A weighted approach for Bearing-Only Tracking of underwater acoustic sources with unbalanced measurements

Tony Punnoose Valayil, Paolo Di Lillo, Gianluca Antonelli

***Abstract*— This paper addresses the Bearing-Only Tracking problem of an underwater acoustic source using multiple Autonomous Underwater Vehicles. The vehicles, equipped with Passive Acoustic Monitoring sensors, must communicate to exchange their local target Direction of Arrival estimates through acoustic communication systems, which are typically characterized by low bandwidth and high latencies. The different availability of local measurements compared to those coming from the other vehicles in the team can create situations where one vehicle is forced to generate an estimate based on measurements that are heavily unbalanced toward one source. This negatively impacts the numerical balance of the regressor matrix used for the estimation, potentially leading to biased results. This paper proposes a weighting method to mitigate this effect, validating it through numerical simulations.**

I. INTRODUCTION

Currently, several automated data acquisition devices are deployed in marine environments to protect ports and marine infrastructures due to increasing threats to civilian targets. The underlying assumption is that sensor data gathered from autonomous underwater vehicles will continue to develop into our navy's most valuable information resource.

Localizing underwater acoustic sources is important for a number of applications, including search and rescue, monitoring and patrolling [1]. Most current underwater acoustic systems for source localization rely on PAM (Passive Acoustic Monitoring) sensors, which give the source's Direction of Arrival (DoA) but cannot measure distance [2], [3]. The process of estimating the position of the acoustic source from measurements of its DoA is known in literature as Bearing-Only Tracking (BOT). In situations characterized by comparable motion capabilities between the target source and the AUV, it is possible to estimate the acoustic source position also by employing only one vehicle [4]. In case the target can move at a much higher velocity with respect to the moving sensors, it is instead necessary to resort to a number of AUV and merge the local information to get a consistent estimate of the target position [5]. It is worth noticing that, also in this case, concerns about the observability of the system can be raised in particular configurations, e.g. when the target and the AUVs are aligned [6].

Several approaches have been studied in the literature for solving the bearing-only tracking problem employing multiple sensors. One of the most significant aspects to take into account when designing estimation algorithms in this domain, is that the information transmitted on the acoustic network is subject to varied delays and potential packet loss, which affects the convergence of any iterative algorithm [7]. For instance, a team of AUVs is proposed in [8] and [9]. However, the challenges presented by communication latency and packet loss are overlooked. In [10], a batch solution is introduced, which similarly shares the limitations noted in the preceding references. Furthermore, in [11], multiple sensors are utilized to monitor a moving target and integrate their estimates. The experimental validation is executed by deploying acoustic sensors within a controlled environment, thus eliminating any latency and packet loss. In [12], a BOT problem is tackled by introducing a particle filter that considers both latency and the potential for non-synchronous data reception.

In [13], the estimation problem is handled by constructing an algorithm that can analyze data coming from various AUVs with a nonuniform time sampling, which has been then extended to estimate and compensate for the sound propagation delay in [14]. The approach is based on a pseudolinear formulation, in which a proper regressor matrix is built based on the exchange of the local DoA through the acoustic communication channel and finally (pseudo)-inverted to generate the target position estimate. In this way, the estimation is robust with respect to communication latency and low bandwidth, demonstrating to be suitable for underwater environments. When applying this algorithm, particular care must be devoted to the data used for the construction of the regressor matrix, which gathers DoA estimates generated locally with those generated by the other AUVs composing the team. One of the problems that can arise is when one AUV owns noticeably more local measurements than those coming from the other ones, even if the motion can be properly optimized to gather optimal data [15]. This is particularly common in underwater environment, in which the communication frequency is significantly lower compared with the generation frequency of DoA estimation algorithms. The result is that the regressor matrix used for the estimation may become numerically unbalanced, potentially producing inaccurate or biased estimates.

This paper investigates how to deal with the aforementioned situation. It is worth noting that discarding some of the incoming observations is not the best option because additional measurements coming from one of the AUVs help decrease the estimation uncertainty in that specific direction. The proposed approach is to appropriately weigh the measurements based on the sensor they originate from to reduce

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the bias in the resulting estimate. The proposed approach is validated with numerical simulations and compared with the standard version of the estimation algorithm.

The paper is organized as follows: in Section II, the needed mathematical background on the modeling of a BOT problem and on the baseline algorithm detailed in [13] is briefly reported; in Section III we describe the proposed weighting solution; in Section IV we show the obtained simulative results; finally, in Section V we report the conclusions and the future research directions.

II. Background

A. Modeling

Let us define as $\boldsymbol{p} = [p_x \; p_y]^{\mathrm{T}} \in \mathbb{R}^2$ the position of a still underwater target to be tracked with respect to an inertial frame and an AUV whose position is expressed by the vector $\boldsymbol{p}_s = [p_{s,x} \; p_{s,y}]^{\mathrm{T}} \in \mathbb{R}^2$, also expressed in an inertial frame. The bearing angle between the vehicle and the target can be defined as:

$$\beta = \text{atan2}(p_y - p_{s,y}, p_x - p_{s,x}) \; . \tag{1}$$

Let us assume that the AUV is equipped with a PAM sensor whose raw measurements are properly preprocessed and fed into a DoA estimation algorithm. For the sake of clarity, we will refer to the output of the DoA estimation algorithm as *measurement* for the rest of the paper. It can be expressed as:

$$y = \beta + \nu \; , \tag{2}$$

where ν is the noise modeled as a uniformly-distributed random variable with bounds $\pm\bar{\nu}$. This output equation may be rewritten in terms of the state as [16]:

$$y = \boldsymbol{C}(\boldsymbol{p}) \, \boldsymbol{p} + f(\boldsymbol{p}, \boldsymbol{p}_s, \nu) \; , \tag{3}$$

with output matrix:

$$\boldsymbol{C}(\boldsymbol{p}) = \begin{bmatrix} \sin\beta & -\cos\beta \end{bmatrix} \; , \tag{4}$$

and noise $f(\boldsymbol{p}, \boldsymbol{p}_s, \nu)$, which is dependent on the relative position between the AUV and the target. Additionally, it is useful to highlight that the measurements (ignoring the noise) may be rewritten as:

$$y = p_{s,x} \sin\beta - p_{s,y} \cos\beta \; . \tag{5}$$

Defining P time steps $t_1, t_2, \ldots, t_P$ and denoting as:

$$\boldsymbol{C}_i = \boldsymbol{C}(\boldsymbol{p}(t_i)) = \begin{bmatrix} \sin\beta(t_i) & -\cos\beta(t_i) \end{bmatrix} \quad i = 1, 2, \ldots, P$$

the output 1×2 matrix corresponding to the generic time step t_i, the measurements can be built in a regressor form by stacking them in a vector $\boldsymbol{y}$ and by stacking the relative output matrices $\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_P$ as:

$$\boldsymbol{y} = \begin{bmatrix} y(t_1) \\ y(t_2) \\ \vdots \\ y(t_P) \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_1 \\ \boldsymbol{C}_2 \\ \vdots \\ \boldsymbol{C}_P \end{bmatrix} \boldsymbol{p}(t_0) = \boldsymbol{\Phi} \, \boldsymbol{p}(\boldsymbol{t_0}) \tag{6}$$

where $\boldsymbol{\Phi} \in \mathbb{R}^{P \times 2}$ is the regressor matrix. It is worth noticing that, since $\boldsymbol{p}$ is defined in a common reference frame to all the AUVs, the generic element of $\boldsymbol{y}$ and the generic regressor's row $\boldsymbol{C}_i$ may also be constructed from information coming from other AUVs.

B. Estimation algorithm

Consider a team of N AUVs located in positions $\boldsymbol{p}_s^i = \begin{bmatrix} p_{s,x}^i \; p_{s,y}^i \end{bmatrix}^{\mathrm{T}}$ with $i = \{1, \ldots, N\}$, each equipped with a PAM sensor and a DoA estimation algorithm. In the proposed solution, each AUV uses its local DoA measurement to create one row of the regressor matrix, including its specific DoA estimation in the measurement vector. This data is then transmitted to the other AUVs in the team via an acoustic communication system, facilitating the sharing of all local measurements.

Let us define the measurement *owned* by AUV j and *generated* by AUV k at time t_i as $y_j^k(t_i)$. Similarly, let us define the 1-row-regressor corresponding to measurement $y_j^k(t_i)$ as $\boldsymbol{C}_{j,i}^k$. Let us assume that, at a certain time step t, the AUV labeled as j owns P_j measurements generated by any of the other AUVs composing the team and arranged such that $t_i < t_{i+1} \forall i$, obtaining the measurement vector:

$$\boldsymbol{y}_j = \begin{bmatrix} y_j^k(t_0) \\ y_j^k(t_1) \\ \vdots \\ y_j^k(t_{P_1}) \end{bmatrix} \qquad k \in \{1, ..., N\} \; . \tag{7}$$

Similarly, the AUV builds its regressor matrix as:

$$\boldsymbol{\Phi}_j = \begin{bmatrix} \boldsymbol{C}_{j,1}^k \\ \boldsymbol{C}_{j,2}^k \\ \vdots \\ \boldsymbol{C}_{j,P_j}^k \end{bmatrix} \qquad k \in \{1, ..., N\} \; . \tag{8}$$

Finally, Eq. (6) can be then particularized for AUV j as:

$$\boldsymbol{y}_j = \boldsymbol{\Phi}_j \boldsymbol{p}(t_0), \tag{9}$$

which is a linear system of equations that can be solved by finding the optimal solution (in a least square sense) minimizing the following objective function:

$$\min_{\boldsymbol{p}} \quad ||(\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{p})||^2 \; . \tag{10}$$

The solution, which represents the estimation of the target position at the past time instant t_0 generated by AUV j, can be computed as:

$$\hat{\boldsymbol{p}}_j(t_0) = \boldsymbol{\Phi}_j^{\dagger} \boldsymbol{y}_j \; , \tag{11}$$

where $\boldsymbol{\Phi}_j^{\dagger}$ is the pseudoinverse of the regressor matrix, defined as:

$$\boldsymbol{\Phi}_j^{\dagger} = (\boldsymbol{\Phi}_j^{\mathrm{T}} \boldsymbol{\Phi}_j)^{-1} \boldsymbol{\Phi}_j^{\mathrm{T}} \; . \tag{12}$$

Under the assumption of a still target, the estimate of the target position at the current time $\boldsymbol{p}(t)$ will be obviously equal to $\boldsymbol{p}(t_0)$.

It is worth pointing out that the aforementioned procedure constructs a regressor in which the rows consistently increase each time a new DoA estimation is either locally acquired or sourced from other AUVs. This leads to the clear drawback of increasing computational load and memory requirements. It is thus necessary to limit the number of rows of the regressor by eliminating the *oldest* sampling times. This is implemented by defining a maximum number of regressor rows P_{max} and each time a new measurement is received the rows of $\boldsymbol{\Phi}$ and the vector $\boldsymbol{y}$ are shifted accordingly.

III. Problem description and proposed solution

The problem that will be addressed in this paper is how to deal with the situation in which the output vector $\boldsymbol{y}$ (and the corresponding regressor matrix rows) is composed of a number of measurements that are highly unbalanced towards one of the AUVs. Typically, the AUV labeled as j will have much more DoA estimation coming from its local algorithm than those coming from the other AUVs. This is because underwater communication is significantly slower than typical DoA estimation algorithms.

Let us define the set of the measurements *owned* by AUV j and *generated* by AUV k present in the output vector $\boldsymbol{y}_j$ at a certain timestep as $\mathcal{Y}_j^k$. The percentage of measurements owned by sensor j and generated by sensor k is then:

$$D_j^k = \frac{|\mathcal{Y}_j^k|}{P_{\text{max}}} \cdot 100 \,, \tag{13}$$

where $|\mathcal{Y}_j^k|$ denotes the cardinality of the set, i.e. the number of its elements and P_{min} is the number of elements of $\boldsymbol{y}_j$, which is equal to the number of rows of $\boldsymbol{\Phi}_j$. When D_j^k is significantly greater than $\frac{1}{N}$, which occurs when there is a highly uneven distribution of the measurements among the N AUVs, the regressor matrix is numerically unbalanced and the estimate generated by employing Eq. (11) results inaccurate and biased toward AUV j. This is because if sensor j provides many more measurements compared with the other ones, the least squares solution in Eq. (11) will prioritize reducing the error for AUV j since it has more terms in the objective function in Eq. (10), at the expense of increasing the error for the measurements coming from the other AUVs. This results in an estimate that is biased toward AUV j, which will anyway minimize the total squared error. On the contrary, if all the sensors contribute equally, the least squares solution balances the errors across all the AUVs, leading to a more accurate estimate.

Figure 1 (top) graphically shows such a situation, taking into account only two AUVs. In detail, the figure shows the estimate obtained by AUV 1 in 100 iterations of the algorithm reporting with different colors the situations in which the percentage of measurements coming from AUV 2 with respect to the local ones are $D_1^2 = \{1\%, 10\%, 50\%\}$. The estimate obtained with $D_1^2 = 1\%$ is highly biased and inaccurate compared to that obtained with $D_1^2 = 50\%$, i.e. with a perfectly even distribution of the measurements between the two AUVs. It is also interesting to evaluate the correlation between the loss in accuracy and the bad conditioning of the regressor matrix, as it is well-known that the minimum singular value of the regressor matrix significantly affects the accuracy of the estimation. In this perspective, Fig. 1 (bottom) reports the average minimum singular value of the regressor matrices obtained in the three conducted simulations. It is clear that when the value of D_1^2 decreases, the same happens to the minimum singular value of the regressor matrix, as expected.

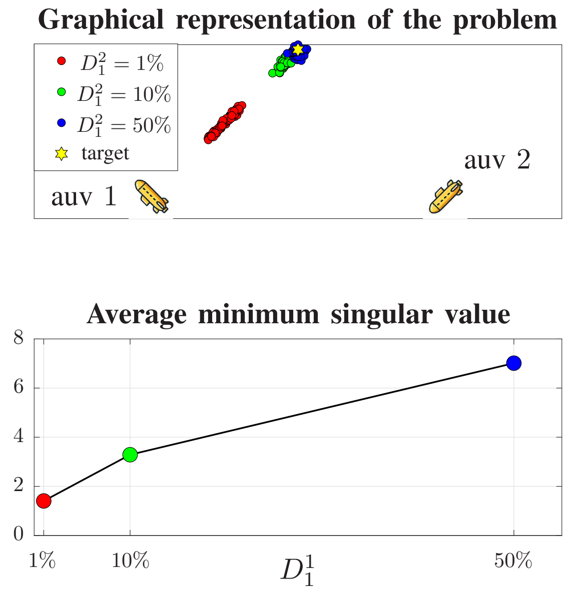


Fig. 1. Top: graphical representation of the problem. Estimate of the target position generated by auv 1 on different values of D_1^2. When the regressor matrix contains many more local DoA measurements than those generated by auv 2, the resulting target position estimate is biased toward auv 1. Bottom: average minimum singular value of the regressor matrix over D_1^2. If the regressor matrix contains many more local DoA measurements than those generated by auv 2, the singular value of the matrix decreases, motivating the loss of accuracy.

The problem is thus originated from the fact that the standard pseudinversion of the regressor matrix considers all the measurements equally in getting the estimate of the target position. We propose to solve the problem by weighting the rows of the regressor differently depending on the total number of available measurements generated by a certain AUV. This is achieved by changing the objective function to optimize as follows:

$$\min_{\boldsymbol{p}} \quad ||\boldsymbol{W}^{\frac{1}{2}}(\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{p})||^2 \,, \tag{14}$$

known as a weighted least squares problem. The solution can be computed by replacing the standard pseudoinverse employed in Eq. (11) to estimate the position of the target with a weighted pseudoinverse, as follows:

$$\boldsymbol{\Phi}_W^{\dagger} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{W}^{-1}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{W}^{-1} \,, \tag{15}$$

where:

$$\boldsymbol{W} = \begin{bmatrix} w_1^k, & 0 & \cdots & 0 \\ 0 & w_2^k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{P_{\text{max}}}^k \end{bmatrix} \qquad k \in \{1, ..., N\} \,, \tag{16}$$

is a diagonal matrix of weights to be properly chosen. It is worth noticing that each element w_i^k is referred to the corresponding i-th row of the regressor matric $\boldsymbol{C}_j^k$. We propose to compute the diagonal elements of the weight matrix $\boldsymbol{W}$ depending on the total number of measurements generated by the specific AUV in the regressor. More in detail, this can be achieved by computing the diagonal elements of the weighting matrix as:

$$w_i^k = \frac{N|\mathcal{Y}_j^k|}{P_{\text{max}}} \,. \tag{17}$$

It is worth noticing that in the case of a perfectly even distribution, all the $w_i^k = 1$, obtaining $\boldsymbol{W} = \boldsymbol{I}$, i.e., the estimate is obtained by employing the standard pseudoinverse. This weighting strategy helps mitigate the bias toward the AUVs that provide more measurements, modifying the objective function to optimize. Finally, all the required algorithmic steps are summarized in Algorithm 1.

Algorithm 1: Proposed estimation algorithm

```
Φ = O_{1×2}   y = 0_2   x̂ = 0_2   t = 0
while 1 do
    if MeasReceived ⇐ t_k, β(t_k), p_s(t_k) then
        y(t_k) = p_{i,x}(t_k) s_{β(t_k)} − p_{i,y}(t_k) c_{β(t_k)}
        C_k = [s_{β(t_k)}  −c_{β(t_k)}]
        if P < P_max then
            t = [t^T  t_k]^T
            y = [y^T  y(t_k)]^T
            Φ = [Φ^T  C_k^T]^T
            t_0 = min(t)
            P = P + 1
        else
            Shift(t), Shift(y), Shift(Φ)
            t = [t^T  t_k]^T
            y = [y^T  y(t_k)]^T
            Φ = [Φ^T  C_k^T]^T
            t_0 = min(t)
            ComputeW(Φ)
            x̂(t_0) = x̂(t) = (Φ^T W^{-1} Φ)^{-1} Φ^T W^{-1} y
            return x̂
        end
    end
end
```

IV. Numerical simulations

In this Section we show numerical simulations aimed at validating the proposed weighting method. To facilitate the analysis of the results, and without loss of generality, we simulate situations in which $N = 2$, i.e. there are only two AUVs available for estimating the target position. Table I reports all the parameters used for the following simulations, including the AUVs and target positions, the maximum number of regressor rows and the level of noise on the measurement of the DoA.

Parameter	Value	Description
P_{max}	100	number of regressor rows
$\bar{\nu}$	$\pm 10°$	noise on the DoA estimation
$\boldsymbol{p}_s^1$	$\begin{bmatrix}-200 & 0\end{bmatrix}^{\text{T}}$ m	auv1 position
$\boldsymbol{p}_s^2$	$\begin{bmatrix}200 & 0\end{bmatrix}^{\text{T}}$ m	auv2 position
$\boldsymbol{p}$	$\begin{bmatrix}0 & 200\end{bmatrix}^{\text{T}}$ m	target position

TABLE I
Parameters used in the reported simulations.

We performed 8 simulations, each one with 100 iterations, by changing the percentage of measurements coming from AUV 2 and comparing the estimate obtained by AUV 1 with the proposed weighted approach with the one obtained with the standard estimation. Figure 2 shows the average error norm obtained with increasing percentages of measurements from AUV 2 with the two algorithms.

It is worth noticing that with $D_1^2 = 50\%$ the error affecting the target position estimation with the two algorithms is the same, proving that when there is a perfectly even distribution between the measurements coming from the two AUVs the proposed weighting strategy computes $\boldsymbol{W} = \boldsymbol{I}$. As D_1^2 decreases, it is instead clear that the proposed approach significantly improves the accuracy of the estimation as the biasing phenomenon on the estimate obtained with the standard algorithm becomes more evident. This is particularly clear with $D_1^2 = 1\%$, which is associated to the regressor matrix with the lowest minimum singular value among the considered tests (see Fig. 1).

In Fig. 3 we report the details of some of the simulations, in particular the ones with $D_1^2 = \{1\%, 3\%, 5\%, 10\%\}$. The top plots report a graphical representation of the results, in which the actual target position is depicted as a yellow hexagram, the position estimation obtained with the standard algorithm is drawn in red and the one obtained with the proposed method in blue. The bottom plots report, instead, the norm of the position estimation error for all the iterations of the algorithms. It is evident that by employing the standard algorithm the position estimation is highly biased, shifting toward the AUV that provided more measurements, i.e. AUV 1. By employing the proposed algorithm, this bias in the estimation is significantly reduced, resulting in a lower error norm and, thus, in a more accurate estimate.

Finally, we tested the reliability of the results by changing the position of the two AUVs. In detail, we have performed 1000 simulations for each of the D_1^2 values already tested in the previous simulations, each one with the standard and the proposed method, changing the position of the AUVs on the x and y coordinates randomly by employing a uniform distribution between -300m and 300m, always keeping the target position at $p = [0\ 0]^{\text{T}}$m. The results are reported in

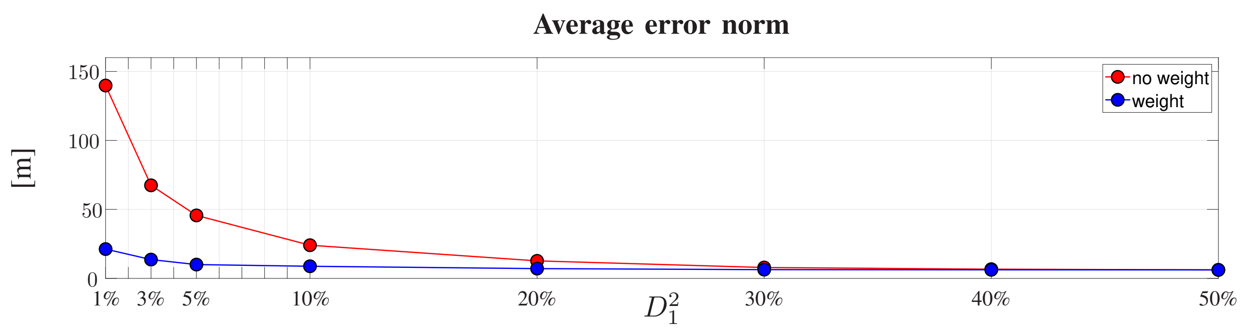
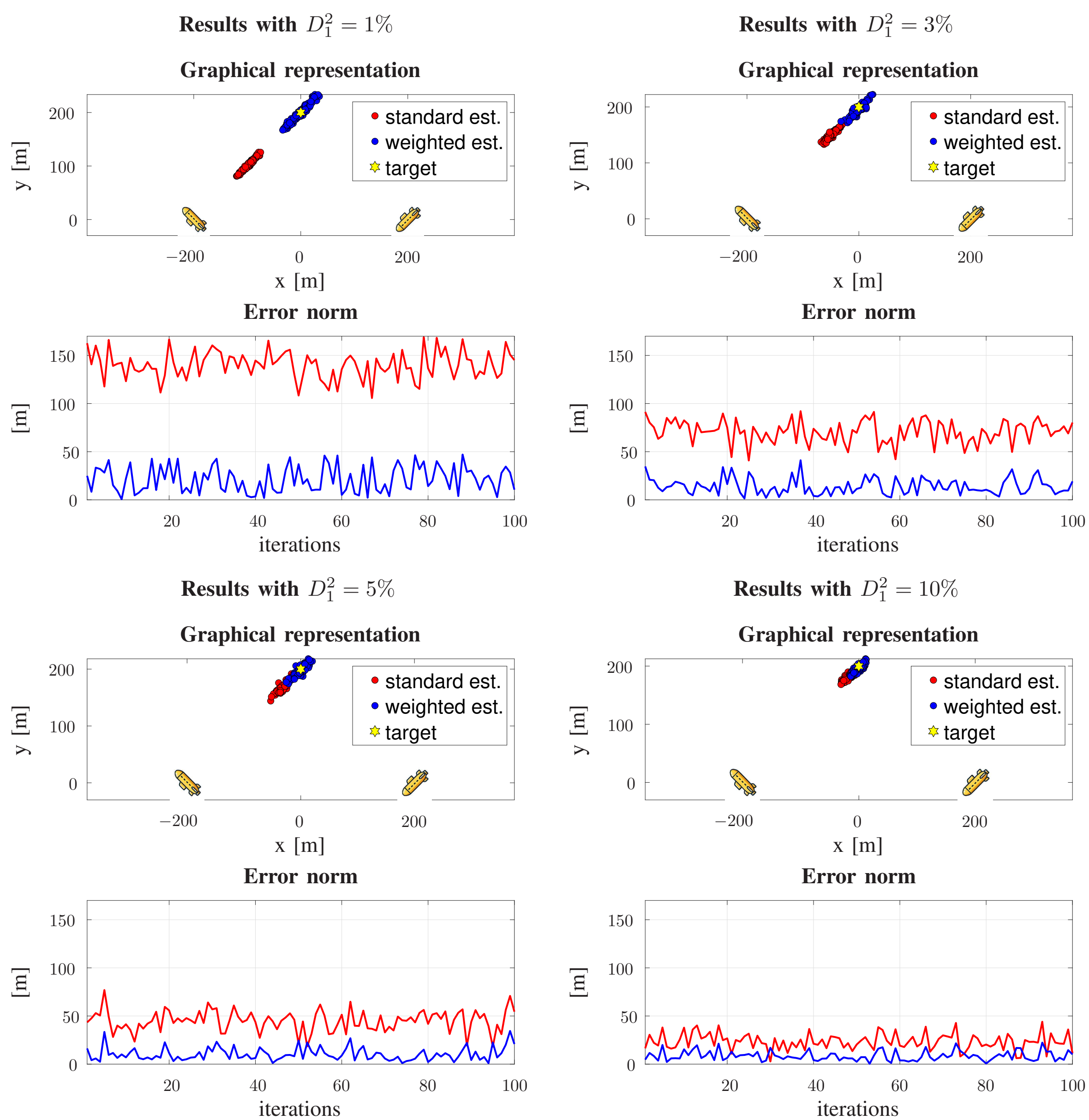


Fig. 2. Average position estimation error norm with $D_1^2 = \{1\%, 3\%, 5\%, 10\%20\%, 30\%, 40\%, 50\%\}$. With $D_1^2 = 50\%$ the measurements are evenly distributed and the proposed approach does not weight the pseudoinversion of the regressor matrix, obtaining the same results of the standard estimation. The difference between the two algorithms gets larger as D_1^2 decreases.



c) 5

d) 10

Fig. 3. Details on the results obtained with the simulations with $D_1^2 = 1\%$, $D_1^2 = 3\%$ $D_1^2 = 5\%$ and $D_1^2 = 10\%$. Top: graphical representation of the results. Bottom: norm of the position estimation error for 100 iterations. In both cases the proposed solution has a significantly lower error with respect to the standard estimation.

Fig. 4, which shows a box plot summarizing the average position error norm for all the conducted simulations. It is clear that the advantages of the proposed approach over the standard one remain consistent also with different relative configurations between the AUVs and the target. It is worth noticing that singular configurations, i.e. with the two AUVs aligned with the target, have been intentionally discarded from the analysis, as the results would have been unavoidably poor [17].

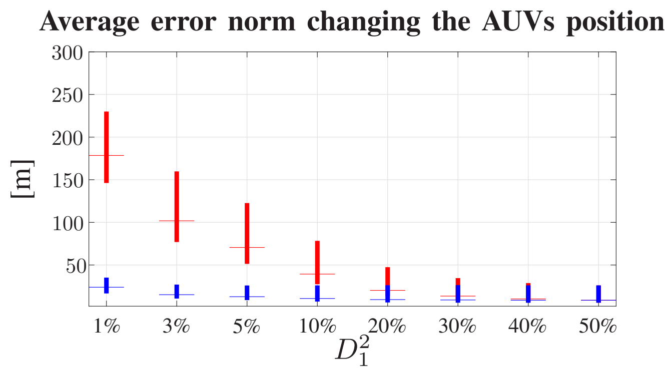


Fig. 4. Results obtained on 1000 simulations by varying the AUVs positions. The advantages of the proposed approach remain consistent also changing the AUVs positions.

V. Conclusions

In this paper, we addressed the situation in which the measurements used for generating the estimate of an acoustic source position rely on heavily unbalanced measurements towards one of the AUVs composing the multi-robot system. The proposed approach is based on the definition of a proper weighting matrix aimed at considering a different number of measurements depending on the sensor they come from and on the total number of measurements sourced by a specific sensor. Finally, numerical simulations show the effectiveness of the proposed method in mitigating the negative effect of the numerical unbalance of the regressor matrix, resulting in a more accurate estimate.

Future efforts will be devoted to two main research directions: i) including other considerations when computing the weights, taking into account the quality of the specific measurement rather than only the quantity; and ii) extending the estimation algorithm to the case in which multiple unknown targets are to be tracked.

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