Equivalent Diagonal Mutual Coupling Matrices for Narrowband ULA Beamformers

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***Abstract*—This work investigates the impact of mutual coupling among antennas in a standard uniform linear array (ULA). Previous research has focused on the mutual coupling matrix (MCM), typically non-diagonal, its effect on the decoupled isotropic matrix, and methods to mitigate its adverse effects. The current investigation stems from the principle that a linear system combining amplified and delayed sinusoidal signals of the same frequency produces a single sinusoid. Utilizing complex weights that correspond to magnitudes and delays in the phasor domain, in addition to initial amplitudes and phases of each signal, allows the determination of the amplitude and phase of the resulting sine wave. This concept suggests using a diagonal matrix to represent the mutual coupling effects. We outline this approach, demonstrate how to derive an equivalent diagonal MCM for a ULA, interpret this matrix using circuit theory, and emphasize the main advantages of the proposed method. Simulation results support our hypothesis.**

***Index Terms*—Antenna arrays, ULA, DoA, beamforming, mutual coupling effects, mutual coupling matrix.**

I. Introduction

Advancements in signal quality, spatial resolution, and system capacity have made antenna arrays essential for modern technologies like 5G networks, radar systems, and satellite communications [1]. These arrays offer significant benefits in beamforming, direction finding, and interference reduction, addressing the increasing demand for high-speed and reliable wireless communication. However, mutual coupling effects can significantly degrade the performance of antenna arrays [2], [3]. This issue arises when induced currents and electromagnetic fields from one antenna element adversely affect neighboring elements, leading to inaccurate beamforming, compromised direction finding, and reduced signal-to-interference-plus-noise ratio (SINR) [2]–[8].

Understanding and mitigating mutual coupling is crucial for enhancing antenna array performance in advanced signal processing applications [2]–[6]. This area of research, at the intersection of antenna and signal processing technologies, promises substantial improvements [2], [4], [6], [7]. Over the years, extensive research has been conducted on this issue, leading to various mitigation methods. The literature includes theoretical analyses, models of mutual coupling phenomena, and practical solutions to compensate for their effects. Comprehensive insights into the underlying electromagnetic interactions between array elements are discussed in [9]–[17], while practical decoupling networks [18]–[21] and compensation algorithms [22]–[27] have been explored. Additionally, works such as those in [3], [22], [24]–[33] address mutual coupling compensation techniques in adaptive beamforming to enhance array performance.

Some contributions have developed robust algorithms that account for mutual coupling effects, thereby enhancing direction finding and beamforming accuracy [22], [25], [28]. More recent advancements include the application of machine learning and optimization techniques for dynamic mutual coupling compensation, as demonstrated in [24], [25], [28]. Notably, investigations into deep learning models for real-time mutual coupling prediction and mitigation highlight the potential of AI-driven approaches in antenna array design and operation [34], [35]. These studies underscore the ongoing efforts to address mutual coupling, emphasizing the need and opening new opportunities for continued research to improve antenna array performance and reliability.

This work assumes that a weighted sum of sinusoids with the same frequency always results in a single sinusoidal signal with a given amplitude and phase. We claim and demonstrate that an equivalent diagonal matrix can represent the MCM from this basic concept. By providing an interpretation for this equivalent diagonal matrix, this paper brings insights that might be useful for a better understanding of the effects of mutual coupling. Another essential contribution of this work is the potential of the proposed equivalent diagonal MCM to reduce the computational complexity in applications involving matrix-vector multiplications while decreasing memory requirements for storage.

The rest of this paper is as follows. Section II covers the fundamentals of array signal processing and outlines the snapshot vector modeling. Section III addresses the rationale behind the proposed method, which is then developed in Section IV while exploring the use of circuit theory tools to interpret the proposed result. Section V presents the experimental results, and Section VI concludes this work.

II. Fundamentals of Array Signal Processing

The diagram in Fig. 1 depicts the snapshot vector of an M-antenna ULA. A single narrowband signal received in the

m-th antenna, assuming an isotropic radiation pattern with no mutual coupling, is given as

$$x_m(t) = s(t)\cos(\Omega_o(t-\Delta t_m)) + n_m(t), \tag{1}$$

where $s(t)$ is the modulating signal and $\Omega_o = 2\pi f_o$, with f_o being the operating frequency. Quantities Δt_m and $n_m(t)$ are the delay to the first antenna and the additive noise of the m-th antenna, respectively. The m-th receiver converts the incoming signal frequency f_o to a much lower intermediate frequency f_I to ensure adequate signal sampling. Following this, the m-th channel's discrete-time domain signal passes through an *analytical signal unit* (ASU) that eliminates the negative part of its spectrum. The resulting analytic signal is represented as

$$x_m(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k}\mathrm{e}^{-\mathrm{j}\Omega_o\Delta t_m} + n_m(k), \tag{2}$$

where $\omega_I = 2\pi f_I/f_s$, with $n_m(k)$ being the discrete-time domain analytic noise and k the discrete time index related to the continuous time index t through $k = tf_s$, where f_s is the sampling frequency. The delay information Δt_m, assumed a fraction of the wavelength divided by c, is still present in this signal, and is such that $s(t-\Delta t_m) \approx s(t)$, $\Delta t_m << \frac{\lambda}{c} = 1/f_o$.

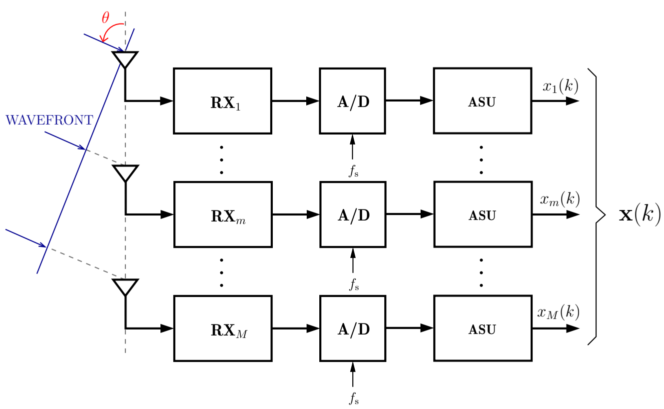

Fig. 1. The snapshot vector $\mathbf{x}(k)$ of an M-antenna ULA, obtained from sampled signals after passing through an analytical signal unit. All receivers are assumed to be synchronized.

The snapshot vector $\mathbf{x}(k) = [x_1(k)\cdots x_m(k)\cdots x_M(k)]^{\mathrm{T}}$ is represented as

$$\mathbf{x}(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k}\mathbf{a}_{\text{ISOTROPIC}}(\theta) + \mathbf{n}(k), \tag{3}$$

where the array manifold vector (AMV) for a ULA with half-wavelength spaced isotropic elements is given as

$$\mathbf{a}_{\text{ISOTROPIC}}(\theta) = \begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{j}\pi\cos(\theta)} \\ \vdots \\ \mathrm{e}^{-\mathrm{j}\pi(M-1)\cos(\theta)} \end{bmatrix}. \tag{4}$$

When mutual coupling distorts the signals in the antenna array, the snapshot model becomes

$$\mathbf{x}(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k}\mathbf{a}(\theta) + \mathbf{n}(k), \tag{5}$$

where the AMV is now represented as $\mathbf{a}(\theta) = \mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta)$; the multiplication by matrix $\mathbf{C}$, the MCM, accounts for the mutual coupling effects and antenna features. Although the MCM usually depends on the angle of arrival θ, this dependency is omitted here for simplicity of notation.

The literature defines the mutual coupling matrix in various forms [36] to [38]. The current study adopts the definition from [36], specifically

$$\mathbf{a}(\theta) = \underbrace{\mathbf{M}\begin{bmatrix} g_{\mathrm{OC},1}(\theta) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & g_{\mathrm{OC},M}(\theta) \end{bmatrix}}_{\mathbf{C}}\mathbf{a}_{\text{ISOTROPIC}}(\theta), \tag{6}$$

where $\mathbf{M} = \mathbf{Z}_{\mathrm{L}}(\mathbf{Z}+\mathbf{Z}_{\mathrm{L}})^{-1}$ is a direction-independent matrix obtained from the load and mutual impedance matrices as defined later in Section IV. The m-th element of the diagonal matrix $g_{\mathrm{OC},m}$ represents the radiation pattern of the m-th antenna when all other antennas are in an open-circuit condition.

III. Rationale Behind the Proposed Method

Consider a system composed of four linear, time-invariant components—A, B, C, and D—as depicted in Fig. 2. Each component introduces a gain α_i and a delay β_i at frequency Ω_0, with i ranging from A to D. The system processes two sinusoidal inputs: $s_1(t) = \cos(\Omega_o(t-\Delta t_1))$ and $s_2(t) = \cos(\Omega_o(t-\Delta t_2))$. The phasor representation for each component is $\alpha_i\mathrm{e}^{-\mathrm{j}\Omega_0\beta_i}$.

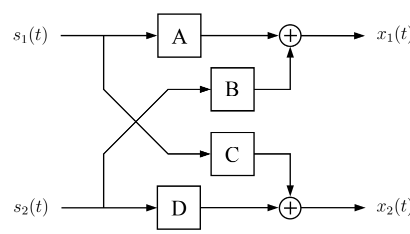

Fig. 2. A linear time-invariant system with two output signals as linear combinations of the two input signals.

Consequently, the output signals $x_1(t)$ and $x_2(t)$ of the system can be expressed as:

$$\begin{cases} x_1(t) = \alpha_{\mathrm{A}}\cos(\Omega_o(t-\Delta t_1-\beta_{\mathrm{A}})) + \alpha_{\mathrm{B}}\cos(\Omega_o(t-\Delta t_2-\beta_{\mathrm{B}})); \\ x_2(t) = \alpha_{\mathrm{C}}\cos(\Omega_o(t-\Delta t_1-\beta_{\mathrm{C}})) + \alpha_{\mathrm{D}}\cos(\Omega_o(t-\Delta t_2-\beta_{\mathrm{D}})). \end{cases} \tag{7}$$

Since $\cos(\theta) = \mathfrak{Re}\left\{\mathrm{e}^{\mathrm{j}\theta}\right\}$, (7) can be written as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathfrak{Re}\left\{\mathrm{e}^{\mathrm{j}\Omega_o t}\underbrace{\begin{bmatrix} \alpha_A\mathrm{e}^{-\mathrm{j}\Omega_o\beta_A} & \alpha_B\mathrm{e}^{-\mathrm{j}\Omega_o\beta_B} \\ \alpha_C\mathrm{e}^{-\mathrm{j}\Omega_o\beta_C} & \alpha_D\mathrm{e}^{-\mathrm{j}\Omega_o\beta_D} \end{bmatrix}}_{\mathbf{Z}}\underbrace{\begin{bmatrix} \mathrm{e}^{-\mathrm{j}\Omega_o\Delta t_1} \\ \mathrm{e}^{-\mathrm{j}\Omega_o\Delta t_2} \end{bmatrix}}_{\mathbf{a}}\right\}. \tag{8}$$

By converting $x_1(t)$ and $x_2(t)$ to discrete-time signals with a sampling frequency f_{s}, their analytic forms (negative frequencies neglected) at time instant k are given by

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \mathrm{e}^{\mathrm{j}\omega_o k}\mathbf{Z}\mathbf{a}, \tag{9}$$

where $\omega_o = \frac{\Omega_o}{f_s}$. We observe that matrix $\mathbf{Z}$, as defined in (8), is non-diagonal. Moreover, the signals in (7), which are weighted sums of sinusoids at the same frequency, can be expressed as cosines with varying amplitudes and phases:

$$\begin{cases} x_1(t) = A_1 \cos(\Omega_o t + \Theta_1) = \Re\mathfrak{e}\left\{ \mathrm{e}^{\mathrm{j}\Omega_o t} A_1 \mathrm{e}^{\mathrm{j}\Theta_1} \right\} \\ x_2(t) = A_2 \cos(\Omega_o t + \Theta_2) = \Re\mathfrak{e}\left\{ \mathrm{e}^{\mathrm{j}\Omega_o t} A_2 \mathrm{e}^{\mathrm{j}\Theta_2} \right\} \end{cases} \quad (10)$$

Hence, from (8), we obtain:

$$\begin{aligned} A_1 &= ((\alpha_A \cos(\Omega_o(\Delta t_1 + \beta_A)) + \alpha_B \cos(\Omega_o(\Delta t_2 + \beta_B)))^2 + \cdots \\ &\quad + (\alpha_A \sin(\Omega_o(\Delta t_1 + \beta_A)) + \alpha_B \sin(\Omega_o(\Delta t_2 + \beta_B)))^2)^{\frac{1}{2}}; \\ A_2 &= ((\alpha_C \cos(\Omega_o(\Delta t_1 + \beta_C)) + \alpha_D \cos(\Omega_o(\Delta t_2 + \beta_D)))^2 + \cdots \\ &\quad + (\alpha_C \sin(\Omega_o(\Delta t_1 + \beta_C)) + \alpha_D \sin(\Omega_o(\Delta t_2 + \beta_D)))^2)^{\frac{1}{2}}; \\ \Theta_1 &= -\tan^{-1} \frac{\alpha_A \sin(\Omega_o(\Delta t_1 + \beta_A)) + \alpha_B \sin(\Omega_o(\Delta t_2 + \beta_B))}{\alpha_A \cos(\Omega_o(\Delta t_1 + \beta_A)) + \alpha_B \cos(\Omega_o(\Delta t_2 + \beta_B))}; \text{ and} \\ \Theta_2 &= -\tan^{-1} \frac{\alpha_C \sin(\Omega_o(\Delta t_1 + \beta_C)) + \alpha_D \sin(\Omega_o(\Delta t_2 + \beta_D))}{\alpha_C \cos(\Omega_o(\Delta t_1 + \beta_C)) + \alpha_D \cos(\Omega_o(\Delta t_2 + \beta_D))}. \end{aligned} \quad (11)$$

From (10), after sampling and converting the signals to analytic, we form vector

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \mathrm{e}^{\mathrm{j}\omega_o k} \underbrace{\begin{bmatrix} A_1 \mathrm{e}^{\mathrm{j}(\Theta_1 + \Omega_o \Delta t_1)} & 0 \\ 0 & A_2 \mathrm{e}^{\mathrm{j}(\Theta_2 + \Omega_o \Delta t_2)} \end{bmatrix}}_{\mathbf{Z}_\mathrm{D}} \underbrace{\begin{bmatrix} \mathrm{e}^{-\mathrm{j}\Omega_o \Delta t_1} \\ \mathrm{e}^{-\mathrm{j}\Omega_o \Delta t_2} \end{bmatrix}}_{\mathbf{a}}, \quad (12)$$

where the product of the last two factors on the right-hand side corresponds to the product $\mathbf{Za}$ on the right-hand side of (9), with matrix $\mathbf{Z}$ replaced with the diagonal matrix $\mathbf{Z}_\mathrm{D}$.

IV. The Proposed Equivalent Diagonal MCM

The mutual coupling matrix $\mathbf{C}$ in $\mathbf{a}(\theta) = \mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta)$ is typically non-diagonal. This matrix captures the effect of mutual coupling, where the m-th continuous-time signal influences the signals on the other antennas, which can be expressed as

$$x_m(t) = \Re\mathfrak{e}\left\{ s(t)\mathrm{e}^{\mathrm{j}\Omega_o t} \left[c_{m,1} \ \cdots \ c_{m,M}\right] \mathbf{a}_{\text{ISOTROPIC}}(\theta) \right\}, \quad (13)$$

In this context, $[c_{m,1} \ \cdots \ c_{m,M}]$ corresponds to the m-th row of $\mathbf{C}$, which includes the mutual coupling and the radiation pattern of each antenna. Based on the rationale discussed in Section III, the previous expression can be rewritten as

$$x_m(t) = A_m \cos(\Omega_o t - \Theta_m) + n_m(t). \quad (14)$$

A. Computing the Equivalent Diagonal MCM

The snapshot vector, after the signals pass through the ADC and the ASU units, can be written as

$$\mathbf{x}(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k} \begin{bmatrix} A_1 \mathrm{e}^{-\mathrm{j}\Theta_1} \\ \vdots \\ A_M \mathrm{e}^{-\mathrm{j}\Theta_M} \end{bmatrix} + \mathbf{n}(k). \quad (15)$$

Comparing the expression in (15) with the traditional MCM expression

$$\mathbf{x}(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k} \mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta) + \mathbf{n}(k), \quad (16)$$

we can rewrite (15) as

$$\mathbf{x}(k) = s(k)\mathrm{e}^{\mathrm{j}\omega_I k} \mathbf{C}_\mathrm{D} \underbrace{\begin{bmatrix} 1 \\ \vdots \\ \mathrm{e}^{-\mathrm{j}\pi(M-1)\cos(\theta)} \end{bmatrix}}_{\mathbf{a}_{\text{ISOTROPIC}}(\theta)} + \mathbf{n}(k), \quad (17)$$

with

$$\mathbf{C}_\mathrm{D} = \begin{bmatrix} A_1 \mathrm{e}^{-\mathrm{j}(\Theta_1 - 0)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_M \mathrm{e}^{-\mathrm{j}(\Theta_M - \pi(M-1)\cos(\theta)} \end{bmatrix} \quad (18)$$

being an equivalent diagonal mutual coupling matrix. The next logical step is to compute the element of matrix $\mathbf{C}_\mathrm{D}$. Deriving A_m and Θ_m from (13), for $1 \le m \le M$, would be more complex and result in a much more complicated expression compared to (11), which offers a straightforward closed-form solution for $M = 2$. However, given that we have the MCM in its non-diagonal form and that the relation $\mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta) = \mathbf{C}_\mathrm{D}\mathbf{a}_{\text{ISOTROPIC}}(\theta)$ holds, computing the diagonal matrix $\mathbf{C}_\mathrm{D}$ becomes significantly simpler as follows.

$$[\mathbf{C}_\mathrm{D}]_{i,i} = \frac{[\mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta)]_i}{[\mathbf{a}_{\text{ISOTROPIC}}(\theta)]_i}, \text{ for } i \text{ from 1 to } M. \quad (19)$$

B. The "Z" Approach

A multiport linear network is a common tool for analyzing and modeling mutual coupling in antenna arrays, as illustrated in Fig. 3. We focus on matrix $\mathbf{M}$, which is a factor of the MCM, as in

$$\mathbf{a}(\theta) = \mathbf{C}\mathbf{a}_{\text{ISOTROPIC}}(\theta) = \overbrace{\mathbf{M}\mathbf{G}(\theta)}^{\mathbf{C}} \underbrace{\mathbf{G}(\theta)\,\mathbf{a}_{\text{ISOTROPIC}}(\theta)}_{\mathbf{a}_\text{oc}(\theta)}, \quad (20)$$

with $\mathbf{G}(\theta)$ being a diagonal matrix including the radiation pattern of each antenna. Matrix $\mathbf{M}$ satisfies the relation $\mathbf{a}(\theta) = \mathbf{M}\mathbf{a}_\text{oc}(\theta)$ and $\mathbf{a}_\text{oc}$ is the open circuit AMV. By multiplying both sides of this expression by $s(k)\mathrm{e}^{\mathrm{j}\omega_I k}$, we obtain the snapshot vectors for the received signals and the open-circuit signals, which relate to the voltages through

$$\mathbf{v} = \mathbf{M}\mathbf{v}_\text{oc}. \quad (21)$$

As seen in Fig. 3, the voltages (represented as phasors) across each of the M antenna terminals are given in vector $\mathbf{v} = [V_1 \ \cdots \ V_m \ \cdots \ V_M]^\mathrm{T}$ while $\mathbf{v}_\text{oc}$ represents the open-circuit voltages for each port.

In a receiving scenario, we express the relationship between the antenna terminal voltages and currents as

$$\mathbf{v} = \begin{bmatrix} Z_{11}I_1 + \ \cdots \ + Z_{1M}I_M + Z_{1s}I_s \\ \vdots \qquad \qquad \vdots \\ Z_{M1}I_1 + \ \cdots \ + Z_{MM}I_M + Z_{Ms}I_s \end{bmatrix} = \mathbf{Zi} + \mathbf{v}_\text{oc}, \quad (22)$$

where $\mathbf{i} = [I_1 \ \cdots \ I_m \ \cdots \ I_M]^\mathrm{T}$ and $Z_{ms}I_s$ represents the open circuit (when all currents are null, i.e., $\mathbf{i} = \mathbf{0}$) voltage V_{oc_m} at the m-th antenna terminal or, equivalently, the m-th element of vector $\mathbf{v}_\text{oc} = [V_{\text{oc}_1} \ \cdots \ V_{\text{oc}_m} \ \cdots \ V_{\text{oc}_M}]^\mathrm{T}$.

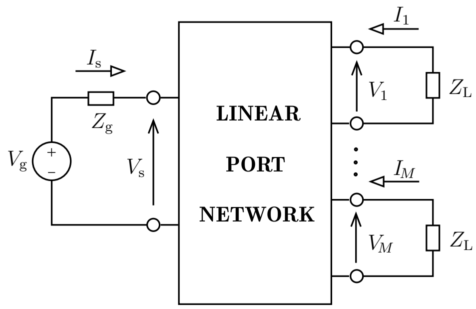

Fig. 3. Antenna array as a linear multiport network.

From Fig. 3, we have $\mathbf{i} = \frac{-1}{Z_L}\mathbf{v}$, where the negative sign reflects the convention that currents are entering the network. Letting $\mathbf{Z}_L = Z_L\mathbf{I}_M$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix, and substituting $\mathbf{i}$ in (22) with $-\mathbf{v}/Z_L$, we obtain

$$\mathbf{v} = \mathbf{Z}_L \left(\mathbf{Z} + \mathbf{Z}_L\right)^{-1} \mathbf{v}_{\mathrm{oc}}, \tag{23}$$

which yields $\mathbf{a}(\theta) = \mathbf{Z}_L \left(\mathbf{Z} + \mathbf{Z}_L\right)^{-1} \mathbf{a}_{\mathrm{oc}}(\theta)$. Hence, we have $\mathbf{M} = \mathbf{Z}_L \left(\mathbf{Z} + \mathbf{Z}_L\right)^{-1}$ as seen in [36].

C. Interpretation and Possible Application

After computing the equivalent MCM factor $\mathbf{M}_{\mathrm{D}}$ in (19), where $\mathbf{C}_{\mathrm{D}} = \mathbf{M}_{\mathrm{D}}\mathbf{G}(\theta)$, its diagonal structure enables us to express it as $\mathbf{M}_{\mathrm{D}} = \mathbf{Z}_{\mathrm{L}} \left(\mathbf{Z}_{\mathrm{L}} + \mathbf{Z}_{\mathrm{D}}\right)^{-1}$, where the diagonal matrix $\mathbf{Z}_{\mathrm{D}}$ is written as

$$\mathbf{Z}_{\mathrm{D}} = \mathbf{Z}_{\mathrm{L}}(\mathbf{I}_M - \mathbf{M}_{\mathrm{D}})\mathbf{M}_{\mathrm{D}}^{-1}. \tag{24}$$

Given that all matrices in the previous equations are diagonal, the m-th element of $\mathbf{Z}_{\mathrm{D}}$ is

$$Z_{\mathrm{D}_m} = \frac{Z_{\mathrm{L}}(1 - [\mathbf{M}_{\mathrm{D}}]_{m,m})}{[\mathbf{M}_{\mathrm{D}}]_{m,m}}. \tag{25}$$

From the definition of the impedance above, the computation of voltage V_m can be obtained as in a typical voltage divider circuit as seen in each of the (uncoupled) M-port networks with loaded terminals in Fig. 4: $V_m = \frac{Z_{\mathrm{L}}}{Z_{\mathrm{L}} + Z_{\mathrm{D}_m}} V_{\mathrm{oc}_m}$, for m varying from 1 to M. Equivalently, we can write $\mathbf{v} = \mathbf{Z}_{\mathrm{L}} \left(\mathbf{Z}_{\mathrm{L}} + \mathbf{Z}_{\mathrm{D}}\right)^{-1} \mathbf{v}_{\mathrm{oc}}$.

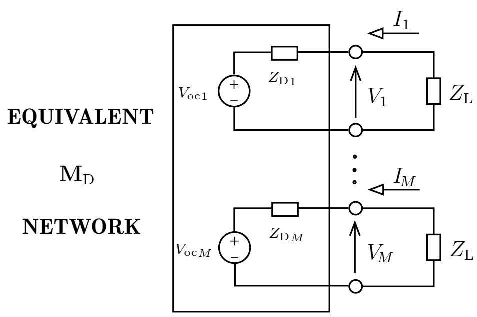

Fig. 4. The M-port network of an equivalent diagonal MCM factor $\mathbf{M}_{\mathrm{D}}$.

A practical and straightforward application of this work is computing the beam pattern of an array over a grid of N directions. The MCM, as a function of θ, is represented by an $M{\times}M{\times}N$ tensor. To simplify computation, we can express this tensor as an $M{\times}N$ matrix, where each column corresponds to the M elements of the diagonal MCM for a specific direction. That allows for more efficient multiplication with an $M \times 1$ vector while reducing the amount of memory for storage.

V. Experimental Results

We simulated mutual coupling in a standard ULA with $M = 6$ antennas, with an incoming signal impinging the array from an angle $\theta = 50°$. The mutual coupling matrix $\mathbf{C}(\mathbf{c}_p)$, adapted from [37], is defined such that $[\mathbf{C}(\mathbf{c}p)]_{i,j} = [\mathbf{c}p]|i-j|$ if $|i-j| < p$, and 0 otherwise. For this simulation, we set $p = 5$ and $\mathbf{c}_p = [1 \quad -0.08 + 0.5\mathrm{j} \quad -0.14 - 0.3\mathrm{j} \quad -0.04 + 0.04\mathrm{j}]$.

Fig. 5 shows that mutual coupling can lead to varying SNRs across channels, even with the same noise level. In this numerical example, the corresponding diagonal MCM, $\mathbf{C}_{\mathrm{D}}$, calculated using (19), is:

$$\begin{bmatrix} 1.73-0.06\mathrm{j} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.32-0.35\mathrm{j} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.2-0.03\mathrm{j} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.2-0.01\mathrm{j} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9-0.06\mathrm{j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.42+0.09\mathrm{j} \end{bmatrix}.$$

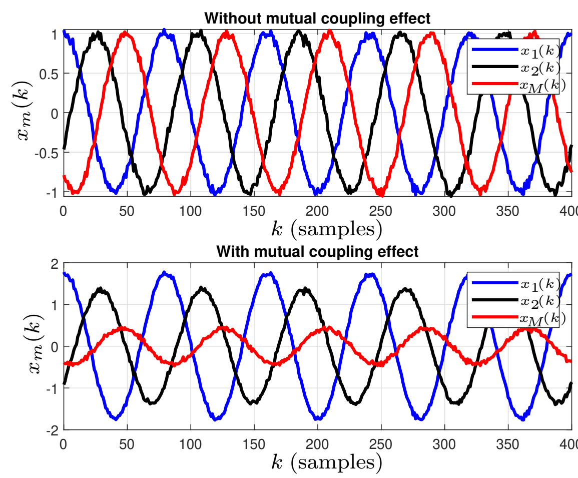

Fig. 5. Simulation results illustrating the effects of mutual coupling on antenna signals.

With $M = 8$ and a grid of 181 angles, a full complex double MCM requires $185,344$ bytes of storage. In contrast, a diagonal MCM reduces the storage requirements to $23,168$ bytes, which is 12.5% of the storage needed for a full MCM.

VI. Conclusion

This work provides insight into representing mutual coupling effects in a uniform linear array using an equivalent diagonal matrix. We extend a basic trigonometric concept to model the snapshot vectors of a uniform linear antenna array and develop a straightforward method for computing the equivalent diagonal matrix. We emphasize the feasibility and reduced memory requirements of the proposed diagonal matrix and provide an interpretation of it through the lens of circuit theory. Finally, the simulation results using the proposed equivalent diagonal MCM readily revealed the distinct impact of mutual coupling on each signal amplitude and phase.

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