sliding Mode Control (NFTSMC) for 2-DOF robotic manipulators, utilizing a smooth scalar sign function. The controller ensures singularity avoidance, chattering elimination, and finite-time convergence despite uncertainties. Lyapunov analysis confirmed its stability. Simulations showed the proposed NFTSMC achieves competitive tracking accuracy with reduced control effort compared to traditional methods, highlighting its efficiency and smoothness. This approach is a strong candidate for real-time robotic applications demanding high precision and robustness.

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